EXERCISE: 12.1 (PAGE NO: 230)

1. The radii of the two circles are 19 cm and 9 cm, respectively. Find the radius of the circle which has a circumference equal to the sum of the circumferences of the two circles.

Solution:

The radius of the 1^{st} circle = 19 cm (given)

∴ circumference of the 1^{st} circle = $2\pi\times19 = 38\pi$ cm

The radius of the 2^{nd} circle = 9 cm (given)

∴ circumference of the 2^{nd} circle = $2\pi\times9 = 18\pi$ cm

So,

The sum of the circumference of two circles = $38\pi+18\pi = 56\pi$ cm

Now, let the radius of the 3^{rd} circle = R

∴ the circumference of the 3^{rd} circle = $2\pi R$

It is given that sum of the circumference of two circles = circumference of the 3^{rd} circle

Hence, $56\pi = 2\pi R$

Or, R = 28 cm.

2. The radii of the two circles are 8 cm and 6 cm, respectively. Find the radius of the circle having an area equal to the sum of the areas of the two circles.

Solution:

The radius of 1^{st} circle = 8 cm (given)

∴ area of 1^{st} circle = $\pi(8)^2 = 64\pi$

The radius of 2^{nd} circle = 6 cm (given)

∴ area of 2^{nd} circle = $\pi(6)^2 = 36\pi$

So,

The sum of 1^{st} and 2^{nd} circle will be = $64\pi+36\pi = 100\pi$

Now, assume that the radius of 3^{rd} circle = R

∴ area of the circle 3^{rd} circle = πR^2

It is given that the area of the circle 3^{rd} circle = Area of 1^{st} circle + Area of 2^{nd} circle

Or, $\pi R^2 = 100\pi cm^2$

$R^2 = 100cm^2$

So, $R = 10cm$

3. Fig. 12.3 depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing the Gold score is 21 cm, and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Solution:

The radius of 1st circle, r_1 = 21/2 cm (as diameter D is given as 21 cm)

So, area of gold region = $\pi r_1^2 = \pi(10.5)^2 = 346.5$ cm^2

Now, it is given that each of the other bands is 10.5 cm wide,

So, the radius of 2nd circle, r_2 = 10.5cm+10.5cm = 21 cm

Thus,

∴ area of red region = Area of 2nd circle − Area of gold region = $(\pi r_2^2 - 346.5)$ cm^2

$= (\pi(21)^2 - 346.5)$ cm^2

$= 1386 - 346.5$

$= 1039.5$ cm^2

Similarly,

The radius of 3rd circle, r_3 = 21 cm+10.5 cm = 31.5 cm

The radius of 4th circle, r_4 = 31.5 cm+10.5 cm = 42 cm

The Radius of 5th circle, r_5 = 42 cm+10.5 cm = 52.5 cm

For the area of nth region,

A = Area of circle n – Area of the circle (n-1)

∴ area of the blue region (n=3) = Area of the third circle – Area of the second circle

$= \pi(31.5)^2 - 1386\ cm^2$

$= 3118.5 - 1386\ cm^2$

$= 1732.5\ cm^2$

∴ area of the black region (n=4) = Area of the fourth circle – Area of the third circle

$= \pi(42)^2 - 1386\ cm^2$

$= 5544 - 3118.5\ cm^2$

$= 2425.5\ cm^2$

∴ area of the white region (n=5) = Area of the fifth circle – Area of the fourth circle

$= \pi(52.5)^2 - 5544\ cm^2$

$= 8662.5 - 5544\ cm^2$

$= 3118.5\ cm^2$

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:

The radius of car's wheel = 80/2 = 40 cm (as D = 80 cm)

So, the circumference of wheels = $2\pi r = 80\,\pi$ cm

Now, in one revolution, the distance covered = circumference of the wheel = $80\,\pi$ cm

It is given that the distance covered by the car in 1 hr = 66km

Converting km into cm, we get,

Distance covered by the car in 1hr = (66×10^5) cm

In 10 minutes, the distance covered will be = $(66\times10^5\times10)/60 = 1100000$ cm/s

∴ distance covered by car = 11×10^5 cm

Now, the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels)

$=(11\times10^5)/80\,\pi = 4375.$

5. Tick the correct solution in the following and justify your choice. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Solution:

Since the perimeter of the circle = area of the circle,

$2\pi r = \pi r^2$

Or, r = 2

So, option (A) is correct, i.e., the radius of the circle is 2 units.

EXERCISE: 12.2 (PAGE NO: 230)

1. Find the area of a sector of a circle with a radius 6 cm if the angle of the sector is 60°.

Solution:

It is given that the angle of the sector is 60°

We know that the area of sector = $(\theta/360°)\times\pi r^2$

∴ area of the sector with angle 60° = $(60°/360°)\times\pi r^2$ cm^2

= $(36/6)\pi$ cm^2

= $6\times22/7$ cm^2 = $132/7$ cm^2

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:

Circumference of the circle, C = 22 cm (given)

It should be noted that a quadrant of a circle is a sector which is making an angle of 90°.

