

EXERCISE 14.1

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1. A survey was conducted by a group of students as a part of their environment awareness program, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

| Number of Plants | 0-2 | 2-4 | 4-6 | 6-8 | 8-10 | 10-12 | 12-14 |
|------------------|-----|-----|-----|-----|------|-------|-------|
| Number of Houses | 1 | 2 | 1 | 5 | 6 | 2 | 3 |

Which method did you use for finding the mean, and why?

Solution:

To find the mean value, we will use the direct method because the numerical value of f_i and x_i are small.

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

| No. of plants (Class interval) | No. of houses Frequency (f_i) | Mid-point (x_i) | $f_i x_i$ |
|-----------------------------------|--------------------------------------|---------------------|---------------------|
| 0-2 | 1 | 1 | 1 |
| 2-4 | 2 | 3 | 6 |
| 4-6 | 1 | 5 | 5 |
| 6-8 | 5 | 7 | 35 |
| 8-10 | 6 | 9 | 54 |
| 10-12 | 2 | 11 | 22 |
| 12-14 | 3 | 13 | 39 |
| | Sum $f_i = 20$ | | Sum $f_i x_i = 162$ |

The formula to find the mean is:

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 162/20$$

$$= 8.1$$

Therefore, the mean number of plants per house is 8.1.

2. Consider the following distribution of daily wages of 50 workers of a factory.

| Daily wages (in Rs.) | 500-520 | 520-540 | 540-560 | 560-580 | 580-600 |
|----------------------|---------|---------|---------|---------|---------|
| Number of workers | 12 | 14 | 8 | 6 | 10 |

Find the mean daily wages of the workers of the factory by using an appropriate method.

Solution:

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

In this case, the value of mid-point (x_i) is very large, so let us assume the mean value, $a = 550$.

Class interval (h) = 20

So, $u_i = (x_i - a)/h$

$u_i = (x_i - 550)/20$

Substitute and find the values as follows:

| Daily wages (Class interval) | Number of workers frequency (f_i) | Mid-point (x_i) | $u_i = (x_i - 550)/20$ | $f_i u_i$ |
|---------------------------------|--|------------------------|------------------------|---------------------|
| 500-520 | 12 | 510 | -2 | -24 |
| 520-540 | 14 | 530 | -1 | -14 |
| 540-560 | 8 | 550 = a | 0 | 0 |
| 560-580 | 6 | 570 | 1 | 6 |
| 580-600 | 10 | 590 | 2 | 20 |
| Total | Sum $f_i = 50$ | | | Sum $f_i u_i = -12$ |

So, the formula to find out the mean is:

$$\text{Mean} = \bar{x} = a + h(\sum f_i u_i / \sum f_i) = 550 + [20 \times (-12/50)] = 550 - 4.8 = 545.20$$

Thus, mean daily wage of the workers = Rs. 545.20

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f .

| Daily Pocket Allowance(in c) | 11-13 | 13-15 | 15-17 | 17-19 | 19-21 | 21-23 | 23-35 |
|------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Number of children | 7 | 6 | 9 | 13 | f | 5 | 4 |

Solution:

To find out the missing frequency, use the mean formula.

Given, mean $\bar{x} = 18$

| Class interval | Number of children (f_i) | Mid-point (x_i) | $f_i x_i$ |
|----------------|------------------------------|---------------------|-------------------------|
| 11-13 | 7 | 12 | 84 |
| 13-15 | 6 | 14 | 84 |
| 15-17 | 9 | 16 | 144 |
| 17-19 | 13 | 18 | 234 |
| 19-21 | f | 20 | 20f |
| 21-23 | 5 | 22 | 110 |
| 23-25 | 4 | 24 | 96 |
| Total | $f_i = 44+f$ | | Sum $f_i x_i = 752+20f$ |

The mean formula is

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{752 + 20f}{44 + f}$$

Now substitute the values and equate to find the missing frequency (f)

$$\Rightarrow 18 = \frac{752 + 20f}{44 + f}$$

$$\Rightarrow 18(44 + f) = (752 + 20f)$$

$$\Rightarrow 792 + 18f = 752 + 20f$$

$$\Rightarrow 792 + 18f = 752 + 20f$$

$$\Rightarrow 792 - 752 = 20f - 18f$$

$$\Rightarrow 40 = 2f$$

$$\Rightarrow f = 20$$

So, the missing frequency, $f = 20$.

