## EXERCISE: 10.2

In Q. 1 to 3, choose the correct option and give a justification.

1. From point $Q$, the length of the tangent to a circle is 24 cm , and the distance of $Q$ from the centre is 25 cm . The radius of the circle is
(A) 7 cm
(B) 12 cm
(C) 15 cm
(D) 24.5 cm

Answer:
First, draw a perpendicular from the centre O of the triangle to a point P on the circle, which is touching the tangent. This line will be perpendicular to the tangent of the circle.


So, OP is perpendicular to PQ , i.e., $\mathrm{OP} \perp \mathrm{PQ}$
From the above figure, it is also seen that $\triangle \mathrm{OPQ}$ is a right-angled triangle.
It is given that
$\mathrm{OQ}=25 \mathrm{~cm}$ and $\mathrm{PQ}=24 \mathrm{~cm}$
By using Pythagoras' theorem in $\triangle \mathrm{OPQ}$,
$\mathrm{OQ}^{2}=\mathrm{OP}^{2}+\mathrm{PQ}^{2}$
$(25)^{2}=\mathrm{OP}^{2}+(24)^{2}$
$\mathrm{OP}^{2}=625-576$
$\mathrm{OP}^{2}=49$
$\mathrm{OP}=7 \mathrm{~cm}$
So, option A, i.e., 7 cm , is the radius of the given circle.
2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre $O$ so that $\angle P O Q=110^{\circ}$, then $\angle P T Q$ is equal to
(A) $60^{\circ}$
(B) $70^{\circ}$
(C) $80^{\circ}$
(D) $90^{\circ}$

Answer:
From the question, it is clear that OP is the radius of the circle to the tangent PT , and OQ is the radius to the tangent TQ.


So, $\mathrm{OP} \perp \mathrm{PT}$ and $\mathrm{TQ} \perp \mathrm{OQ}$
$\therefore \angle \mathrm{OPT}=\angle \mathrm{OQT}=90^{\circ}$
Now, in the quadrilateral POQT, we know that the sum of the interior angles is $360^{\circ}$.
So, $\angle \mathrm{PTQ}+\angle \mathrm{POQ}+\angle \mathrm{OPT}+\angle \mathrm{OQT}=360^{\circ}$
Now, by putting the respective values, we get
$\angle \mathrm{PTQ}+90^{\circ}+110^{\circ}+90^{\circ}=360^{\circ}$
$\angle \mathrm{PTQ}=70^{\circ}$
So, $\angle \mathrm{PTQ}$ is $70^{\circ}$ which is option B .
3. If tangents $P A$ and $P B$ from a point $P$ to a circle with centre $O$ are inclined to each other at an angle of $80^{\circ}$, then $\angle P O A$ is equal to
(A) $50^{\circ}$
(B) $60^{\circ}$
(C) $70^{\circ}$
(D) $80^{\circ}$

Answer:
First, draw the diagram according to the given statement.


Now, in the above diagram, OA is the radius to tangent PA , and OB is the radius to tangent PB .
So, OA is perpendicular to PA , and OB is perpendicular to PB , i.e., $\mathrm{OA} \perp \mathrm{PA}$ and $\mathrm{OB} \perp \mathrm{PB}$.
So, $\angle \mathrm{OBP}=\angle \mathrm{OAP}=90^{\circ}$
Now, in the quadrilateral AOBP,
The sum of all the interior angles will be $360^{\circ}$.
So, $\angle \mathrm{AOB}+\angle \mathrm{OAP}+\angle \mathrm{OBP}+\angle \mathrm{APB}=360^{\circ}$
Putting their values, we get
$\angle \mathrm{AOB}+260^{\circ}=360^{\circ}$
$\angle \mathrm{AOB}=100^{\circ}$
Now, consider the triangles $\triangle \mathrm{OPB}$ and $\triangle \mathrm{OPA}$. Here,
$\mathrm{AP}=\mathrm{BP}$ (Since the tangents from a point are always equal)
$\mathrm{OA}=\mathrm{OB}$ (Which are the radii of the circle)
$\mathrm{OP}=\mathrm{OP}$ (It is the common side)
Now, we can say that triangles OPB and OPA are similar using SSS congruency.
$\therefore \triangle \mathrm{OPB} \cong \triangle \mathrm{OPA}$
So, $\angle \mathrm{POB}=\angle \mathrm{POA}$
$\angle \mathrm{AOB}=\angle \mathrm{POA}+\angle \mathrm{POB}$
$2(\angle \mathrm{POA})=\angle \mathrm{AOB}$
By putting the respective values, we get
$=>\angle \mathrm{POA}=100^{\circ} / 2=50^{\circ}$
As the angle $\angle \mathrm{POA}$ is $50^{\circ}$, option A is the correct option.
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

## Answer:

First, draw a circle and connect two points, A and B , such that AB becomes the diameter of the circle. Now, draw two tangents, PQ and RS , at points A and B , respectively.