Let the radius of the circle = r

As $C = 2\pi r = 22$,

$R = 22/2\pi$ cm = $7/2$ cm

∴ area of the quadrant = $(\theta/360°) \times \pi r^2$

Here, $\theta = 90°$

So, $A = (90°/360°) \times \pi\, r^2$ cm^2

= $(49/16)\, \pi$ cm^2

= $77/8$ cm^2 = 9.6 cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

Length of minute hand = radius of the clock (circle)

∴ Radius (r) of the circle = 14 cm (given)

Angle swept by minute hand in 60 minutes = 360°

So, the angle swept by the minute hand in 5 minutes = $360° \times 5/60 = 30°$

We know,

Area of a sector = $(\theta/360°) \times \pi r^2$

Now, the area of the sector making an angle of 30° = $(30°/360°) \times \pi r^2$ cm^2

= $(1/12) \times \pi 14^2$

= $(49/3)\times(22/7)$ cm^2

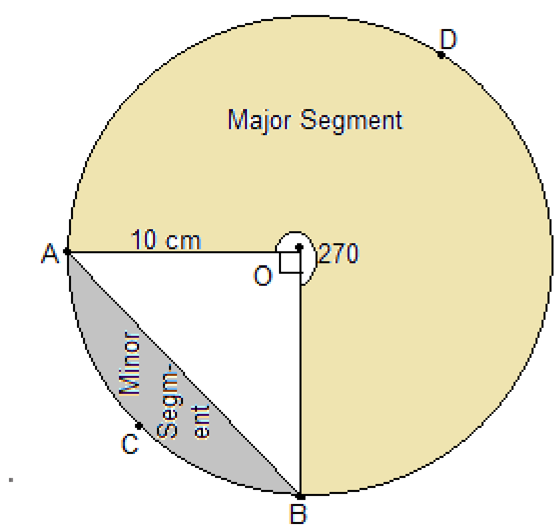
= $154/3$ cm^2

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment

(ii) major sector. (Use π = 3.14)

Solution:



Here, AB is the chord which is subtending an angle 90° at the centre O.

It is given that the radius (r) of the circle = 10 cm

(i) Area of minor sector = $(90/360°)\times\pi r^2$

= $(¼)\times(22/7)\times10^2$

Or, the Area of the minor sector = 78.5 cm^2

Also, the area of ΔAOB = ½×OB×OA

Here, OB and OA are the radii of the circle, i.e., = 10 cm

So, the area of ΔAOB = ½×10×10

= 50 cm^2

Now, area of minor segment = area of the minor sector – the area of ΔAOB

= 78.5 – 50

= 28.5 cm^2

(ii) Area of major sector = Area of the circle – Area of he minor sector

= (3.14×10^2)-78.5

= 235.5 cm^2

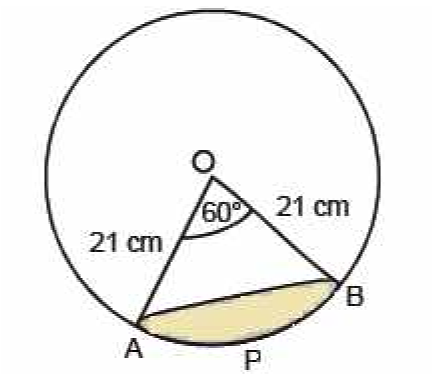
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Solution:



Given,

Radius = 21 cm

θ = 60°

(i) Length of an arc = θ/360°×Circumference(2πr)

∴ Length of an arc AB = (60°/360°)×2×(22/7)×21

= (1/6)×2×(22/7)×21

Or Arc AB Length = 22cm

(ii) It is given that the angle subtended by the arc = 60°

So, the area of the sector making an angle of 60° = (60°/360°)×π r^2 cm^2

= 441/6×22/7 cm^2

Or, the area of the sector formed by the arc APB is 231 cm^2

(iii) Area of segment APB = Area of sector OAPB – Area of ΔOAB

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔOAB is an equilateral triangle. So, its area will be √3/4×a^2 sq. Units.