4. Thirty women were examined in a hospital by a doctor, and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

| Number of heart beats per minute | 65-68 | 68-71 | 71-74 | 74-77 | 77-80 | 80-83 | 83-86 |
|----------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Number of women | 2 | 4 | 3 | 8 | 7 | 4 | 2 |

Solution:

From the given data, let us assume the mean as $a = 75.5$

$$x_i = (\text{Upper limit} + \text{Lower limit})/2$$

$$\text{Class size (h)} = 3$$

Now, find the u_i and $f_i u_i$ as follows:

| Class Interval | Number of women (f_i) | Mid-point (x_i) | $u_i = (x_i - 75.5)/h$ | $f_i u_i$ |
|----------------|---------------------------|---------------------|------------------------|-------------------|
| 65-68 | 2 | 66.5 | -3 | -6 |
| 68-71 | 4 | 69.5 | -2 | -8 |
| 71-74 | 3 | 72.5 | -1 | -3 |
| 74-77 | 8 | 75.5 = a | 0 | 0 |
| 77-80 | 7 | 78.5 | 1 | 7 |
| 80-83 | 4 | 81.5 | 2 | 8 |
| 83-86 | 2 | 84.5 | 3 | 6 |
| Sum $f_i = 30$ | | | | Sum $f_i u_i = 4$ |

$$\text{Mean} = \bar{x} = a + h(\sum f_i u_i / \sum f_i)$$

$$= 75.5 + 3 \times (4/30)$$

$$= 75.5 + (4/10)$$

$$= 75.5 + 0.4$$

$$= 75.9$$

Therefore, the mean heart beats per minute for these women is 75.9

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

| Number of mangoes | 50-52 | 53-55 | 56-58 | 59-61 | 62-64 |
|-------------------|-------|-------|-------|-------|-------|
| Number of boxes | 15 | 110 | 135 | 115 | 25 |

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Solution:

The given data is not continuous, so we add 0.5 to the upper limit and subtract 0.5 from the lower limit as the gap between two intervals is 1.

Here, assumed mean (a) = 57

Class size (h) = 3

Here, the step deviation is used because the frequency values are big.

| Class Interval | Number of boxes (f_i) | Mid-point (x_i) | $u_i = (x_i - 57)/h$ | $f_i u_i$ |
|----------------|---------------------------|---------------------|----------------------|--------------------|
| 49.5-52.5 | 15 | 51 | -2 | -30 |
| 52.5-55.5 | 110 | 54 | -1 | -110 |
| 55.5-58.5 | 135 | 57 = a | 0 | 0 |
| 58.5-61.5 | 115 | 60 | 1 | 115 |
| 61.5-64.5 | 25 | 63 | 2 | 50 |
| | Sum $f_i = 400$ | | | Sum $f_i u_i = 25$ |

The formula to find out the Mean is:

$$\text{Mean} = \bar{x} = a + h(\sum f_i u_i / \sum f_i)$$

$$= 57 + 3(25/400)$$

$$= 57 + 0.1875$$

$$= 57.19$$

Therefore, the mean number of mangoes kept in a packing box is 57.19

6. The table below shows the daily expenditure on food of 25 households in a locality.

| Daily expenditure(in c) | 100-150 | 150-200 | 200-250 | 250-300 | 300-350 |
|-------------------------|---------|---------|---------|---------|---------|
| Number of households | 4 | 5 | 12 | 2 | 2 |

Find the mean daily expenditure on food by a suitable method.

