1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 – 7x +3 = 0$

(ii) $2x^2 + x – 4 = 0$
(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solutions:

(i) $2x^2 – 7x + 3 = 0$

$\Rightarrow 2x^2 – 7x = – 3$

Dividing by 2 on both sides, we get

$\Rightarrow x^2 -7x/2 = -3/2$

$\Rightarrow x^2 -2 \times x \times 7/4 = -3/2$

On adding $(7/4)^2$ to both sides of the equation, we get

$\Rightarrow (x)^2-2\times x\times7/4 +(7/4)^2 = (7/4)^2-3/2$

$\Rightarrow (x-7/4)^2 = (49/16) – (3/2)$

$\Rightarrow(x-7/4)^2 = 25/16$

$\Rightarrow(x-7/4)^2 = \pm5/4$

$\Rightarrow x = 7/4 \pm 5/4$

$\Rightarrow x = 7/4 + 5/4$ or $x = 7/4 – 5/4$

$\Rightarrow x = 12/4$ or $x = 2/4$

$\Rightarrow x = 3$ or $x = 1/2$

(ii) $2x^2 + x – 4 = 0$

$\Rightarrow 2x^2 + x = 4$

Dividing both sides of the equation by 2, we get

$\Rightarrow x^2 +x/2 = 2$

Now on adding $(1/4)^2$ to both sides of the equation, we get,

$\Rightarrow (x)^2+ 2 \times x \times 1/4 + (1/4)^2 = 2 + (1/4)^2$

$\Rightarrow (x + 1/4)^2 = 33/16$

$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$

$\Rightarrow x = \pm \sqrt{33}/4 - 1/4$

$\Rightarrow x = (\pm \sqrt{33}-1)/4$

Therefore, either $x = (\sqrt{33}-1)/4$ or $x = (-\sqrt{33}-1)/4$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Converting the equation into $a^2+2ab+b^2$ form, we get,

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$

$\Rightarrow (2x + \sqrt{3})^2 = 0$

$\Rightarrow (2x + \sqrt{3}) = 0$ and $(2x + \sqrt{3}) = 0$

Therefore, either $x = -\sqrt{3}/2$ or $x = -\sqrt{3}/2$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

Dividing both sides of the equation by 2, we get

$\Rightarrow x^2 + 1/2x = 2$

$\Rightarrow x^2 + 2 \times x \times 1/4 = -2$

By adding $(1/4)^2$ to both sides of the equation, we get

$\Rightarrow (x)^2 + 2 \times x \times 1/4 + (1/4)^2 = (1/4)^2 - 2$

$\Rightarrow (x + 1/4)^2 = 1/16 - 2$

$\Rightarrow (x + 1/4)^2 = -31/16$

As we know, the square of numbers cannot be negative.

Therefore, there is no real root for the given equation, $2x^2 + x + 4 = 0$

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(i) $2x^2 – 7x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

$a = 2, b = -7$ and $c = 3$

By using the quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = (7±√(49 – 24))/4

⇒ x = (7±√25)/4

⇒ x = (7±5)/4

⇒ x = (7+5)/4 or x = (7-5)/4

⇒ x = 12/4 or 2/4

∴ x = 3 or 1/2

(ii) 2x² + x – 4 = 0

On comparing the given equation with ax² + bx + c = 0, we get,

a = 2, b = 1 and c = -4

By using the quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒x = (-1±√1+32)/4

⇒x = (-1±√33)/4

∴ x = (-1+√33)/4 or x = (-1-√33)/4

(iii) 4x² + 4√3x + 3 = 0

On comparing the given equation with ax² + bx + c = 0, we get

a = 4, b = 4√3 and c = 3

By using the quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = (-4√3±√48-48)/8

⇒ x = (-4√3±0)/8

∴ x = -√3/2 or x = -√3/2

(iv) 2x² + x + 4 = 0

On comparing the given equation with ax² + bx + c = 0, we get,

a = 2, b = 1 and c = 4

By using the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = (-1±√1-32)/4

⇒ x = (-1±√-31)/4

As we know, the square of a number can never be negative. Therefore, there is no real solution for the given equation.

