EXERCISE: 12.3 (PAGE NO: 234)

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

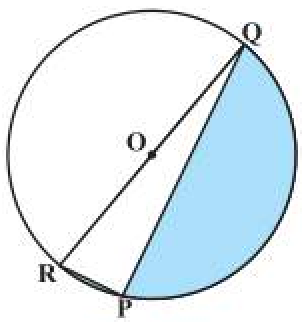


Fig. 12.19

Solution:

Here, P is in the semi-circle, and so,

P = 90°

So, it can be concluded that QR is the hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using the Pythagorean theorem,

$QR^2 = PR^2+PQ^2$

Or, $QR^2 = 7^2+24^2$

QR= 25 cm = Diameter

Hence, the radius of the circle = 25/2 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7)\times(25/2)\times(25/2)/2$ cm^2

= 13750/56 cm^2 = 245.54 cm^2

Also, the area of the ΔPQR = ½×PR×PQ

=(½)×7×24 cm^2

= 84 cm^2

Hence, the area of the shaded region = 245.54 cm^2-84 cm^2

= 161.54 cm^2

2. Find the area of the shaded region in Fig. 12.20, if the radii of the two concentric circles with centre O are 7 cm and 14 cm, respectively and AOC = 40°.

Solution:

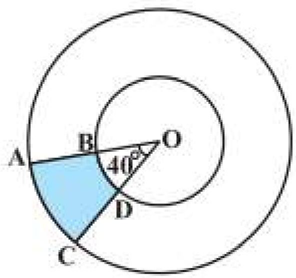


Fig. 12.20

Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = $(\theta/360°)\times\pi r^2$

So, Area of OAC = $(40°/360°)\times\pi r^2$ cm^2

= 68.44 cm^2

Area of the sector OBD = $(40°/360°)\times\pi r^2$ cm^2

= $(1/9)\times(22/7)\times7^2$ = 17.11 cm^2

Now, the area of the shaded region ABDC = Area of OAC – Area of the OBD

= 68.44 cm^2 – 17.11 cm^2 = 51.33 cm^2

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

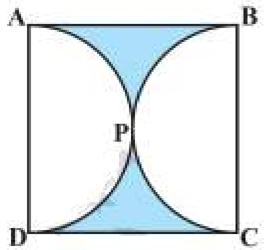


Fig. 12.21

Solution:

Side of the square ABCD (as given) = 14 cm

So, the Area of ABCD = a^2

= 14×14 cm^2 = 196 cm^2

We know that the side of the square = diameter of the circle = 14 cm

So, the side of the square = diameter of the semicircle = 14 cm

∴ the radius of the semicircle = 7 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7\times7\times7)/2$ cm^2

= 77 cm^2

∴ he area of two semicircles = 2×77 cm^2 = 154 cm^2

Hence, the area of the shaded region = Area of the Square – Area of two semicircles

= 196 cm^2 -154 cm^2

= 42 cm^2

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as the centre.

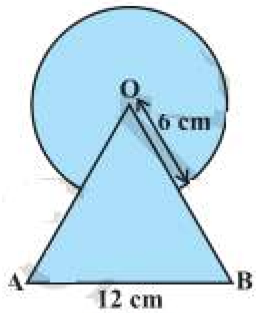


Fig. 12.22

Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

The area of the sector is common in both.

The radius of the circle = 6 cm

Side of the triangle = 12 cm

Area of the equilateral triangle = $(\sqrt{3}/4)\ (OA)^2 = (\sqrt{3}/4)\times 12^2 = 36\sqrt{3}$ cm²

Area of the circle = $\pi R^2 = (22/7)\times 6^2 = 792/7$ cm²

Area of the sector making angle 60° = $(60°/360°)\times \pi r^2$ cm²

= $(1/6)\times(22/7)\times 6^2$ cm² = 132/7 cm²

Area of the shaded region = Area of the equilateral triangle + Area of the circle – Area of the sector

= $36\sqrt{3}$ cm² +792/7 cm²-132/7 cm²

= $(36\sqrt{3}+660/7)$ cm²

5. From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

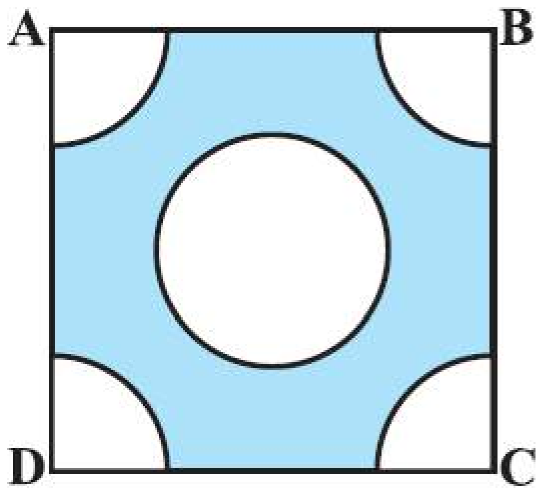


Fig. 12.23

Solution:

Side of the square = 4 cm

The radius of the circle = 1 cm

Four quadrants of a circle are cut from the corner, and one circle of radius are cut from the middle.