Solution:

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

Let us assume the mean (a) = 225

Class size (h) = 50

| Class Interval | Number of households (f_i) | Mid-point (x_i) | $d_i = x_i - A$ | $u_i = d_i/50$ | $f_i u_i$ |
|----------------|--------------------------------|---------------------|-----------------|----------------|--------------------|
| 100-150 | 4 | 125 | -100 | -2 | -8 |
| 150-200 | 5 | 175 | -50 | -1 | -5 |
| 200-250 | 12 | 225 = a | 0 | 0 | 0 |
| 250-300 | 2 | 275 | 50 | 1 | 2 |
| 300-350 | 2 | 325 | 100 | 2 | 4 |
| | Sum $f_i = 25$ | | | | Sum $f_i u_i = -7$ |

$$\text{Mean} = \bar{x} = a + h(\sum f_i u_i / \sum f_i)$$

$$= 225 + 50(-7/25)$$

$$= 225 - 14$$

$$= 211$$

Therefore, the mean daily expenditure on food is 211.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

| Concentration of SO_2 (in ppm) | Frequency |
|--|-----------|
|--|-----------|

| | |
|-------------|---|
| 0.00 – 0.04 | 4 |
| 0.04 – 0.08 | 9 |
| 0.08 – 0.12 | 9 |
| 0.12 – 0.16 | 2 |
| 0.16 – 0.20 | 4 |
| 0.20 – 0.24 | 2 |

Find the mean concentration of SO₂ in the air.

Solution:

To find out the mean, first find the midpoint of the given frequencies as follows:

| Concentration of SO ₂ (in ppm) | Frequency (f _i) | Mid-point (x _i) | f _i x _i |
|---|-----------------------------|-----------------------------|---|
| 0.00-0.04 | 4 | 0.02 | 0.08 |
| 0.04-0.08 | 9 | 0.06 | 0.54 |
| 0.08-0.12 | 9 | 0.10 | 0.90 |
| 0.12-0.16 | 2 | 0.14 | 0.28 |
| 0.16-0.20 | 4 | 0.18 | 0.72 |
| 0.20-0.24 | 2 | 0.22 | 0.44 |
| Total | Sum f _i = 30 | | Sum (f _i x _i) = 2.96 |

The formula to find out the mean is

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 2.96/30$$

$$= 0.099 \text{ ppm}$$

Therefore, the mean concentration of SO₂ in the air is 0.099 ppm.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

| Number of days | 0-6 | 6-10 | 10-14 | 14-20 | 20-28 | 28-38 | 38-40 |
|--------------------|-----|------|-------|-------|-------|-------|-------|
| Number of students | 11 | 10 | 7 | 4 | 4 | 3 | 1 |

Solution:

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

| Class interval | Frequency (f_i) | Mid-point (x_i) | $f_i x_i$ |
|----------------|---------------------|---------------------|---------------------|
| 0-6 | 11 | 3 | 33 |
| 6-10 | 10 | 8 | 80 |
| 10-14 | 7 | 12 | 84 |
| 14-20 | 4 | 17 | 68 |
| 20-28 | 4 | 24 | 96 |
| 28-38 | 3 | 33 | 99 |
| 38-40 | 1 | 39 | 39 |
| | Sum $f_i = 40$ | | Sum $f_i x_i = 499$ |

The mean formula is,

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 499/40$$

$$= 12.48 \text{ days}$$

Therefore, the mean number of days a student was absent = 12.48.

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

| Literacy rate (in %) | 45-55 | 55-65 | 65-75 | 75-85 | 85-98 |
|----------------------|-------|-------|-------|-------|-------|
| Number of cities | 3 | 10 | 11 | 8 | 3 |

Solution:

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

In this case, the value of mid-point (x_i) is very large, so let us assume the mean value, $a = 70$.