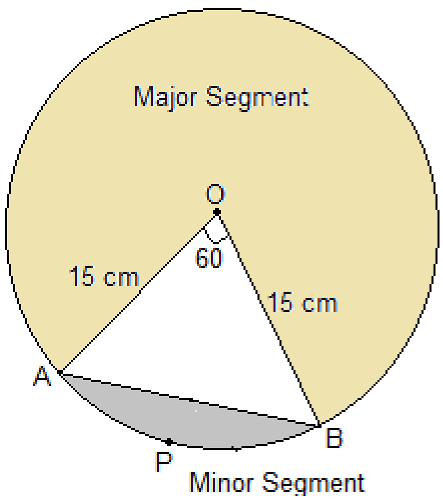
The area of segment APB = 231-(√3/4)×$(OA)^2$

= 231-(√3/4)×21^2

Or, the area of segment APB = [231-(441×√3)/4] cm^2

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:



Given,

Radius = 15 cm

θ = 60°

So,

Area of sector OAPB = (60°/360°)×πr^2 cm^2

= 225/6 πcm^2

Now, ΔAOB is equilateral as two sides are the radii of the circle and hence equal and one angle is 60°

So, Area of ΔAOB = $(\sqrt{3}/4) \times a^2$

Or, $(\sqrt{3}/4) \times 15^2$

∴ Area of ΔAOB = 97.31 cm^2

Now, the area of minor segment APB = Area of OAPB – Area of ΔAOB

Or, the area of minor segment APB = $((225/6)\pi - 97.31)$ cm^2 = 20.43 cm^2

And,

Area of major segment = Area of the circle – Area of the segment APB

Or, area of major segment = $(\pi \times 15^2) - 20.4$ = 686.06 cm^2

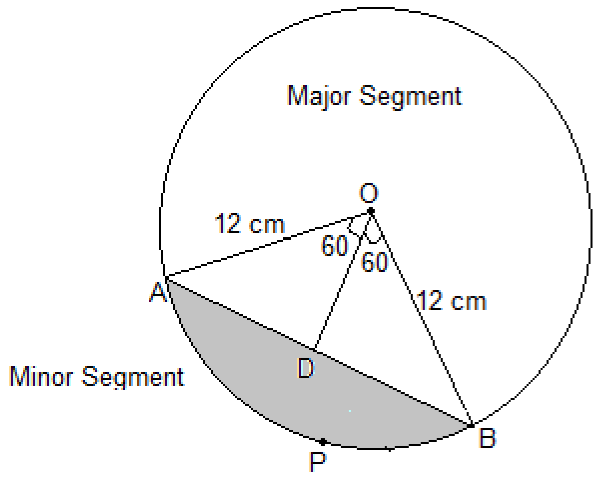
7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:

Radius, r = 12 cm

Now, draw a perpendicular OD on chord AB, and it will bisect chord AB.

So, AD = DB



Now, the area of the minor sector = $(\theta/360°) \times \pi r^2$

= $(120/360) \times (22/7) \times 12^2$

= 150.72 cm^2

Consider the ΔAOB,

$\angle$ OAB = 180°-(90°+60°) = 30°

Now, cos 30° = AD/OA

$\sqrt{3}/2$ = AD/12

Or, AD = $6\sqrt{3}$ cm

We know OD bisects AB. So,

AB = 2×AD = $12\sqrt{3}$ cm

Now, sin 30° = OD/OA

Or, ½ = OD/12

∴ OD = 6 cm

So, the area of ΔAOB = ½ × base × height

Here, base = AB = $12\sqrt{3}$ and

Height = OD = 6

So, area of ΔAOB = ½×$12\sqrt{3}$×6 = $36\sqrt{3}$ cm = 62.28 cm^2

∴ area of the corresponding Minor segment = Area of the Minor sector – Area of ΔAOB

= 150.72 cm^2– 62.28 cm^2 = 88.44 cm^2

8. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

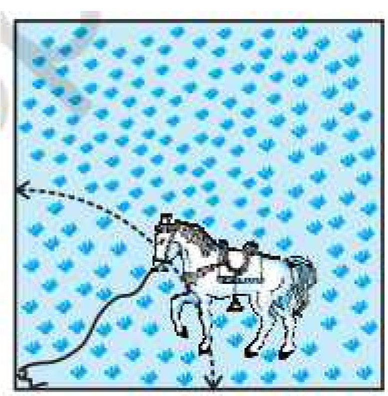


Fig. 12.11

Solution:

As the horse is tied at one end of a square field, it will graze only a quarter (i.e. sector with θ = 90°) of the field with a radius 5 m.