Area of the square = $(\text{side})^2 = 4^2 = 16\ \text{cm}^2$

Area of the quadrant = $(\pi R^2)/4\ \text{cm}^2 = (22/7)\times(1^2)/4 = 11/14\ \text{cm}^2$

∴ Total area of the 4 quadrants = $4\times(11/14)\ \text{cm}^2 = 22/7\ \text{cm}^2$

Area of the circle = $\pi R^2\ \text{cm}^2 = (22/7\times1^2) = 22/7\ \text{cm}^2$

Area of the shaded region = Area of the square – (Area of the 4 quadrants + Area of the circle)

$= 16\ \text{cm}^2 - (22/7)\ \text{cm}^2 - (22/7)\ \text{cm}^2$

$= 68/7\ \text{cm}^2$

6. In a circular table cover of radius 32 cm, a design is formed, leaving an equilateral triangle ABC in the middle, as shown in Fig. 12.24. Find the area of the design.

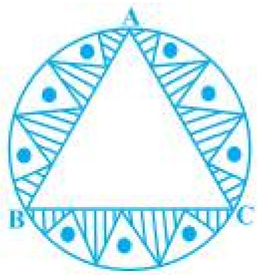


Fig. 12.24

Solution:

The radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

$\Rightarrow$ BD = AB/2

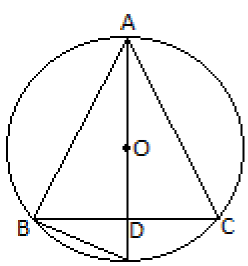
Since, AD is the median of the triangle

$\therefore$ AO = Radius of the circle = (2/3) AD

$\Rightarrow$ (2/3)AD = 32 cm

$\Rightarrow$ AD = 48 cm

In ΔADB,



By Pythagoras' theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = 48^2 + (AB/2)^2$

$\Rightarrow AB^2 = 2304 + AB^2/4$

$\Rightarrow 3/4\ (AB^2) = 2304$

$\Rightarrow AB^2 = 3072$

$\Rightarrow AB = 32\sqrt{3}$ cm

Area of ΔADB = $\sqrt{3}/4 \times (32\sqrt{3})^2$ cm^2 = $768\sqrt{3}$ cm^2

Area of the circle = πR^2 = (22/7)×32×32 = 22528/7 cm^2

Area of the design = Area of the circle – Area of ΔADB

= ($22528/7 - 768\sqrt{3}$) cm^2

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

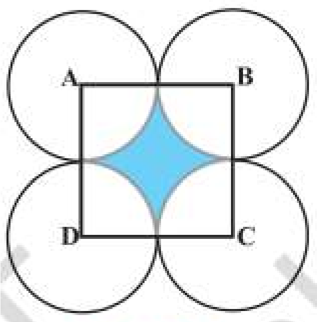


Fig. 12.25

Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = 14^2 = 196 cm^2

Area of the quadrant = $(\pi R^2)/4$ cm^2 = (22/7) ×7^2/4 cm^2

= 77/2 cm^2

Total area of the quadrant = 4×77/2 cm^2 = 154cm^2

Area of the shaded region = Area of the square ABCD – Area of the quadrant

= 196 cm^2 – 154 cm^2

= 42 cm^2

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find

(i) the distance around the track along its inner edge

(ii) the area of the track.

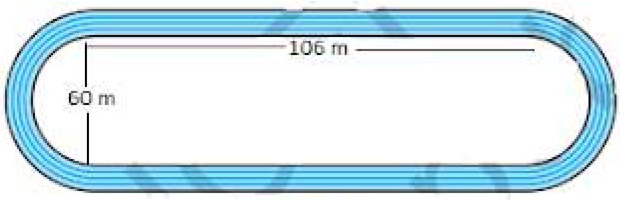


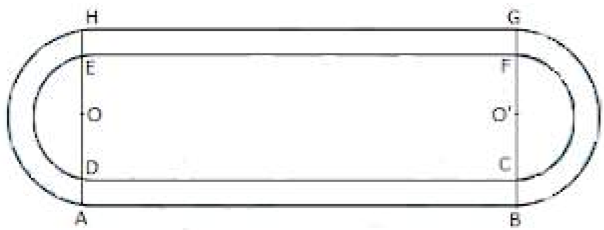
Fig. 12.26

Solution:

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m



DE = CF = 60 m

The radius of the inner semicircle, r = OD = O'C

= 60/2 m = 30 m

The radius of the outer semicircle, R = OA = O'B

= 30+10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD+EF+2×(Circumference of inner semicircle)

= 106+106+(2×πr) m = 212+(2×22/7×30) m

= 212+1320/7 m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) – 2 × (area of inner semicircle)

= (AB×CD)+(EF×GH)+2×(πr^2/2) -2×(πR^2/2) m^2

= (106×10)+(106×10)+2×π/2(r^2-R^2) m^2

= 2120+22/7×70×10 m^2

= 4320 m^2

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other, and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

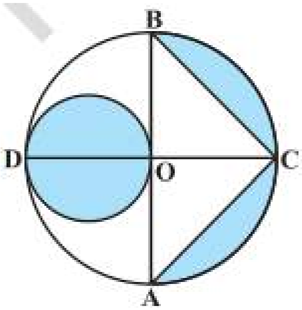


Fig. 12.27

Solution:

The radius of larger circle, R = 7 cm

The radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = 1/2 × AB × OC = (½)×7×14 = 49 cm^2

Area of larger circle = πR^2 = $(22/7)\times 7^2$ = 154 cm^2

Area of larger semicircle = 154/2 cm^2 = 77 cm^2

Area of smaller circle = πr^2 = (22/7)×(7/2)×(7/2) = 77/2 cm^2

Area of the shaded region = Area of the larger circle – Area of the triangle – Area of the larger semicircle + Area of the smaller circle

Area of the shaded region = (154-49-77+77/2) cm^2

= 133/2 cm^2 = 66.5 cm^2

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as the centre, a circle is drawn with a radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region (Use π = 3.14 and $\sqrt{3}$ = 1.73205).

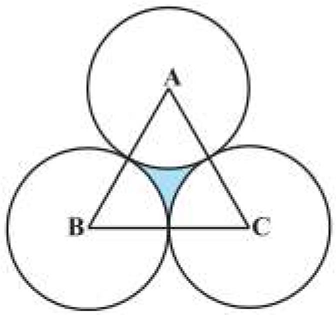


Fig. 12.28

Solution:

ABC is an equilateral triangle.

∴ ∠ A = ∠ B = ∠ C = 60°

There are three sectors, each making 60°.

Area of ΔABC = 17320.5 cm^2

⇒ √3/4 ×$(side)^2$ = 17320.5

⇒ $(side)^2$ =17320.5×4/1.73205

⇒ $(side)^2$ = 4×10^4

⇒ side = 200 cm

Radius of the circles = 200/2 cm = 100 cm

Area of the sector = (60°/360°)×π r^2 cm^2

= 1/6×3.14×$(100)^2$ cm^2

= 15700/3cm^2

Area of 3 sectors = 3×15700/3 = 15700 cm^2

Thus, the area of the shaded region = Area of an equilateral triangle ABC – Area of 3 sectors

= 17320.5-15700 cm^2 = 1620.5 cm^2

11. On a square handkerchief, nine circular designs, each of a radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

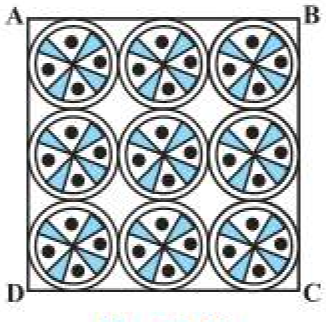


Fig. 12.29

Solution:

Number of circular designs = 9

The radius of the circular design = 7 cm

There are three circles on one side of the square handkerchief.