Class interval (h) = 10

So, $u_i = (x_i - a)/h$

$u_i = (x_i - 70)/10$

Substitute and find the values as follows:

| Class Interval | Frequency (f_i) | (x_i) | $u_i = (x_i - 70)/10$ | $f_i u_i$ |
|----------------|---------------------|-----------|-----------------------|--------------------|
| 45-55 | 3 | 50 | -2 | -6 |
| 55-65 | 10 | 60 | -1 | -10 |
| 65-75 | 11 | 70 = a | 0 | 0 |
| 75-85 | 8 | 80 | 1 | 8 |
| 85-95 | 3 | 90 | 2 | 6 |
| | Sum $f_i = 35$ | | | Sum $f_i u_i = -2$ |

So, Mean = $\bar{x} = a + (\sum f_i u_i / \sum f_i) \times h$

= $70 + (-2/35) \times 10$

= 69.43

Therefore, the mean literacy part = 69.43%

EXERCISE 14.2

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1. The following table shows the ages of the patients admitted to a hospital during a year:

| Age (in years) | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
|--------------------|------|-------|-------|-------|-------|-------|
| Number of patients | 6 | 11 | 21 | 23 | 14 | 5 |

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Solution:

To find out the modal class, let us consider the class interval with high frequency.

Here, the greatest frequency = 23, so the modal class = 35 – 45,

Lower limit of modal class = $l = 35$,

class width (h) = 10,

$f_m = 23$,

$f_1 = 21$ and $f_2 = 14$

The formula to find the mode is

$$\text{Mode} = l + [(f_m - f_1) / (2f_m - f_1 - f_2)] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 35 + [(23 - 21) / (46 - 21 - 14)] \times 10$$

$$= 35 + (20/11)$$

$$= 35 + 1.8$$

$$= 36.8 \text{ years}$$

So the mode of the given data = 36.8 years

Calculation of Mean:

First find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit})/2$

| Class Interval | Frequency (f_i) | Mid-point (x_i) | $f_i x_i$ |
|----------------|---------------------|---------------------|-----------|
| 5-15 | 6 | 10 | 60 |

| | | | |
|-------|----------------|----|----------------------|
| 15-25 | 11 | 20 | 220 |
| 25-35 | 21 | 30 | 630 |
| 35-45 | 23 | 40 | 920 |
| 45-55 | 14 | 50 | 700 |
| 55-65 | 5 | 60 | 300 |
| | Sum $f_i = 80$ | | Sum $f_i x_i = 2830$ |

The mean formula is

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 2830/80$$

$$= 35.375 \text{ years}$$

Therefore, the mean of the given data = 35.375 years

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

| Lifetime (in hours) | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 |
|---------------------|------|-------|-------|-------|--------|---------|
| Frequency | 10 | 35 | 52 | 61 | 38 | 29 |

Determine the modal lifetimes of the components.

Solution:

From the given data the modal class is 60–80.

Lower limit of modal class = $l = 60$,

The frequencies are:

$$f_m = 61, f_1 = 52, f_2 = 38 \text{ and } h = 20$$

The formula to find the mode is

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 60 + \left[\frac{(61 - 52)}{(122 - 52 - 38)} \right] \times 20$$

$$\text{Mode} = 60 + [(9 \times 20)/32]$$

$$\text{Mode} = 60 + (45/8) = 60 + 5.625$$

Therefore, modal lifetime of the components = 65.625 hours.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

| Expenditure (in Rs.) | Number of families |
|----------------------|--------------------|
| 1000-1500 | 24 |
| 1500-2000 | 40 |
| 2000-2500 | 33 |
| 2500-3000 | 28 |
| 3000-3500 | 30 |
| 3500-4000 | 22 |
| 4000-4500 | 16 |
| 4500-5000 | 7 |

Solution:

Given data:

Modal class = 1500-2000,

$$l = 1500,$$

Frequencies:

$$f_m = 40 \quad f_1 = 24, \quad f_2 = 33 \text{ and}$$

$$h = 500$$

Mode formula:

$$\text{Mode} = l + [(f_m - f_1) / (2f_m - f_1 - f_2)] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 1500 + [(40 - 24) / (80 - 24 - 33)] \times 500$$

$$\text{Mode} = 1500 + [(16 \times 500) / 23]$$

$$\text{Mode} = 1500 + (8000 / 23) = 1500 + 347.83$$

Therefore, modal monthly expenditure of the families = Rupees 1847.83

Calculation for mean:

First find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit}) / 2$

Let us assume a mean, (a) be 2750.

| Class Interval | f_i | x_i | $d_i = x_i - a$ | $u_i = d_i / h$ | $f_i u_i$ |
|----------------|-------------|----------|-----------------|-----------------|-----------------|
| 1000-1500 | 24 | 1250 | -1500 | -3 | -72 |
| 1500-2000 | 40 | 1750 | -1000 | -2 | -80 |
| 2000-2500 | 33 | 2250 | -500 | -1 | -33 |
| 2500-3000 | 28 | 2750 = a | 0 | 0 | 0 |
| 3000-3500 | 30 | 3250 | 500 | 1 | 30 |
| 3500-4000 | 22 | 3750 | 1000 | 2 | 44 |
| 4000-4500 | 16 | 4250 | 1500 | 3 | 48 |
| 4500-5000 | 7 | 4750 | 2000 | 4 | 28 |
| | $f_i = 200$ | | | | $f_i u_i = -35$ |

The formula to calculate the mean,

$$\text{Mean} = \bar{x} = a + (\sum f_i u_i / \sum f_i) \times h$$

Substitute the values in the given formula

$$= 2750 + (-35 / 200) \times 500$$

$$= 2750 - 87.50$$

$$= 2662.50$$

So, the mean monthly expenditure of the families = Rs. 2662.50

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures

| No of students per teacher | Number of states / U.T |
|----------------------------|------------------------|
| 15-20 | 3 |
| 20-25 | 8 |
| 25-30 | 9 |
| 30-35 | 10 |
| 35-40 | 3 |
| 40-45 | 0 |
| 45-50 | 0 |
| 50-55 | 2 |

Solution:

Given data:

Modal class = 30 – 35,

$l = 30$,

Class width (h) = 5,

$f_m = 10$, $f_1 = 9$ and $f_2 = 3$

Mode Formula:

$$\text{Mode} = l + [(f_m - f_1) / (2f_m - f_1 - f_2)] \times h$$

Substitute the values in the given formula

$$\text{Mode} = 30 + [(10 - 9) / (20 - 9 - 3)] \times 5$$

$$= 30 + (5/8)$$

$$= 30 + 0.625$$

$$= 30.625$$

Therefore, the mode of the given data = 30.625

Calculation of mean:

Find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit}) / 2$

| Class Interval | Frequency (f_i) | Mid-point (x_i) | $f_i x_i$ |
|----------------|---------------------|---------------------|------------------------|
| 15-20 | 3 | 17.5 | 52.5 |
| 20-25 | 8 | 22.5 | 180.0 |
| 25-30 | 9 | 27.5 | 247.5 |
| 30-35 | 10 | 32.5 | 325.0 |
| 35-40 | 3 | 37.5 | 112.5 |
| 40-45 | 0 | 42.5 | 0 |
| 45-50 | 0 | 47.5 | 0 |
| 50-55 | 2 | 52.5 | 105.0 |
| | Sum $f_i = 35$ | | Sum $f_i x_i = 1022.5$ |

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 1022.5/35$$

$$= 29.2 \text{ (approx)}$$

Therefore, mean = 29.2

5. The given distribution shows the number of runs scored by some top batsmen of the world in one- day international cricket matches.

| Run Scored | Number of Batsman |
|------------|-------------------|
| 3000-4000 | 4 |
| 4000-5000 | 18 |
| 5000-6000 | 9 |
| 6000-7000 | 7 |
| 7000-8000 | 6 |
| 8000-9000 | 3 |
| 9000-10000 | 1 |

10000-11000

1

Find the mode of the data.