Here, the length of the rope will be the radius of the circle, i.e. r = 5 m

It is also known that the side of the square field = 15 m

(i) Area of circle = $\pi r^2 = 22/7 \times 5^2 = 78.5$ m^2

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle) = 78.5/4 = 19.625 m^2

(ii) If the rope is increased to 10 m,

Area of circle will be = $\pi r^2 = 22/7 \times 10^2 = 314$ m^2

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle)

= 314/4 = 78.5 m^2

∴ increase in the grazing area = 78.5 m^2 – 19.625 m^2 = 58.875 m^2

9. A brooch is made with silver wire in the form of a circle with a diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors, as shown in Fig. 12.12. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

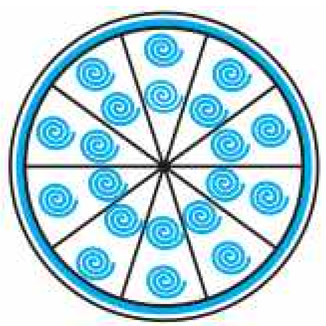


Fig. 12.12

Solution:

Diameter (D) = 35 mm

Total number of diameters to be considered= 5

Now, the total length of 5 diameters that would be required = 35×5 = 175

Circumference of the circle = $2\pi r$

Or, $C = \pi D = 22/7 \times 35 = 110$

Area of the circle $= \pi r^2$

Or, $A = (22/7) \times (35/2)^2 = 1925/2$ mm^2

(i) Total length of silver wire required = Circumference of the circle + Length of 5 diameter

= 110+175 = 285 mm

(ii) Total Number of sectors in the brooch = 10

So, the area of each sector = total area of the circle/number of sectors

∴ Area of each sector = $(1925/2) \times 1/10 = 385/4$ mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming the umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Fig. 12.13

Solution:

The radius (r) of the umbrella when flat = 45 cm

So, the area of the circle (A) $= \pi r^2 = (22/7) \times (45)^2 = 6364.29$ cm^2

Total number of ribs (n) = 8

∴ The area between the two consecutive ribs of the umbrella = A/n

6364.29/8 cm^2

Or, The area between the two consecutive ribs of the umbrella = 795.5 cm^2

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

Given,

Radius (r) = 25 cm

Sector angle (θ) = 115°

Since there are 2 blades,

The total area of the sector made by wiper = $2\times(\theta/360°)\times\pi\ r^2$

$= 2\times(115/360)\times(22/7)\times25^2$

$= 2\times158125/252\ cm^2$

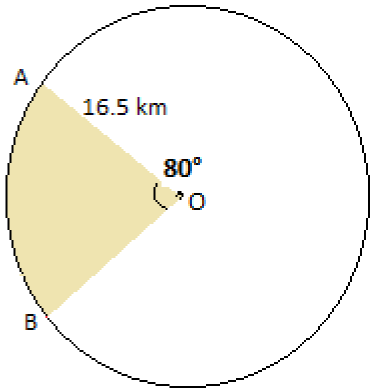
$= 158125/126 = 1254.96\ cm^2$

12. To warn ships of underwater rocks, a lighthouse spreads a red-coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Solution:

Let O bet the position of the lighthouse.



Here, the radius will be the distance over which light spreads.

Given radius (r) = 16.5 km

Sector angle (θ) = 80°

Now, the total area of the sea over which the ships are warned = Area made by the sector

Or, Area of sector = $(\theta/360°)\times\pi r^2$

$= (80°/360°)\times\pi r^2\ km^2$

$= 189.97\ km^2$

13. A round table cover has six equal designs, as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use √3 = 1.7)

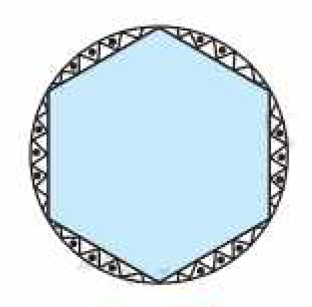


Fig. 12.14

Solution:

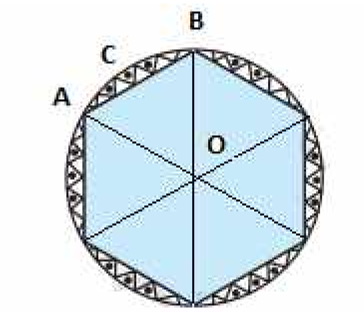


Fig. 12.14

Total number of equal designs = 6

AOB= 360°/6 = 60°

The radius of the cover = 28 cm

Cost of making design = ₹ 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be $(\sqrt{3}/4)\times a^2$ sq. units

Here, a = OA

∴ Area of equilateral ΔAOB = $(\sqrt{3}/4)\times 28^2 = 333.2$ cm^2

Area of sector ACB = $(60°/360°)\times\pi r^2$ cm^2

= 410.66 cm^2

So, the area of a single design = the area of sector ACB – the area of ΔAOB

= 410.66 cm^2 – 333.2 cm^2 = 77.46 cm^2

∴ area of 6 designs = 6×77.46 cm^2 = 464.76 cm^2

So, total cost of making design = 464.76 cm^2 ×Rs.0.35 per cm^2

= Rs. 162.66

14. Tick the correct solution in the following:

The area of a sector of angle p (in degrees) of a circle with radius R is

(A) $p/180 \times 2\pi R$

(B) $p/180 \times \pi R^2$

(C) $p/360 \times 2\pi R$

(D) $p/720 \times 2\pi R^2$

Solution:

The area of a sector = $(\theta/360°)\times\pi r^2$

Given, $\theta = p$

So, the area of sector = $p/360\times\pi R^2$

Multiplying and dividing by 2 simultaneously,

= $(p/360)\times 2/2\times\pi R^2$

= $(2p/720)\times 2\pi R^2$

So, option (D) is correct.

