EXERCISE 4.2 PAGE: 76

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 – 3x – 10 = 0$
(ii) $2x^2 + x – 6 = 0$
(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 – x + 1/8 = 0$
(v) $100x^2 – 20x + 1 = 0$

Solutions:

(i) Given, $x^2 – 3x – 10 = 0$

Taking L.H.S.,

$\Rightarrow x^2 – 5x + 2x – 10$

$\Rightarrow x(x – 5) + 2(x – 5)$

$\Rightarrow (x – 5)(x + 2)$

The roots of this equation, $x^2 – 3x – 10 = 0$ are the values of x for which $(x – 5)(x + 2) = 0$

Therefore, $x – 5 = 0$ or $x + 2 = 0$

$\Rightarrow x = 5$ or $x = -2$

(ii) Given, $2x^2 + x – 6 = 0$

Taking L.H.S.,

$\Rightarrow 2x^2 + 4x – 3x – 6$

$\Rightarrow 2x(x + 2) – 3(x + 2)$

$\Rightarrow (x + 2)(2x – 3)$

The roots of this equation, $2x^2 + x – 6=0$ are the values of x for which $(x\ x + 2)(2x – 3) = 0$

Therefore, $x + 2 = 0$ or $2x – 3 = 0$

$\Rightarrow x = -2$ or $x = 3/2$

(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}=0$

Taking L.H.S.,

$\Rightarrow \sqrt{2}\, x^2 + 5x + 2x + 5\sqrt{2}$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = (\sqrt{2}x + 5)(x + \sqrt{2})$

The roots of this equation, $\sqrt{2}x^2 + 7x + 5\sqrt{2}=0$ are the values of x for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Therefore, $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

=> $x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(iv) $2x^2 – x +1/8 = 0$

Taking L.H.S.,

$=1/8 (16x^2 – 8x + 1)$

$= 1/8 (16x^2 – 4x -4x + 1)$

$= 1/8 (4x(4x – 1) -1(4x – 1))$

$= 1/8 (4x – 1)^2$

The roots of this equation, $2x^2 – x + 1/8 = 0$, are the values of x for which $(4x – 1)^2= 0$

Therefore, $(4x – 1) = 0$ or $(4x – 1) = 0$

$\Rightarrow x = 1/4$ or $x = 1/4$

(v) Given, $100x^2 – 20x + 1=0$

Taking L.H.S.,

$= 100x^2 – 10x – 10x + 1$

$= 10x(10x – 1) -1(10x – 1)$

$= (10x – 1)^2$

The roots of this equation, $100x^2 – 20x + 1=0$, are the values of x for which $(10x – 1)^2= 0$

$\therefore (10x – 1) = 0$ or $(10x – 1) = 0$

$\Rightarrow x = 1/10$ or $x = 1/10$

2. Solve the problems given in Example 1.

Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Solutions:

(i) Let us say the number of marbles John has = x

Therefore, the number of marble Jivanti has = $45 – x$

After losing 5 marbles each,

Number of marbles John has = $x – 5$

Number of marble Jivanti has = $45 – x – 5 = 40 – x$

Given that the product of their marbles is 124.

$\therefore (x – 5)(40 – x) = 124$

$\Rightarrow x^2 – 45x + 324 = 0$

$\Rightarrow x^2 – 36x – 9x + 324 = 0$

$\Rightarrow x(x – 36) -9(x – 36) = 0$

$\Rightarrow (x – 36)(x – 9) = 0$

Thus, we can say,

$x – 36 = 0$ or $x – 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

Therefore,

If John's marbles = 36

Then, Jivanti's marbles = 45 – 36 = 9

And if John's marbles = 9

Then, Jivanti's marbles = 45 – 9 = 36

(ii) Let us say the number of toys produced in a day is x.

Therefore, cost of production of each toy = Rs$(55 – x)$

Given the total cost of production of the toys = Rs 750

$\therefore x(55 – x) = 750$

$\Rightarrow x^2 – 55x + 750 = 0$

$\Rightarrow x^2 – 25x – 30x + 750 = 0$

$\Rightarrow x(x – 25) -30(x – 25) = 0$

⇒ $(x – 25)(x – 30) = 0$

Thus, either x -25 = 0 or $x – 30 = 0$

⇒ $x = 25$ or $x = 30$

Hence, the number of toys produced in a day will be either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Solution:

Let us say the first number is x, and the second number is 27 – x.

Therefore, the product of two numbers

$x(27 – x) = 182$

⇒ $x^2 – 27x – 182 = 0$

⇒ $x^2 – 13x – 14x + 182 = 0$

⇒ $x(x – 13) -14(x – 13) = 0$

⇒ $(x – 13)(x -14) = 0$

Thus, either, x = -13 = 0 or x – 14 = 0

⇒ x = 13 or x = 14

Therefore, if first number = 13, then second number = 27 – 13 = 14

And if first number = 14, then second number = 27 – 14 = 13

Hence, the numbers are 13 and 14.

4. Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let us say the two consecutive positive integers are x and $x + 1$.

Therefore, as per the given questions,

$x^2 + (x + 1)^2 = 365$

⇒ $x^2 + x^2 + 1 + 2x = 365$

⇒ $2x^2 + 2x – 364 = 0$

⇒ $x^2 + x – 182 = 0$

⇒ $x^2 + 14x – 13x – 182 = 0$

⇒ $x(x + 14) -13(x + 14) = 0$

⇒ $(x + 14)(x – 13) = 0$

Thus, either, $x + 14 = 0$ or $x – 13 = 0$,

⇒ $x = – 14$ or $x = 13$

Since the integers are positive, x can be 13 only.

∴ $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let us say the base of the right triangle is x cm.

Given, the altitude of right triangle = (x – 7) cm

From Pythagoras' theorem, we know,

$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

∴ $x^2 + (x – 7)^2 = 13^2$

⇒ $x^2 + x^2 + 49 – 14x = 169$

⇒ $2x^2 – 14x – 120 = 0$

⇒ $x^2 – 7x – 60 = 0$

⇒ $x^2 – 12x + 5x – 60 = 0$

⇒ $x(x – 12) + 5(x – 12) = 0$

⇒ $(x – 12)(x + 5) = 0$

Thus, either $x – 12 = 0$ or $x + 5 = 0$,

⇒ $x = 12$ or $x = – 5$

Since sides cannot be negative, x can only be 12.

Therefore, the base of the given triangle is 12 cm, and the altitude of this triangle will be (12 – 7) cm = 5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on

that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Solution:

Let us say the number of articles produced is x.

Therefore, cost of production of each article = Rs $(2x + 3)$

Given the total cost of production is Rs.90

$\therefore x(2x + 3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow (2x + 15)(x - 6) = 0$

Thus, either $2x + 15 = 0$ or $x - 6 = 0$

$\Rightarrow x = -15/2$ or $x = 6$

As the number of articles produced can only be a positive integer, x can only be 6.

Hence, the number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15