Solution:

Given data:

Modal class = 4000 – 5000,

$l = 4000$,

class width (h) = 1000,

$f_m = 18$, $f_1 = 4$ and $f_2 = 9$

Mode Formula:

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values

$$\text{Mode} = 4000 + \left[\frac{(18 - 4)}{(36 - 4 - 9)} \right] \times 1000$$

$$= 4000 + (14000/23)$$

$$= 4000 + 608.695$$

$$= 4608.695$$

$$= 4608.7 \text{ (approximately)}$$

Thus, the mode of the given data is 4608.7 runs.

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data:

| Number of cars | Frequency |
|----------------|-----------|
| 0-10 | 7 |
| 10-20 | 14 |
| 20-30 | 13 |
| 30-40 | 12 |
| 40-50 | 20 |

| | |
|-------|----|
| 50-60 | 11 |
| 60-70 | 15 |
| 70-80 | 8 |

Solution:

Given Data:

Modal class = 40 – 50, $l = 40$,

Class width (h) = 10, $f_m = 20$, $f_1 = 12$ and $f_2 = 11$

$$\text{Mode} = l + [(f_m - f_1) / (2f_m - f_1 - f_2)] \times h$$

Substitute the values

$$\text{Mode} = 40 + [(20 - 12) / (40 - 12 - 11)] \times 10$$

$$= 40 + (80/17)$$

$$= 40 + 4.7$$

$$= 44.7$$

Thus, the mode of the given data is 44.7 cars.

EXERCISE 14.3

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1. The following frequency distribution gives the monthly consumption of an electricity of 68 consumers in a locality. Find the median, mean and mode of the data and compare them.

| Monthly consumption(in units) | No. of customers |
|-------------------------------|------------------|
| 65-85 | 4 |
| 85-105 | 5 |
| 105-125 | 13 |
| 125-145 | 20 |
| 145-165 | 14 |
| 165-185 | 8 |
| 185-205 | 4 |

Solution:

Find the cumulative frequency of the given data as follows:

| Class Interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 65-85 | 4 | 4 |
| 85-105 | 5 | 9 |
| 105-125 | 13 | 22 |
| 125-145 | 20 | 42 |
| 145-165 | 14 | 56 |
| 165-185 | 8 | 64 |
| 185-205 | 4 | 68 |
| | N = 68 | |

From the table, it is observed that, $N = 68$ and hence $N/2 = 34$

Hence, the median class is 125-145 with cumulative frequency = 42

Where, $l = 125$, $N = 68$, $cf = 22$, $f = 20$, $h = 20$

Median is calculated as follows:

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 125 + [(34 - 22)/20] \times 20$$

$$= 125 + 12$$

$$= 137$$

Therefore, median = 137

To calculate the mode:

Modal class = 125-145,

f_m or $f_1 = 20$, $f_0 = 13$, $f_2 = 14$ & $h = 20$

Mode formula:

$$\text{Mode} = l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$$

$$\text{Mode} = 125 + [(20 - 13)/(40 - 13 - 14)] \times 20$$

$$= 125 + (140/13)$$

$$= 125 + 10.77$$

$$= 135.77$$

Therefore, mode = 135.77

Calculate the Mean:

| Class Interval | f_i | x_i | $d_i = x_i - a$ | $u_i = d_i/h$ | $f_i u_i$ |
|----------------|-------|---------|-----------------|---------------|-----------|
| 65-85 | 4 | 75 | -60 | -3 | -12 |
| 85-105 | 5 | 95 | -40 | -2 | -10 |
| 105-125 | 13 | 115 | -20 | -1 | -13 |
| 125-145 | 20 | 135 = a | 0 | 0 | 0 |
| 145-165 | 14 | 155 | 20 | 1 | 14 |

| | | | | | |
|---------|----------------|-----|----|---|-------------------|
| 165-185 | 8 | 175 | 40 | 2 | 16 |
| 185-205 | 4 | 195 | 60 | 3 | 12 |
| | Sum $f_i = 68$ | | | | Sum $f_i u_i = 7$ |

$$\bar{x} = a + h (\sum f_i u_i / \sum f_i) = 135 + 20 (7/68)$$

Mean = 137.05

In this case, mean, median and mode are more/less equal in this distribution.

