EXERCISE 1.1 PAGE: 7

1. Use Euclid's division algorithm to find the HCF of:

i. 135 and 225

ii. 196 and 38220

iii. 867 and 255

Solutions:

i. 135 and 225

As you can see from the question, 225 is greater than 135. Therefore, by Euclid's division algorithm, we have,

$225 = 135 \times 1 + 90$

Now, remainder $90 \neq 0$, thus again using division lemma for 90, we get,

$135 = 90 \times 1 + 45$

Again, $45 \neq 0$, repeating the above step for 45, we get,

$90 = 45 \times 2 + 0$

The remainder is now zero, so our method stops here. Since, in the last step, the divisor is 45, therefore, HCF (225,135) = HCF (135, 90) = HCF (90, 45) = 45.

Hence, the HCF of 225 and 135 is 45.

ii. 196 and 38220

In this given question, 38220>196, therefore the by applying Euclid's division algorithm and taking 38220 as divisor, we get,

$38220 = 196 \times 195 + 0$

We have already got the remainder as 0 here. Therefore, HCF(196, 38220) = 196.

Hence, the HCF of 196 and 38220 is 196.

iii. 867 and 255

As we know, 867 is greater than 255. Let us apply now Euclid's division algorithm on 867, to get,

$867 = 255 \times 3 + 102$

Remainder $102 \neq 0$, therefore taking 255 as divisor and applying the division lemma method, we get,

$255 = 102 \times 2 + 51$

Again, $51 \neq 0$. Now 102 is the new divisor, so repeating the same step we get,

$102 = 51 \times 2 + 0$

The remainder is now zero, so our procedure stops here. Since, in the last step, the divisor is 51, therefore, HCF (867,255) = HCF(255,102) = HCF(102,51) = 51.

Hence, the HCF of 867 and 255 is 51.

2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Solution:

Let a be any positive integer and b = 6. Then, by Euclid's algorithm, a = 6q + r, for some integer $q \geq 0$, and r = 0, 1, 2, 3, 4, 5, because $0 \leq r < 6$.

Now substituting the value of r, we get,

If r = 0, then a = 6q

Similarly, for r= 1, 2, 3, 4 and 5, the value of a is 6q+1, 6q+2, 6q+3, 6q+4 and 6q+5, respectively.

If a = 6q, 6q+2, 6q+4, then a is an even number and divisible by 2. A positive integer can be either even or odd Therefore, any positive odd integer is of the form of 6q+1, 6q+3 and 6q+5, where q is some integer.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Given,

Number of army contingent members = 616

Number of army band members = 32

If the two groups have to march in the same column, we have to find out the highest common factor between the two groups. HCF(616, 32), gives the maximum number of columns in which they can march.

By using Euclid's algorithm to find their HCF, we get,

Since, 616>32, therefore,

$616 = 32 \times 19 + 8$

Since, $8 \neq 0$, therefore, taking 32 as new divisor, we have,

$32 = 8 \times 4 + 0$

Now we have got remainder as 0, therefore, HCF (616, 32) = 8.

Hence, the maximum number of columns in which they can march is 8.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solutions:

Let x be any positive integer and y = 3.

By Euclid's division algorithm, then,

$x = 3q + r$ for some integer $q \geq 0$ and r = 0, 1, 2, as $r \geq 0$ and $r < 3$.

Therefore, x = 3q, 3q+1 and 3q+2

Now as per the question given, by squaring both the sides, we get,

$x^2 = (3q)^2 = 9q^2 = 3 \times 3q^2$

Let $3q^2 = m$

Therefore, $x^2 = 3m$ ……………………..(1)

$x^2 = (3q + 1)^2 = (3q)^2+1^2+2\times3q\times1 = 9q^2 + 1 +6q = 3(3q^2+2q) +1$

Substitute, $3q^2+2q = m$, to get,

$x^2 = 3m + 1$ ……………………………. (2)

$x^2 = (3q + 2)^2 = (3q)^2+2^2+2\times3q\times2 = 9q^2 + 4 + 12q = 3 (3q^2 + 4q + 1)+1$

Again, substitute, $3q^2+4q+1 = m$, to get,

$x^2 = 3m + 1$……………………………. (3)

Hence, from equation 1, 2 and 3, we can say that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Solution:

Let x be any positive integer and y = 3.

By Euclid's division algorithm, then,

$x = 3q+r$, where $q\geq0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, putting the value of r, we get,

$x = 3q$

or

$x = 3q + 1$

or

$x = 3q + 2$

Now, by taking the cube of all the three above expressions, we get,

Case (i): When r = 0, then,

$x^2 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$; where $m = 3q^3$

Case (ii): When r = 1, then,

$x^3 = (3q+1)^3 = (3q)^3 +1^3+3\times3q\times1(3q+1) = 27q^3+1+27q^2+9q$

Taking 9 as common factor, we get,

$x^3 = 9(3q^3+3q^2+q)+1$

Putting = m, we get,

Putting $(3q^3+3q^{2+}q) = m$, we get ,

$x^3 = 9m+1$

Case (iii): When r = 2, then,

$x^3 = (3q+2)^3 = (3q)^3+2^3+3\times3q\times2(3q+2) = 27q^3+54q^2+36q+8$

Taking 9 as common factor, we get,

$x^3=9(3q^3+6q^2+4q)+8$

Putting $(3q^3+6q^2+4q) = m$, we get ,

$x^3 = 9m+8$

Therefore, from all the three cases explained above, it is proved that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.