3. Find the roots of the following equations:

(i) x-1/x = 3, x ≠ 0
(ii) 1/x+4 – 1/x-7 = 11/30, x = -4, 7

Solution:

(i) x-1/x = 3

⇒ x² – 3x -1 = 0

On comparing the given equation with ax² + bx + c = 0, we get

a = 1, b = -3 and c = -1

By using the quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = (3±√9+4)/2

⇒ x = (3±√13)/2

∴ x = (3+√13)/2 or x = (3-√13)/2

(ii) 1/x+4 – 1/x-7 = 11/30

⇒ x-7-x-4/(x+4)(x-7) = 11/30

⇒ -11/(x+4)(x-7) = 11/30

⇒ (x+4)(x-7) = -30

⇒ $x^2 – 3x – 28 = 30$

⇒ $x^2 – 3x + 2 = 0$

We can solve this equation by factorisation method now.

⇒ $x^2 – 2x – x + 2 = 0$

⇒ x(x – 2) – 1(x – 2) = 0

⇒ (x – 2)(x – 1) = 0

⇒ x = 1 or 2

4. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Solution:

Let us say the present age of Rahman is x years.

Three years ago, Rehman's age was (x – 3) years.

Five years after, his age will be (x + 5) years.

Given the sum of the reciprocals of Rehman's ages 3 years ago and after 5 years is equal to 1/3.

∴ 1/x-3 + 1/x-5 = 1/3

(x+5+x-3)/(x-3)(x+5) = 1/3

(2x+2)/(x-3)(x+5) = 1/3

⇒ 3(2x + 2) = (x-3)(x+5)

⇒ 6x + 6 = x^2 + 2x – 15

⇒ x^2 – 4x – 21 = 0

⇒ x^2 – 7x + 3x – 21 = 0

⇒ x(x – 7) + 3(x – 7) = 0

$\Rightarrow (x - 7)(x + 3) = 0$

$\Rightarrow x = 7, -3$

As we know, age cannot be negative.

Therefore, Rahman's present age is 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let us say the marks of Shefali in Maths be x.

Then, the marks in English will be 30 – x.

As per the given question,

$(x + 2)(30 - x - 3) = 210$

$(x + 2)(27 - x) = 210$

$\Rightarrow -x^2 + 25x + 54 = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 12x - 13x + 156 = 0$

$\Rightarrow x(x - 12) -13(x - 12) = 0$

$\Rightarrow (x - 12)(x - 13) = 0$

$\Rightarrow x = 12, 13$

Therefore, if the marks in Maths are 12, then marks in English will be 30 – 12 = 18, and if the marks in Maths are 13, then marks in English will be 30 – 13 = 17**.**

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solution:

Let us say the shorter side of the rectangle is *x* m.

Then, larger side of the rectangle $= (x + 30)$ m

Diagonal of the rectangle $= \sqrt{x^2 + (x + 30)^2}$

As given, the length of the diagonal is = x + 30 m

Therefore,

$$\sqrt{x^2 + (x + 30)^2} = x + 60$$

$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 – 60x – 2700 = 0$

$\Rightarrow x^2 – 90x + 30x – 2700 = 0$

$\Rightarrow x(x – 90) + 30(x \text{ -}90) = 0$

$\Rightarrow (x – 90)(x + 30) = 0$

$\Rightarrow x = 90, \text{-}30$

However, the side of the field cannot be negative. Therefore, the length of the shorter side will be 90 m.

And the length of the larger side will be (90 + 30) m = 120 m.

7. The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:

Let us say the larger and smaller number be x and y, respectively.

As per the question given,

$x^2 – y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 – 8x = 180$

$\Rightarrow x^2 – 8x – 180 = 0$

$\Rightarrow x^2 – 18x + 10x – 180 = 0$

$\Rightarrow x(x – 18) +10(x – 18) = 0$

$\Rightarrow (x – 18)(x + 10) = 0$

$\Rightarrow x = 18, \text{-}10$

However, the larger number cannot be considered a negative number, as 8 times the larger number will be negative, and hence, the square of the smaller number will be negative, which is not possible.

