1. The following table shows the ages of the patients admitted to a hospital during a year:

| Age (in years) | $5-15$ | $15-25$ | $25-35$ | $35-45$ | $45-55$ | $55-65$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of patients | 6 | 11 | 21 | 23 | 14 | 5 |

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Solution:
To find out the modal class, let us the consider the class interval with high frequency.
Here, the greatest frequency $=23$, so the modal class $=35-45$,
Lower limit of modal class $=1=35$,
class width $(\mathrm{h})=10$,
$\mathrm{f}_{\mathrm{m}}=23$,
$\mathrm{f}_{1}=21$ and $\mathrm{f}_{2}=14$
The formula to find the mode is
Mode $=l+\left[\left(f_{m}-f_{l}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values in the formula, we get
Mode $=35+[(23-21) /(46-21-14)] \times 10$
$=35+(20 / 11)$
$=35+1.8$
$=36.8$ years
So the mode of the given data $=36.8$ years
Calculation of Mean:
First find the midpoint using the formula, $\mathrm{x}_{\mathrm{i}}=($ upper limit + lower limit $) / 2$

| Class Interval | Frequency $\left(\mathrm{f}_{\mathrm{i}}\right)$ | Mid-point $\left(\mathrm{x}_{\mathrm{i}}\right)$ | $\mathrm{f}_{\mathrm{i} \mathrm{X}_{\mathrm{i}}}$ |
| :--- | :--- | :--- | :--- |
| $5-15$ | 6 | 10 | 60 |


| $15-25$ | 11 | 20 | 220 |
| :--- | :--- | :--- | :--- |
| $25-35$ | 21 | 30 | 630 |
| $35-45$ | 23 | 40 | 920 |
| $45-55$ | 14 | 50 | 700 |
| $55-65$ | 5 | 60 | 300 |
|  | Sum $\mathrm{f}_{\mathrm{i}}=80$ |  | Sum $\mathrm{f}_{\mathrm{i} \mathrm{x}_{\mathrm{i}}=2830}$ |

The mean formula is

Mean $=\overline{\mathrm{x}}=\sum \mathrm{f}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} / \sum \mathrm{f}_{\mathrm{i}}$
$=2830 / 80$
$=35.375$ years
Therefore, the mean of the given data $=35.375$ years
2. The following data gives the information on the observed lifetimes (in hours) of $\mathbf{2 2 5}$
electrical components:

| Lifetime (in hours) | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ | $100-120$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 10 | 35 | 52 | 61 | 38 | 29 |

Determine the modal lifetimes of the components.
Solution:
From the given data the modal class is 60-80.
Lower limit of modal class $=1=60$,
The frequencies are:
$\mathrm{f}_{\mathrm{m}}=61, \mathrm{f}_{1}=52, \mathrm{f}_{2}=38$ and $\mathrm{h}=20$

The formula to find the mode is
Mode $=l+\left[\left(f_{m}-f_{l}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values in the formula, we get
Mode $=60+[(61-52) /(122-52-38)] \times 20$

Mode $=60+[(9 \times 20) / 32]$
Mode $=60+(45 / 8)=60+5.625$
Therefore, modal lifetime of the components $=65.625$ hours .
3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

| Expenditure (in Rs.) | Number of families |
| :--- | :--- |
| $1000-1500$ | 24 |
| $1500-2000$ | 40 |
| $2000-2500$ | 33 |
| $2500-3000$ | 28 |
| $3000-3500$ | 30 |
| $3500-4000$ | 16 |
| $4000-4500$ | 7 |

Solution:
Given data:
Modal class $=1500-2000$,
$l=1500$,
Frequencies:
$\mathrm{f}_{\mathrm{m}}=40 \mathrm{f}_{1}=24, \mathrm{f}_{2}=33$ and
$h=500$

Mode formula:
Mode $=l+\left[\left(f_{m}-f_{l}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values in the formula, we get

Mode $=1500+[(40-24) /(80-24-33)] \times 500$
Mode $=1500+[(16 \times 500) / 23]$
Mode $=1500+(8000 / 23)=1500+347.83$
Therefore, modal monthly expenditure of the families $=$ Rupees 1847.83
Calculation for mean:
First find the midpoint using the formula, $\mathrm{x}_{\mathrm{i}}=($ upper limit + lower limit $) / 2$
Let us assume a mean, (a) be 2750.

| Class Interval | $\mathrm{f}_{\mathrm{i}}$ | $\mathrm{X}_{\mathrm{i}}$ | $\mathrm{d}_{\mathrm{i}}=\mathrm{X}_{\mathrm{i}}-\mathrm{a}$ | $\mathrm{u}_{\mathrm{i}}=\mathrm{d}_{\mathrm{i}} / \mathrm{h}$ | $\mathrm{f}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1000-1500 | 24 | 1250 | -1500 | -3 | -72 |
| 1500-2000 | 40 | 1750 | -1000 | -2 | -80 |
| 2000-2500 | 33 | 2250 | -500 | -1 | -33 |
| 2500-3000 | 28 | $2750=\mathrm{a}$ | 0 | 0 | 0 |
| 3000-3500 | 30 | 3250 | 500 | 1 | 30 |
| 3500-4000 | 22 | 3750 | 1000 | 2 | 44 |
| 4000-4500 | 16 | 4250 | 1500 | 3 | 48 |
| 4500-5000 | 7 | 4750 | 2000 | 4 | 28 |
|  | $\mathrm{f}_{\mathrm{i}}=200$ |  |  |  | $\mathrm{f}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}}=-35$ |