Now, both radii, i.e. AO and OB , are perpendicular to the tangents.
So, OB is perpendicular to RS, and OA is perpendicular to PQ .
So, $\angle \mathrm{OAP}=\angle \mathrm{OAQ}=\angle \mathrm{OBR}=\angle \mathrm{OBS}=90^{\circ}$
From the above figure, angles OBR and OAQ are alternate interior angles.
Also, $\angle \mathrm{OBR}=\angle \mathrm{OAQ}$ and $\angle \mathrm{OBS}=\angle \mathrm{OAP}$ (Since they are also alternate interior angles)
So, it can be said that line PQ and line RS will be parallel to each other (Hence Proved).
5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

## Solution:

Let, $O$ is the centre of the given circle.
A tangent PR has been drawn touching the circle at point P .
Draw $\mathrm{QP} \perp \mathrm{RP}$ at point P , such that point Q lies on the circle.

$\angle \mathrm{OPR}=90^{\circ}$ (radius $\perp$ tangent)
Also, $\angle \mathrm{QPR}=90^{\circ}$ (Given)
$\therefore \angle \mathrm{OPR}=\angle \mathrm{QPR}$
Now, the above case is possible only when centre O lies on the line QP.
Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.
6. The length of a tangent from point $A$ at a distance 5 cm from the centre of the circle is 4 cm . Find the radius of the circle.

## Answer:

Draw the diagram as shown below.


Here, AB is the tangent that is drawn on the circle from point A .
So, the radius OB will be perpendicular to AB , i.e., $\mathrm{OB} \perp \mathrm{AB}$
We know, $\mathrm{OA}=5 \mathrm{~cm}$ and $\mathrm{AB}=4 \mathrm{~cm}$
Now, In $\triangle \mathrm{ABO}$,
$\mathrm{OA}^{2}=\mathrm{AB}^{2}+\mathrm{BO}^{2}($ Using Pythagoras' theorem $)$
$5^{2}=4^{2}+\mathrm{BO}^{2}$
$\mathrm{BO}^{2}=25-16$
$\mathrm{BO}^{2}=9$
$\mathrm{BO}=3$
So, the radius of the given circle, i.e., BO , is 3 cm .
7. Two concentric circles are of radii 5 cm and 3 cm . Find the length of the chord of the larger circle which touches the smaller circle.

Answer:
Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle, which touches the smaller circle at a point P , as shown in the figure below.


From the above diagram, AB is tangent to the smaller circle to point P .
$\therefore \mathrm{OP} \perp \mathrm{AB}$
Using Pythagoras' theorem in triangle OPA,
$\mathrm{OA}^{2}=\mathrm{AP}^{2}+\mathrm{OP}^{2}$
$5^{2}=\mathrm{AP}^{2}+3^{2}$
$\mathrm{AP}^{2}=25-9$
$\mathrm{AP}=4$
Now, as $\mathrm{OP} \perp \mathrm{AB}$,
Since the perpendicular from the centre of the circle bisects the chord, AP will be equal to PB .
So, $\mathrm{AB}=2 \mathrm{AP}=2 \times 4=8 \mathrm{~cm}$
So, the length of the chord of the larger circle is 8 cm .
8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that $\mathrm{AB}+\mathrm{CD}=\mathrm{AD}+\mathrm{BC}$ Answer:

The figure given is:


From the figure, we can conclude a few points, which are
(i) $\mathrm{DR}=\mathrm{DS}$
(ii) $\mathrm{BP}=\mathrm{BQ}$
(iii) $\mathrm{AP}=\mathrm{AS}$
(iv) $\mathrm{CR}=\mathrm{CQ}$

Since they are tangents on the circle from points D, B, A, and C, respectively.
Now, adding the LHS and RHS of the above equations, we get,
$\mathrm{DR}+\mathrm{BP}+\mathrm{AP}+\mathrm{CR}=\mathrm{DS}+\mathrm{BQ}+\mathrm{AS}+\mathrm{CQ}$
By rearranging them, we get,
$(\mathrm{DR}+\mathrm{CR})+(\mathrm{BP}+\mathrm{AP})=(\mathrm{CQ}+\mathrm{BQ})+(\mathrm{DS}+\mathrm{AS})$
By simplifying,
$A D+B C=C D+A B$
9. In Fig. 10.13, $X Y$ and $X^{\prime} Y^{\prime}$ are two parallel tangents to a circle with centre $O$ and another tangent $A B$ with the point of contact $C$ intersecting $X Y$ at $A$ and $X^{\prime} Y^{\prime}$ at $B$. Prove that $\angle A O B=90^{\circ}$.