EXERCISE: 12.3 (PAGE NO: 234)

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

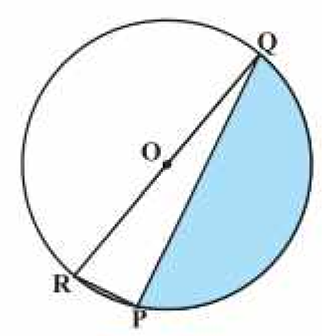


Fig. 12.19

Solution:

Here, P is in the semi-circle, and so,

P = 90°

So, it can be concluded that QR is the hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using the Pythagorean theorem,

$QR^2 = PR^2+PQ^2$

Or, $QR^2 = 7^2+24^2$

QR= 25 cm = Diameter

Hence, the radius of the circle = 25/2 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= (22/7)×(25/2)×(25/2)/2 cm^2

= 13750/56 cm^2 = 245.54 cm^2

Also, the area of the ΔPQR = ½×PR×PQ

=(½)×7×24 cm^2

= 84 cm^2

Hence, the area of the shaded region = 245.54 cm^2-84 cm^2

= 161.54 cm^2

2. Find the area of the shaded region in Fig. 12.20, if the radii of the two concentric circles with centre O are 7 cm and 14 cm, respectively and AOC = 40°.

Solution:

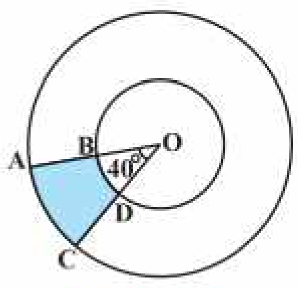


Fig. 12.20

Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = $(\theta/360°)\times\pi r^2$

So, Area of OAC = $(40°/360°)\times\pi r^2$ cm^2

= 68.44 cm^2

Area of the sector OBD = $(40°/360°)\times\pi r^2$ cm^2

= $(1/9)\times(22/7)\times7^2$ = 17.11 cm^2

Now, the area of the shaded region ABDC = Area of OAC – Area of the OBD

= 68.44 cm^2 – 17.11 cm^2 = 51.33 cm^2

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

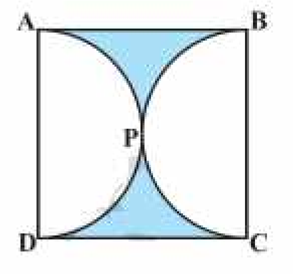


Fig. 12.21

Solution:

Side of the square ABCD (as given) = 14 cm

So, the Area of ABCD = a^2

= 14×14 cm^2 = 196 cm^2

We know that the side of the square = diameter of the circle = 14 cm

So, the side of the square = diameter of the semicircle = 14 cm

∴ the radius of the semicircle = 7 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7\times7\times7)/2$ cm^2

= 77 cm^2

∴ he area of two semicircles = 2×77 cm^2 = 154 cm^2

Hence, the area of the shaded region = Area of the Square – Area of two semicircles

= 196 cm^2 -154 cm^2

= 42 cm^2

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as the centre.

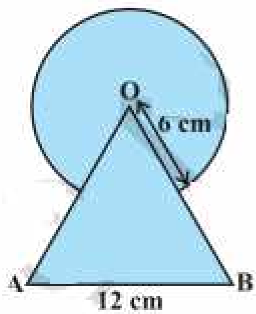


Fig. 12.22

Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

The area of the sector is common in both.

The radius of the circle = 6 cm

Side of the triangle = 12 cm

Area of the equilateral triangle = $(\sqrt{3}/4)\,(OA)^2 = (\sqrt{3}/4)\times 12^2 = 36\sqrt{3}$ cm^2

Area of the circle = $\pi R^2 = (22/7)\times 6^2 = 792/7$ cm^2

Area of the sector making angle 60° = $(60°/360°)\times \pi r^2$ cm^2

$= (1/6)\times(22/7)\times 6^2$ cm^2 $= 132/7$ cm^2

Area of the shaded region = Area of the equilateral triangle + Area of the circle – Area of the sector