The formula to calculate the mean,
Mean $=\bar{x}=\mathrm{a}+\left(\sum \mathrm{f}_{\mathrm{i}} \mathrm{u}_{\mathrm{i}} / \sum \mathrm{f}_{\mathrm{i}}\right) \times \mathrm{h}$
Substitute the values in the given formula
$=2750+(-35 / 200) \times 500$
$=2750-87.50$
$=2662.50$
So, the mean monthly expenditure of the families $=$ Rs. 2662.50
4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures

| No of students per teacher | Number of states / U.T |
| :--- | :--- |
| $15-20$ | 3 |
| $20-25$ | 8 |
| $25-30$ | 9 |
| $30-35$ | 10 |
| $35-40$ | 3 |
| $40-45$ | 0 |
| $45-50$ | 0 |
| $50-55$ | 2 |

Solution:
Given data:
Modal class $=30-35$,
$l=30$,
Class width $(\mathrm{h})=5$,
$\mathrm{f}_{\mathrm{m}}=10, \mathrm{f}_{1}=9$ and $\mathrm{f}_{2}=3$
Mode Formula:
Mode $=l+\left[\left(f_{m}-f_{1}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values in the given formula
Mode $=30+[(10-9) /(20-9-3)] \times 5$
$=30+(5 / 8)$
$=30+0.625$
$=30.625$
Therefore, the mode of the given data $=30.625$
Calculation of mean:
Find the midpoint using the formula, $\mathrm{x}_{\mathrm{i}}=($ upper limit + lower limit $) / 2$

| Class Interval | Frequency $\left(\mathrm{f}_{\mathrm{i}}\right)$ | Mid-point $\left(\mathrm{x}_{\mathrm{i}}\right)$ | $\mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}$ |
| :--- | :--- | :--- | :--- |
| $15-20$ | 3 | 17.5 | 52.5 |
| $20-25$ | 8 | 22.5 | 180.0 |
| $25-30$ | 9 | 27.5 | 247.5 |
| $30-35$ | 10 | 32.5 | 325.0 |
| $35-40$ | 3 | 37.5 | 112.5 |
| $40-45$ | 0 | 42.5 | 0 |
| $45-50$ | 2 | 47.5 | 0 |
| $50-55$ | Sum $\mathrm{f}_{\mathrm{i}}=35$ | 52.5 | 105.0 |

> Mean $=\overline{\mathrm{x}}=\sum \mathrm{f}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}} \sum \mathrm{f}_{\mathrm{i}}$
> $=1022.5 / 35$
> $=29.2$ (approx)

Therefore, mean $=29.2$
5. The given distribution shows the number of runs scored by some top batsmen of the world in one- day international cricket matches.

| Run Scored | Number of Batsman |
| :--- | :--- |
| $3000-4000$ | 4 |
| $4000-5000$ | 18 |
| $5000-6000$ | 9 |
| $6000-7000$ | 7 |
| $7000-8000$ | 6 |
| $8000-9000$ | 3 |
| $9000-10000$ | 1 |


| $10000-11000$ | 1 |
| :--- | :--- |

Find the mode of the data.
Solution:
Given data:
Modal class $=4000-5000$,
$1=4000$,
class width $(\mathrm{h})=1000$,
$\mathrm{f}_{\mathrm{m}}=18, \mathrm{f}_{1}=4$ and $\mathrm{f}_{2}=9$
Mode Formula:
Mode $=l+\left[\left(f_{m}-f_{l}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values
Mode $=4000+[(18-4) /(36-4-9)] \times 1000$
$=4000+(14000 / 23)$
$=4000+608.695$
$=4608.695$
$=4608.7$ (approximately)
Thus, the mode of the given data is 4608.7 runs.
6. A student noted the number of cars passing through a spot on a road for 100 periods each of $\mathbf{3}$ minutes and summarized it in the table given below. Find the mode of the data:

| Number of cars | Frequency |
| :--- | :--- |
| $\mathbf{0 - 1 0}$ | 7 |
| $10-20$ | 14 |
| $20-30$ | 13 |
| $30-40$ | 12 |
| $40-50$ | 20 |


| $50-60$ | 11 |
| :--- | :--- |
| $60-70$ | 15 |
| $70-80$ | 8 |

Solution:
Given Data:
Modal class $=40-50,1=40$,
Class width $(h)=10, \mathrm{f}_{\mathrm{m}}=20, \mathrm{f}_{1}=12$ and $\mathrm{f}_{2}=11$
Mode $=l+\left[\left(f_{m}-f_{l}\right) /\left(2 f_{m}-f_{l}-f_{2}\right)\right] \times h$
Substitute the values
Mode $=40+[(20-12) /(40-12-11)] \times 10$
$=40+(80 / 17)$
$=40+4.7$
$=44.7$
Thus, the mode of the given data is 44.7 cars.