∴ side of the square = 3×diameter of circle = 3×14 = 42 cm

Area of the square = 42×42 cm^2 = 1764 cm^2

Area of the circle = π r^2 = (22/7)×7×7 = 154 cm^2

Total area of the design = 9×154 = 1386 cm^2

Area of the remaining portion of the handkerchief = Area of the square – Total area of the design = 1764 – 1386 = 378 cm^2

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and a radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB

(ii) shaded region

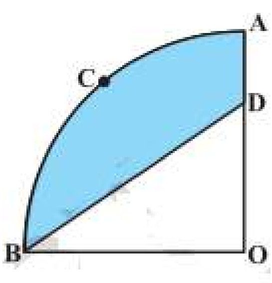


Fig. 12.30

Solution:

Radius of the quadrant = 3.5 cm = 7/2 cm

(i) Area of the quadrant OACB = $(\pi R^2)/4$ cm^2

= $(22/7)\times(7/2)\times(7/2)/4$ cm^2

= $77/8$ cm^2

(ii) Area of the triangle BOD = $(½)\times(7/2)\times2$ cm^2

= $7/2$ cm^2

Area of the shaded region = Area of the quadrant – Area of the triangle BOD

= $(77/8)-(7/2)$ cm^2 = $49/8$ cm^2

= 6.125 cm^2

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

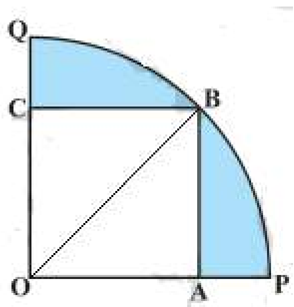


Fig. 12.31

Solution:

Side of square = OA = AB = 20 cm

The radius of the quadrant = OB

OAB is the right-angled triangle

By Pythagoras' theorem in ΔOAB,

$OB^2 = AB^2+OA^2$

$\Rightarrow OB^2 = 20^2+20^2$

$\Rightarrow OB^2 = 400+400$

$\Rightarrow OB^2 = 800$

$\Rightarrow OB= 20\sqrt{2}$ cm

Area of the quadrant = $(\pi R^2)/4$ cm^2 = $(3.14/4)\times(20\sqrt{2})^2$ cm^2 = 628cm^2

Area of the square = 20×20 = 400 cm^2

Area of the shaded region = Area of the quadrant – Area of the square

= 628-400 cm^2 = 228cm^2

14. AB and CD are, respectively, arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.

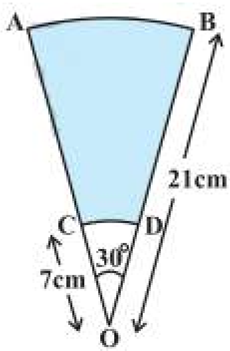


Fig. 12.32

Solution:

The radius of the larger circle, R = 21 cm

The radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = (30°/360°)×πR^2 cm^2

= (1/12)×(22/7)×21^2 cm^2

= 231/2cm^2

Area of the smaller circle = (30°/360°)×πr^2 cm^2

= 1/12×22/7×7^2 cm^2

=77/6 cm^2

Area of the shaded region = (231/2) – (77/6) cm^2

= 616/6 cm^2 = 308/3cm^2

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm, and a semicircle is drawn with BC as a diameter. Find the area of the shaded region.

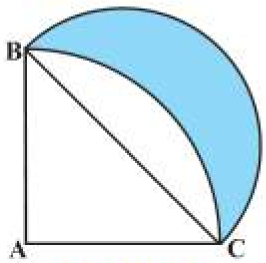


Fig. 12.33

Solution:

The radius of the quadrant ABC of the circle = 14 cm

AB = AC = 14 cm

BC is the diameter of the semicircle.

ABC is the right-angled triangle.

By Pythagoras' theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 14^2 + 14^2$

$\Rightarrow BC = 14\sqrt{2}$ cm

Radius of the semicircle = $14\sqrt{2}/2$ cm = $7\sqrt{2}$ cm

Area of the ΔABC =(½)×14×14 = 98 cm^2

Area of the quadrant = (¼)×(22/7)×(14×14) = 154 cm^2

Area of the semicircle = (½)×(22/7)×$7\sqrt{2}$×$7\sqrt{2}$ = 154 cm^2

Area of the shaded region =Area of the semicircle + Area of the ΔABC – Area of the quadrant

= 154 +98-154 cm^2 = 98cm^2

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

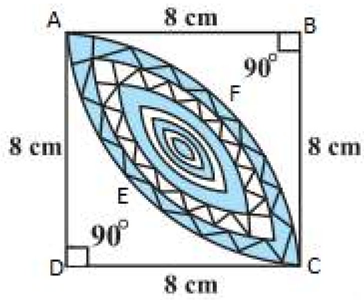


Fig. 12.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = (½)×8×8 = 32 cm^2

Area of quadrant AECB = Area of quadrant AFCD = (¼)×22/7×8^2

= 352/7 cm^2

Area of shaded region = (Area of quadrant AECB – Area of ΔABC) = (Area of quadrant AFCD – Area of ΔADC)

= (352/7 -32)+(352/7- 32) cm^2

= 2×(352/7-32) cm^2

= 256/7 cm^2