$= 36\sqrt{3}$ cm^2 $+792/7$ cm^2 $-132/7$ cm^2

$= (36\sqrt{3}+660/7)$ cm^2

5. From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

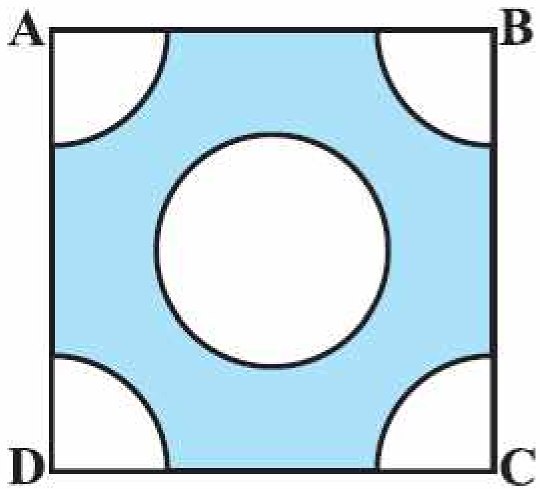


Fig. 12.23

Solution:

Side of the square = 4 cm

The radius of the circle = 1 cm

Four quadrants of a circle are cut from the corner, and one circle of radius are cut from the middle.

Area of the square = $(\text{side})^2 = 4^2 = 16\ \text{cm}^2$

Area of the quadrant = $(\pi R^2)/4\ \text{cm}^2 = (22/7)\times(1^2)/4 = 11/14\ \text{cm}^2$

$\therefore$ Total area of the 4 quadrants = $4 \times (11/14)\ \text{cm}^2 = 22/7\ \text{cm}^2$

Area of the circle = $\pi R^2\ \text{cm}^2 = (22/7\times1^2) = 22/7\ \text{cm}^2$

Area of the shaded region = Area of the square – (Area of the 4 quadrants + Area of the circle)

$= 16\ \text{cm}^2 - (22/7)\ \text{cm}^2 – (22/7)\ \text{cm}^2$

$= 68/7\ \text{cm}^2$

6. In a circular table cover of radius 32 cm, a design is formed, leaving an equilateral triangle ABC in the middle, as shown in Fig. 12.24. Find the area of the design.

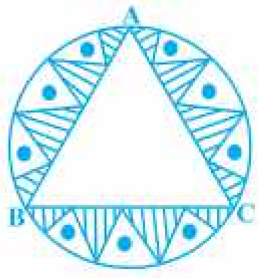


Fig. 12.24

Solution:

The radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

⇒ BD = AB/2

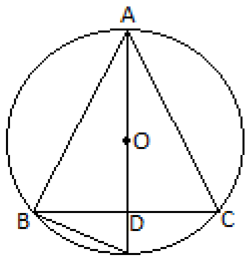
Since, AD is the median of the triangle

∴ AO = Radius of the circle = (2/3) AD

⇒ (2/3)AD = 32 cm

⇒ AD = 48 cm

In ΔADB,



By Pythagoras' theorem,

$AB^2 = AD^2 + BD^2$

⇒ $AB^2 = 48^2 + (AB/2)^2$

⇒ $AB^2 = 2304 + AB^2/4$

⇒ $3/4\ (AB^2) = 2304$

⇒ $AB^2 = 3072$

⇒ $AB = 32\sqrt{3}$ cm

Area of $\Delta ADB = \sqrt{3}/4 \times (32\sqrt{3})^2$ cm^2 = $768\sqrt{3}$ cm^2

Area of the circle = πR^2 = (22/7)×32×32 = 22528/7 cm^2

Area of the design = Area of the circle – Area of ΔADB

= $(22528/7 - 768\sqrt{3})$ cm^2

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

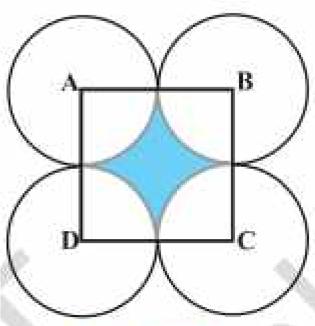


Fig. 12.25

Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = 14^2 = 196 cm^2

Area of the quadrant = $(\pi R^2)/4$ cm^2 = (22/7) ×7^2/4 cm^2

= 77/2 cm^2

Total area of the quadrant = 4×77/2 cm^2 = 154cm^2

Area of the shaded region = Area of the square ABCD – Area of the quadrant

= 196 cm^2 – 154 cm^2

= 42 cm^2

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find

(i) the distance around the track along its inner edge

(ii) the area of the track.