Answer:
From the figure given in the textbook, join OC. Now, the diagram will be as


Now, the triangles $\triangle \mathrm{OPA}$ and $\triangle \mathrm{OCA}$ are similar using SSS congruency as
(i) $\mathrm{OP}=\mathrm{OC}$ They are the radii of the same circle
(ii) $\mathrm{AO}=\mathrm{AO}$ It is the common side
(iii) $\mathrm{AP}=\mathrm{AC}$ These are the tangents from point A

So, $\triangle \mathrm{OPA} \cong \triangle \mathrm{OCA}$
Similarly,
$\triangle \mathrm{OQB} \cong \triangle \mathrm{OCB}$
So,
$\angle \mathrm{POA}=\angle \mathrm{COA} \ldots$ (Equation i)
And, $\angle \mathrm{QOB}=\angle \mathrm{COB} \ldots$ (Equation ii)
Since the line POQ is a straight line, it can be considered as the diameter of the circle.
So, $\angle \mathrm{POA}+\angle \mathrm{COA}+\angle \mathrm{COB}+\angle \mathrm{QOB}=180^{\circ}$
Now, from equations (i) and equation (ii), we get,
$2 \angle \mathrm{COA}+2 \angle \mathrm{COB}=180^{\circ}$
$\angle \mathrm{COA}+\angle \mathrm{COB}=90^{\circ}$
$\therefore \angle \mathrm{AOB}=90^{\circ}$
10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

## Answer:

First, draw a circle with centre O . Choose an external point P and draw two tangents, PA and PB , at point A and point $B$, respectively. Now, join $A$ and $B$ to make $A B$ in a way that subtends $\angle A O B$ at the centre of the circle. The diagram is as follows:


From the above diagram, it is seen that the line segments OA and PA are perpendicular.
So, $\angle \mathrm{OAP}=90^{\circ}$
In a similar way, the line segments $\mathrm{OB} \perp \mathrm{PB}$ and so, $\angle \mathrm{OBP}=90^{\circ}$

Now, in the quadrilateral OAPB,
$\therefore \angle \mathrm{APB}+\angle \mathrm{OAP}+\angle \mathrm{PBO}+\angle \mathrm{BOA}=360^{\circ}$ (since the sum of all interior angles will be $360^{\circ}$ )
By putting the values, we get,
$\angle \mathrm{APB}+180^{\circ}+\angle \mathrm{BOA}=360^{\circ}$
So, $\angle \mathrm{APB}+\angle \mathrm{BOA}=180^{\circ}$ (Hence proved).
11. Prove that the parallelogram circumscribing a circle is a rhombus.

Answer:
Consider a parallelogram $A B C D$ which is circumscribing a circle with a centre $O$. Now, since $A B C D$ is a parallelogram, $\mathrm{AB}=\mathrm{CD}$ and $\mathrm{BC}=\mathrm{AD}$.


From the above figure, it is seen that,
(i) $\mathrm{DR}=\mathrm{DS}$
(ii) $\mathrm{BP}=\mathrm{BQ}$
(iii) $\mathrm{CR}=\mathrm{CQ}$
(iv) $\mathrm{AP}=\mathrm{AS}$

These are the tangents to the circle at $\mathrm{D}, \mathrm{B}, \mathrm{C}$, and A , respectively.
Adding all these, we get
$\mathrm{DR}+\mathrm{BP}+\mathrm{CR}+\mathrm{AP}=\mathrm{DS}+\mathrm{BQ}+\mathrm{CQ}+\mathrm{AS}$
By rearranging them, we get
$(\mathrm{BP}+\mathrm{AP})+(\mathrm{DR}+\mathrm{CR})=(\mathrm{CQ}+\mathrm{BQ})+(\mathrm{DS}+\mathrm{AS})$
Again by rearranging them, we get
$\mathrm{AB}+\mathrm{CD}=\mathrm{BC}+\mathrm{AD}$
Now, since $\mathrm{AB}=\mathrm{CD}$ and $\mathrm{BC}=\mathrm{AD}$, the above equation becomes
$2 \mathrm{AB}=2 \mathrm{BC}$
$\therefore \mathrm{AB}=\mathrm{BC}$
Since $\mathrm{AB}=\mathrm{BC}=\mathrm{CD}=\mathrm{DA}$, it can be said that ABCD is a rhombus.
12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments $B D$ and $D C$ into which $B C$ is divided by the point of contact $D$ are of lengths 8 cm and 6 cm , respectively (see Fig. 10.14). Find the sides $A B$ and $A C$.