Therefore, the larger number will be 18 only.

$x = 18$

∴ $y^2 = 8x = 8 \times 18 = 144$

⇒ $y = \pm\sqrt{144} = \pm12$

∴ Smaller number $= \pm12$

Therefore, the numbers are 18 and 12 or 18 and -12.

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

It is given that

Distance $= 360$ km

Consider x as the speed, then the time taken

$t = 360/x$

If the speed is increased by 5 km/h, the speed will be $(x + 5)$ km/h.

Distance will be the same.

$t = 360/(x + 5)$

We know that

Time with original speed – Time with increased speed $= 1$

$360/x – 360/(x + 5) = 1$

$LCM = x (x + 5)$

$[360 (x + 5) – 360x]/x(x + 5) = 1$
$360 x + 1800 – 360x = x (x + 5)$

$x^2 + 5x = 1800$

$x^2 + 5x – 1800 = 0$

$x^2 + 45x – 40x – 1800 = 0$

$x (x + 45) – 40 (x + 45) = 0$

$(x – 40) (x + 45) = 0$

$x = 40$ km/hr

As we know, the value of speed cannot be negative.

Therefore, the speed of the train is 40 km/h.

9. Two water taps together can fill a tank in

$9\frac{3}{8}$ **hours. The tap of a larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time at which each tap can separately fill the tank.**

Solution:

Let the time taken by the smaller pipe to fill the tank = x hr.

Time taken by the larger pipe = $(x – 10)$ hr

Part of the tank filled by smaller pipe in 1 hour = $1/x$

Part of the tank filled by larger pipe in 1 hour = $1/(x – 10)$

As given, the tank can be filled in

$9\frac{3}{8}$ = 75/8 hours by both the pipes together.

Therefore,

$1/x + 1/x\text{-}10 = 8/75$

$x\text{-}10+x/x(x\text{-}10) = 8/75$

⇒ $2x\text{-}10/x(x\text{-}10) = 8/75$

⇒ $75(2x – 10) = 8x^2 – 80x$

⇒ $150x – 750 = 8x^2 – 80x$

⇒ $8x^2 – 230x +750 = 0$

⇒ $8x^2 – 200x – 30x + 750 = 0$

⇒ $8x(x – 25) \text{-}30(x – 25) = 0$

⇒ $(x – 25)(8x \text{-}30) = 0$

⇒ $x = 25, 30/8$

Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible.

Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 =15 hours, respectively.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let us say the average speed of the passenger train = x km/h.

Average speed of express train = $(x + 11)$ km/h

Given the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance. Therefore,

$(132/x) – (132/(x+11)) = 1$

$132(x+11-x)/(x(x+11)) = 1$

$132 \times 11 /(x(x+11)) = 1$

$\Rightarrow 132 \times 11 = x(x + 11)$

$\Rightarrow x^2 + 11x – 1452 = 0$

$\Rightarrow x^2 + 44x -33x -1452 = 0$

$\Rightarrow x(x + 44) -33(x + 44) = 0$

$\Rightarrow (x + 44)(x – 33) = 0$

$\Rightarrow x = – 44, 33$

As we know, speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be $33 + 11 = 44$ km/h.

11. Sum of the areas of two squares is 468 m². If the difference between their perimeters is 24 m, find the sides of the two squares.

Solution:

Let the sides of the two squares be x m and y m.

Therefore, their perimeter will be $4x$ and $4y$, respectively

And the area of the squares will be x^2 and y^2. respectively.

Given,

$4x – 4y = 24$

$x – y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y^2) + y^2 = 468$

⇒ $36 + y^2 + 12y + y^2 = 468$

⇒ $2y^2 + 12y + 432 = 0$

⇒ $y^2 + 6y - 216 = 0$

⇒ $y^2 + 18y - 12y - 216 = 0$

⇒ $y(y +18) -12(y + 18) = 0$

⇒ $(y + 18)(y - 12) = 0$

⇒ $y = -18, 12$

As we know, the side of a square cannot be negative.

Hence, the sides of the squares are 12 m and $(12 + 6)$ m $= 18$ m.