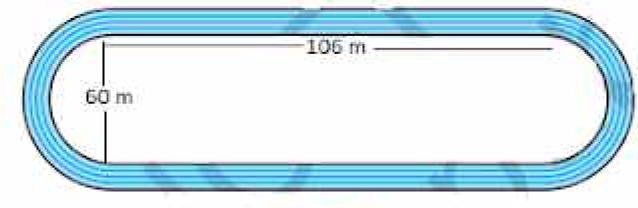


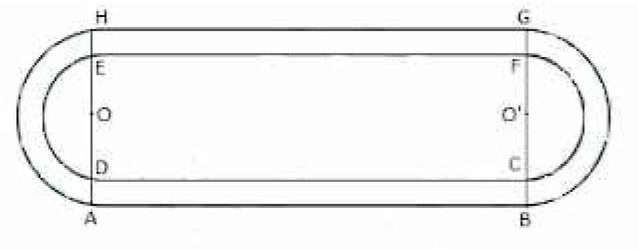
Fig. 12.26

Solution:

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m



DE = CF = 60 m

The radius of the inner semicircle, r = OD = O'C

= 60/2 m = 30 m

The radius of the outer semicircle, R = OA = O'B

= 30+10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD+EF+2×(Circumference of inner semicircle)

= 106+106+(2×πr) m = 212+(2×22/7×30) m

= 212+1320/7 m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) – 2 × (area of inner semicircle)

= $(AB\times CD)+(EF\times GH)+2\times(\pi r^2/2)\ -2\times(\pi R^2/2)\ m^2$

= $(106\times10)+(106\times10)+2\times\pi/2(r^2-R^2)\ m^2$

= $2120+22/7\times70\times10\ m^2$

$= 4320\ m^2$

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other, and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

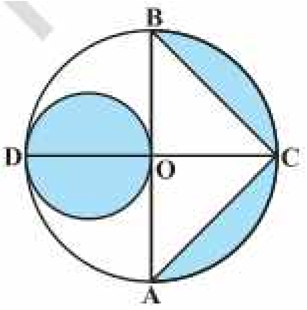


Fig. 12.27

Solution:

The radius of larger circle, R = 7 cm

The radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = 1/2 × AB × OC = (½)×7×14 = 49 cm^2

Area of larger circle = $\pi R^2 = (22/7)\times 7^2 = 154\ cm^2$

Area of larger semicircle = $154/2\ cm^2 = 77\ cm^2$

Area of smaller circle = $\pi r^2 = (22/7)\times(7/2)\times(7/2) = 77/2\ cm^2$

Area of the shaded region = Area of the larger circle – Area of the triangle – Area of the larger semicircle + Area of the smaller circle

Area of the shaded region = (154-49-77+77/2) cm^2

$= 133/2\ cm^2 = 66.5\ cm^2$

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as the centre, a circle is drawn with a radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region (Use π = 3.14 and √3 = 1.73205).

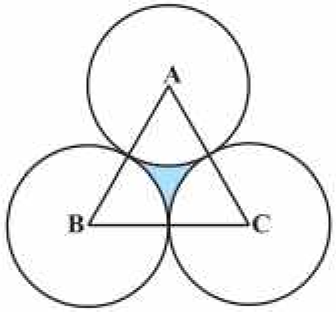


Fig. 12.28

Solution:

ABC is an equilateral triangle.

∴ ∠ A = ∠ B = ∠ C = 60°

There are three sectors, each making 60°.

Area of ΔABC = 17320.5 cm^2

⇒ √3/4 ×$(side)^2$ = 17320.5

⇒ $(side)^2$ =17320.5×4/1.73205

⇒ $(side)^2$ = 4×10^4

⇒ side = 200 cm

Radius of the circles = 200/2 cm = 100 cm

Area of the sector = (60°/360°)×π r^2 cm^2

= 1/6×3.14×$(100)^2$ cm^2

= 15700/3cm^2

Area of 3 sectors = 3×15700/3 = 15700 cm^2

Thus, the area of the shaded region = Area of an equilateral triangle ABC – Area of 3 sectors

= 17320.5-15700 cm^2 = 1620.5 cm^2

11. On a square handkerchief, nine circular designs, each of a radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

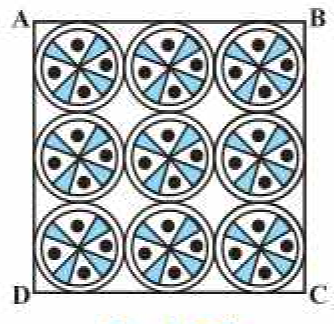


Fig. 12.29

Solution:

Number of circular designs = 9

The radius of the circular design = 7 cm

There are three circles on one side of the square handkerchief.

