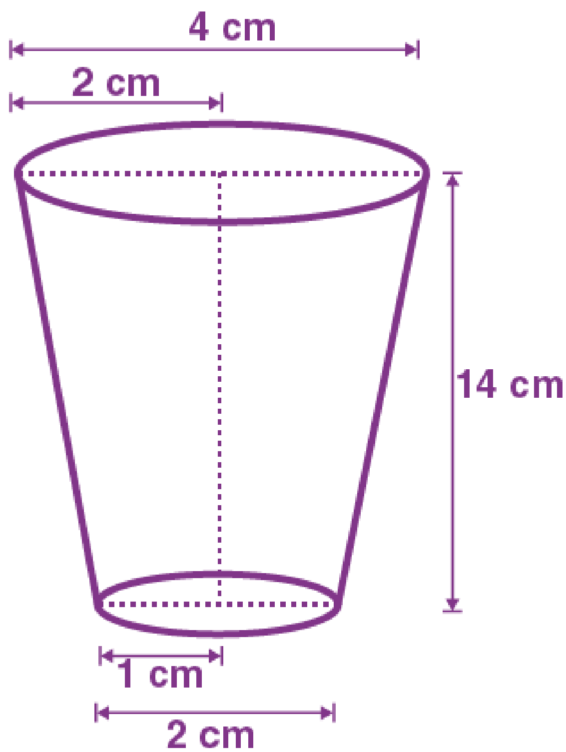
EXERCISE: 13.4 (PAGE NO: 257)

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution:



Radius (r_1) of the upper base = 4/2 = 2 cm

Radius (r_2) of lower the base = 2/2 = 1 cm

Height = 14 cm

Now, the capacity of glass = Volume of the frustum of the cone

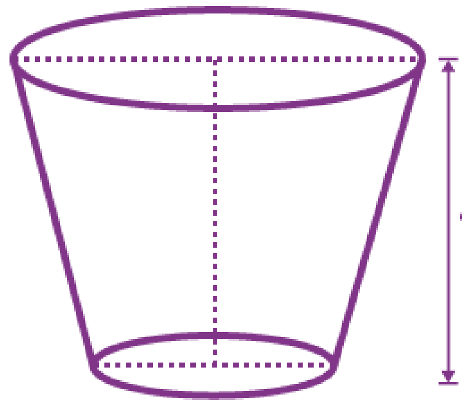
So, Capacity of glass = $(\frac{1}{3})\times\pi\times h(r_1^2+r_2^2+r_1r_2)$

= $(\frac{1}{3})\times\pi\times(14)(2^2+1^2+(2)(1))$

∴ The capacity of the glass = $102\times(\frac{2}{3})$ cm^3

2. The slant height of a frustum of a cone is 4 cm, and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the surface area of the frustum.

Solution:



Given,

Slant height (l) = 4 cm

Circumference of upper circular end of the frustum = 18 cm

∴ $2\pi r_1 = 18$

Or, $r_1 = 9/\pi$

Similarly, the circumference of the lower end of the frustum = 6 cm

∴ $2\pi r_2 = 6$

Or, $r_2 = 3/\pi$

Now, the surface area of the frustum = $\pi(r_1+r_2) \times l$

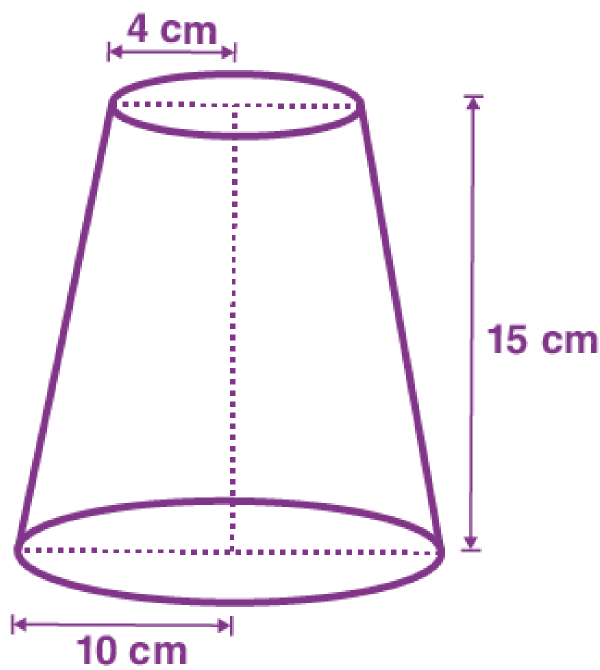
$= \pi(9/\pi+3/\pi) \times 4$

$= 12\times4 = 48\ cm^2$

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, the radius at the upper base is 4 cm, and its slant height is 15 cm, find the area of material used for making it.