Answer:
The figure given is as follows:


Consider the triangle ABC ,
We know that the length of any two tangents which are drawn from the same point to the circle is equal.
So,
(i) $\mathrm{CF}=\mathrm{CD}=6 \mathrm{~cm}$
(ii) $\mathrm{BE}=\mathrm{BD}=8 \mathrm{~cm}$
(iii) $\mathrm{AE}=\mathrm{AF}=x$

Now, it can be observed that,
(i) $\mathrm{AB}=\mathrm{EB}+\mathrm{AE}=8+\mathrm{x}$
(ii) $\mathrm{CA}=\mathrm{CF}+\mathrm{FA}=6+x$
(iii) $\mathrm{BC}=\mathrm{DC}+\mathrm{BD}=6+8=14$

Now the semi-perimeter "s" will be calculated as follows
$2 \mathrm{~s}=\mathrm{AB}+\mathrm{CA}+\mathrm{BC}$
By putting the respective values, we get,
$2 \mathrm{~s}=28+2 x$
$s=14+x$

Area of $\triangle \mathrm{ABC}=\sqrt{s(s-a)(s-b)(s-c)}$
By solving this, we get,
$=\sqrt{ }(14+x) 48 x$ $\qquad$
Again, the area of $\triangle \mathrm{ABC}=2 \times$ area of $(\triangle \mathrm{AOF}+\triangle \mathrm{COD}+\triangle \mathrm{DOB})$
$=2 \times[(1 / 2 \times \mathrm{OF} \times \mathrm{AF})+(1 / 2 \times \mathrm{CD} \times \mathrm{OD})+(1 / 2 \times \mathrm{DB} \times \mathrm{OD})]$
$=2 \times 1 / 2(4 x+24+32)=56+4 x$
Now from (i) and (ii), we get,
$\sqrt{ }(14+x) 48 x=56+4 x$
Now, square both sides,
$48 x(14+x)=(56+4 x)^{2}$
$48 x=[4(14+\mathrm{x})]^{2} /(14+x)$
$48 x=16(14+x)$
$48 x=224+16 x$
$32 x=224$
$x=7 \mathrm{~cm}$
So, $A B=8+x$
i.e. $A B=15 \mathrm{~cm}$

And, CA $=x+6=13 \mathrm{~cm}$.
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

## Answer:

First, draw a quadrilateral $A B C D$ which will circumscribe a circle with its centre O in a way that it touches the circle at points P, Q, R, and S. Now, after joining the vertices of ABCD, we get the following figure:


Now, consider the triangles OAP and OAS.
$\mathrm{AP}=\mathrm{AS}$ (They are the tangents from the same point A )
$\mathrm{OA}=\mathrm{OA}$ (It is the common side)
$\mathrm{OP}=\mathrm{OS}$ (They are the radii of the circle)
So, by $S S S$ congruency $\triangle \mathrm{OAP} \cong \triangle \mathrm{OAS}$
So, $\angle \mathrm{POA}=\angle \mathrm{AOS}$
Which implies that $\angle 1=\angle 8$
Similarly, other angles will be
$\angle 4=\angle 5$
$\angle 2=\angle 3$
$\angle 6=\angle 7$
Now by adding these angles, we get
$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^{\circ}$
Now by rearranging,
$(\angle 1+\angle 8)+(\angle 2+\angle 3)+(\angle 4+\angle 5)+(\angle 6+\angle 7)=360^{\circ}$
$2 \angle 1+2 \angle 2+2 \angle 5+2 \angle 6=360^{\circ}$
Taking 2 as common and solving, we get
$(\angle 1+\angle 2)+(\angle 5+\angle 6)=180^{\circ}$
Thus, $\angle \mathrm{AOB}+\angle \mathrm{COD}=180^{\circ}$

Similarly, it can be proved that $\angle \mathrm{BOC}+\angle \mathrm{DOA}=180^{\circ}$
Therefore, the opposite sides of any quadrilateral which is circumscribing a given circle will subtend supplementary angles at the centre of the circle.