∴ side of the square = 3×diameter of circle = 3×14 = 42 cm

Area of the square = 42×42 cm^2 = 1764 cm^2

Area of the circle = πr^2 = (22/7)×7×7 = 154 cm^2

Total area of the design = 9×154 = 1386 cm^2

Area of the remaining portion of the handkerchief = Area of the square – Total area of the design = 1764 – 1386 = 378 cm^2

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and a radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB

(ii) shaded region

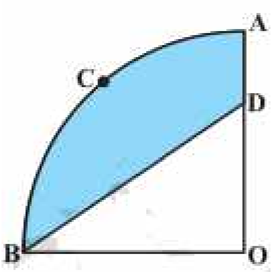


Fig. 12.30

Solution:

Radius of the quadrant = 3.5 cm = 7/2 cm

(i) Area of the quadrant OACB = $(\pi R^2)/4$ cm^2

= $(22/7)\times(7/2)\times(7/2)/4$ cm^2

= $77/8$ cm^2

(ii) Area of the triangle BOD = $(½)\times(7/2)\times2$ cm^2

= $7/2$ cm^2

Area of the shaded region = Area of the quadrant – Area of the triangle BOD

= $(77/8)-(7/2)$ cm^2 = $49/8$ cm^2

= 6.125 cm^2

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

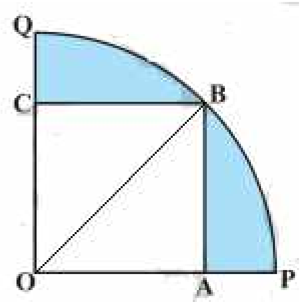


Fig. 12.31

Solution:

Side of square = OA = AB = 20 cm

The radius of the quadrant = OB

OAB is the right-angled triangle

By Pythagoras' theorem in ΔOAB,

$OB^2 = AB^2+OA^2$

$\Rightarrow OB^2 = 20^2+20^2$

$\Rightarrow OB^2 = 400+400$

$\Rightarrow OB^2 = 800$

$\Rightarrow OB = 20\sqrt{2}$ cm

Area of the quadrant = $(\pi R^2)/4$ cm^2 = $(3.14/4)\times(20\sqrt{2})^2$ cm^2 = 628cm^2

Area of the square = 20×20 = 400 cm^2

Area of the shaded region = Area of the quadrant – Area of the square

= 628-400 cm^2 = 228cm^2

14. AB and CD are, respectively, arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.

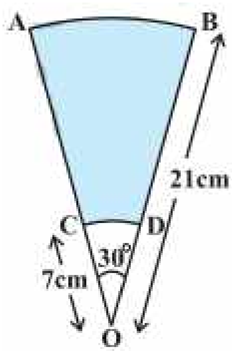


Fig. 12.32

Solution:

The radius of the larger circle, R = 21 cm

The radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = (30°/360°)×πR^2 cm^2

= (1/12)×(22/7)×21^2 cm^2

= 231/2cm^2

Area of the smaller circle = (30°/360°)×πr^2 cm^2

= 1/12×22/7×7^2 cm^2

=77/6 cm^2

Area of the shaded region = (231/2) – (77/6) cm^2

= 616/6 cm^2 = 308/3cm^2

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm, and a semicircle is drawn with BC as a diameter. Find the area of the shaded region.

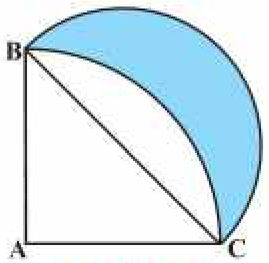


Fig. 12.33

Solution:

The radius of the quadrant ABC of the circle = 14 cm

AB = AC = 14 cm

BC is the diameter of the semicircle.

ABC is the right-angled triangle.

By Pythagoras' theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

⇒ $BC^2 = 14^2 + 14^2$

⇒ BC = 14√2 cm

Radius of the semicircle = 14√2/2 cm = 7√2 cm

Area of the ΔABC =(½)×14×14 = 98 cm^2

Area of the quadrant = (¼)×(22/7)×(14×14) = 154 cm^2

Area of the semicircle = (½)×(22/7)×7√2×7√2 = 154 cm^2

Area of the shaded region =Area of the semicircle + Area of the ΔABC – Area of the quadrant

= 154 +98-154 cm^2 = 98cm^2

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

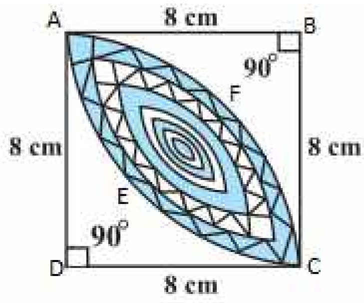


Fig. 12.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = (½)×8×8 = 32 cm^2

Area of quadrant AECB = Area of quadrant AFCD = (¼)×22/7×8^2

= 352/7 cm^2

Area of shaded region = (Area of quadrant AECB – Area of ΔABC) = (Area of quadrant AFCD – Area of ΔADC)

= (352/7 -32)+(352/7- 32) cm^2

= 2×(352/7-32) cm^2

= 256/7 cm^2