Fig. 13.24

Solution:



Given,

For the lower circular end, radius (r_1) = 10 cm

For the upper circular end, radius (r_2) = 4 cm

Slant height (l) of frustum = 15 cm

Now,

The area of material to be used for making the fez = CSA of frustum + Area of the upper circular end

CSA of frustum = $\pi(r_1+r_2)\times l$

$= 210\pi$

And, the Area of the upper circular end $= \pi r_2^2$

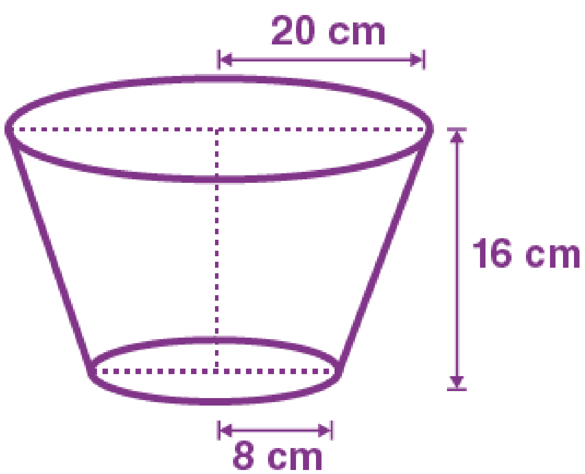
$= 16\pi$

The area of material to be used for making the fez = $210\pi + 16\pi = (226 \times 22)/7 = 710\ 2/7$

∴ The area of material used = 710 2/7 cm^2

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 20 per litre. Also, find the cost of the metal sheet used to make the container if it costs Rs. 8 per 100 cm^2.

Solution:



Given,

$r_1 = 20$ cm,

$r_2 = 8$ cm and

h = 16 cm

∴ Volume of the frustum $= (1/3)\times\pi\times h(r_1^2+r_2^2+r_1r_2)$

$= 1/3 \times 3.14 \times 16((20)^2+(8)^2+(20)(8))$

$= 1/3 \times 3.14 \times 16(400 + 64 + 160) = 10449.92\ cm^3 = 10.45$ lit

It is given that the rate of milk = Rs. 20/litre

So, the cost of milk = 20×volume of the frustum

= 20×10.45

= Rs. 209

Now, the slant height will be

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{16^2 + 12^2}$$

l = 20 cm

So, CSA of the container = $\pi(r_1+r_2)\times l$

$$= \frac{314}{100}(20+8) \times 20 \text{ cm}^2$$

= 1758.4 cm^2

Hence, the total metal that would be required to make the container will be = 1758.4 + (Area of the bottom circle)

= $1758.4+\pi r^2 = 1758.4+\pi(8)^2$

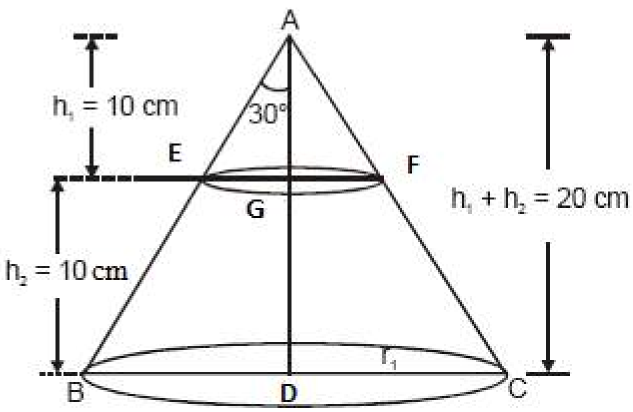
= 1758.4+201 = 1959.4 cm^2

∴ Total cost of metal = Rs. (8/100) × 1959.4 = Rs. 157

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained is drawn into a wire of diameter 1/16 cm, find the length of the wire.

Solution:

The diagram will be as follows



Consider AEG

$$\frac{EG}{AG} = \tan 30°$$

$$EG = \frac{10}{\sqrt{3}} cm = \frac{10\sqrt{3}}{3}$$

In ΔABD,

$$\frac{BD}{AD} = \tan 30°$$

$$BD = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} cm$$

Radius (r_1) of upper end of frustum = (10√3)/3 cm

Radius (r_2) of lower end of container = (20√3)/3 cm

Height (r_3) of container = 10 cm

Now,

Volume of the frustum = (⅓)×π×h($r_1^2+r_2^2+r_1r_2$)

$$= \frac{1}{3} \times \pi \times 10 \left[\left(\frac{10\sqrt{3}}{3} \right)^2 + \left(\frac{20\sqrt{3}}{3} \right)^2 + \frac{(10\sqrt{3})(20\sqrt{3})}{3 \times 3} \right]$$

Solving this, we get

Volume of the frustum = 22000/9 cm^3

The radius (r) of wire = (1/16)×(½) = 1/32 cm

Now,

Let the length of the wire be "l".

The volume of wire = Area of cross-section x Length

= (πr^2)xl

= $\pi(1/32)^2$x l

Now, Volume of frustum = Volume of wire

22000/9 = (22/7)x$(1/32)^2$x l

Solving this, we get,

l = 7964.44 m