2. If the median of a distribution given below is 28.5, find the value of x & y.

| Class Interval | Frequency |
|----------------|-----------|
| 0-10 | 5 |
| 10-20 | x |
| 20-30 | 20 |
| 30-40 | 15 |
| 40-50 | y |
| 50-60 | 5 |
| Total | 60 |

Solution:

Given data, $n = 60$

Median of the given data = 28.5

| CI | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|----------------------|------|-------|-------|-------|--------|--------|
| Frequency | 5 | x | 20 | 15 | y | 5 |
| Cumulative frequency | 5 | 5+x | 25+x | 40+x | 40+x+y | 45+x+y |

Where, $N/2 = 30$

Median class is 20 – 30 with a cumulative frequency = $25 + x$

Lower limit of median class, $l = 20$,

$cf = 5 + x$,

$$f = 20 \text{ \& h = 10}$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Substitute the values

$$28.5 = 20 + [(30 - 5 - x)/20] \times 10$$

$$8.5 = (25 - x)/2$$

$$17 = 25 - x$$

Therefore, $x = 8$.

Now, from cumulative frequency, we can identify the value of $x + y$ as follows:

Since,

$$60 = 45 + x + y$$

Now, substitute the value of x , to find y

$$60 = 45 + 8 + y$$

$$y = 60 - 53$$

$$y = 7$$

Therefore, the value of $x = 8$ and $y = 7$.

3. The life insurance agent found the following data for the distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

| Age (in years) | Number of policy holder |
|----------------|-------------------------|
| Below 20 | 2 |
| Below 25 | 6 |
| Below 30 | 24 |
| Below 35 | 45 |
| Below 40 | 78 |
| Below 45 | 89 |

| | |
|----------|-----|
| Below 50 | 92 |
| Below 55 | 98 |
| Below 60 | 100 |

Solution:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 15-20 | 2 | 2 |
| 20-25 | 4 | 6 |
| 25-30 | 18 | 24 |
| 30-35 | 21 | 45 |
| 35-40 | 33 | 78 |
| 40-45 | 11 | 89 |
| 45-50 | 3 | 92 |
| 50-55 | 6 | 98 |
| 55-60 | 2 | 100 |

Given data: $N = 100$ and $N/2 = 50$

Median class = 35-40

Then, $l = 35$, $cf = 45$, $f = 33$ & $h = 5$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\text{Median} = 35 + [(50 - 45)/33] \times 5$$

$$= 35 + (25/33)$$

$$= 35.76$$

Therefore, the median age = 35.76 years.

4. The lengths of 40 leaves in a plant are measured correctly to the nearest millimeter, and the data obtained is represented as in the following table:

| Length (in mm) | Number of leaves |
|----------------|------------------|
| 118-126 | 3 |
| 127-135 | 5 |
| 136-144 | 9 |
| 145-153 | 12 |
| 154-162 | 5 |
| 163-171 | 4 |
| 172-180 | 2 |

Find the median length of the leaves.

(Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5, . . . , 171.5 – 180.5.)

Solution:

Since the data are not continuous reduce 0.5 in the lower limit and add 0.5 in the upper limit.

| Class Interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 117.5-126.5 | 3 | 3 |
| 126.5-135.5 | 5 | 8 |
| 135.5-144.5 | 9 | 17 |
| 144.5-153.5 | 12 | 29 |
| 153.5-162.5 | 5 | 34 |
| 162.5-171.5 | 4 | 38 |
| 171.5-180.5 | 2 | 40 |

So, the data obtained are:

$$N = 40 \text{ and } N/2 = 20$$

Median class = 144.5-153.5

then, $l = 144.5$,

$$cf = 17, f = 12 \text{ \& } h = 9$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\text{Median} = 144.5 + [(20 - 17) / 12] \times 9$$

$$= 144.5 + (9/4)$$

$$= 146.75 \text{ mm}$$

Therefore, the median length of the leaves = 146.75 mm.

