EXERCISE 1.3 PAGE: 14

1. Prove that √5 is irrational.

Solutions: Let us assume, that √5 is rational number.

i.e. √5 = x/y (where, x and y are co-primes)

y√5= x

Squaring both the sides, we get,

$(y\sqrt{5})^2 = x^2$

$\Rightarrow 5y^2 = x^2$……………………………….. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,

$5y^2 = (5k)^2$

$\Rightarrow y^2 = 5k^2$

is divisible by 5 it means y is divisible by 5.

Clearly, x and y are not co-primes. Thus, our assumption about √5 is rational is incorrect.

Hence, √5 is an irrational number.

2. Prove that 3 + 2√5 + is irrational.

Solutions: Let us assume 3 + 2√5 is rational.

Then we can find co-prime x and y (y ≠ 0) such that 3 + 2√5 = x/y

Rearranging, we get,

$$2\sqrt{5} = \frac{x}{y} - 3$$
$$\sqrt{5} = \frac{1}{2}\left(\frac{x}{y} - 3\right)$$

Since, x and y are integers, thus,

$\frac{1}{2}\left(\frac{x}{y} - 3\right)$ is a rational number.

Therefore, √5 is also a rational number. But this contradicts the fact that √5 is irrational.

So, we conclude that 3 + 2√5 is irrational.

3. Prove that the following are irrationals:

(i) 1/√2

(ii) 7√5

(iii) 6 + √2

Solutions:

(i) 1/√2

Let us assume 1/√2 is rational.

Then we can find co-prime x and y (y ≠ 0) such that 1/√2 = x/y

Rearranging, we get,

√2 = y/x

Since, x and y are integers, thus, √2 is a rational number, which contradicts the fact that √2 is irrational.

Hence, we can conclude that 1/√2 is irrational.

(ii) 7√5

Let us assume 7√5 is a rational number.

Then we can find co-prime a and b (b ≠ 0) such that 7√5 = x/y

Rearranging, we get,

√5 = x/7y

Since, x and y are integers, thus, √5 is a rational number, which contradicts the fact that √5 is irrational.

Hence, we can conclude that 7√5 is irrational.

(iii) 6 +√2

Let us assume 6 +√2 is a rational number.

Then we can find co-primes x and y (y ≠ 0) such that 6 +√2 = x/y·

Rearranging, we get,

√2 = (x/y) – 6

Since, x and y are integers, thus (x/y) – 6 is a rational number and therefore, √2 is rational. This contradicts the fact that √2 is an irrational number.

Hence, we can conclude that 6 +√2 is irrational.