5. The following table gives the distribution of a lifetime of 400 neon lamps.

| Lifetime (in hours) | Number of lamps |
|---------------------|-----------------|
| 1500-2000 | 14 |
| 2000-2500 | 56 |
| 2500-3000 | 60 |
| 3000-3500 | 86 |
| 3500-4000 | 74 |
| 4000-4500 | 62 |
| 4500-5000 | 48 |

Find the median lifetime of a lamp.

Solution:

| Class Interval | Frequency | Cumulative |
|----------------|-----------|------------|
| 1500-2000 | 14 | 14 |
| 2000-2500 | 56 | 70 |
| 2500-3000 | 60 | 130 |
| 3000-3500 | 86 | 216 |
| 3500-4000 | 74 | 290 |

| | | |
|-----------|----|-----|
| 4000-4500 | 62 | 352 |
| 4500-5000 | 48 | 400 |

Data:

$$N = 400 \text{ \& } N/2 = 200$$

$$\text{Median class} = 3000 - 3500$$

$$\text{Therefore, } l = 3000, cf = 130,$$

$$f = 86 \text{ \& } h = 500$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\text{Median} = 3000 + [(200 - 130)/86] \times 500$$

$$= 3000 + (35000/86)$$

$$= 3000 + 406.98$$

$$= 3406.98$$

Therefore, the median lifetime of the lamps = 3406.98 hours

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

| Number of letters | 1-4 | 4-7 | 7-10 | 10-13 | 13-16 | 16-19 |
|--------------------|-----|-----|------|-------|-------|-------|
| Number of surnames | 6 | 30 | 40 | 16 | 4 | 4 |

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Solution:

To calculate median:

| Class Interval | Frequency | Cumulative Frequency |
|----------------|-----------|----------------------|
| 1-4 | 6 | 6 |
| 4-7 | 30 | 36 |
| 7-10 | 40 | 76 |

| | | |
|-------|----|-----|
| 10-13 | 16 | 92 |
| 13-16 | 4 | 96 |
| 16-19 | 4 | 100 |

Given:

$$N = 100 \text{ \& } N/2 = 50$$

$$\text{Median class} = 7-10$$

$$\text{Therefore, } l = 7, cf = 36, f = 40 \text{ \& } h = 3$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\text{Median} = 7 + [(50 - 36)/40] \times 3$$

$$\text{Median} = 7 + (4/40)$$

$$\text{Median} = 8.05$$

Calculate the Mode:

$$\text{Modal class} = 7-10,$$

$$\text{Where, } l = 7, f_1 = 40, f_0 = 30, f_2 = 16 \text{ \& } h = 3$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$\text{Mode} = 7 + [(40 - 30)/(2 \times 40 - 30 - 16)] \times 3$$

$$= 7 + (30/34)$$

$$= 7.88$$

$$\text{Therefore mode} = 7.88$$

Calculate the Mean:

| Class Interval | f_i | x_i | $f_i x_i$ |
|----------------|-------|-------|-----------|
| 1-4 | 6 | 2.5 | 15 |
| 4-7 | 30 | 5.5 | 165 |

| | | | |
|-------|-----------------|------|---------------------|
| 7-10 | 40 | 8.5 | 340 |
| 10-13 | 16 | 11.5 | 184 |
| 13-16 | 4 | 14.5 | 58 |
| 16-19 | 4 | 17.5 | 70 |
| | Sum $f_i = 100$ | | Sum $f_i x_i = 832$ |

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\text{Mean} = 832/100 = 8.32$$

Therefore, mean = 8.32

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

| Weight(in kg) | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|--------------------|-------|-------|-------|-------|-------|-------|-------|
| Number of students | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

Solution:

| Class Interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 40-45 | 2 | 2 |
| 45-50 | 3 | 5 |
| 50-55 | 8 | 13 |
| 55-60 | 6 | 19 |
| 60-65 | 6 | 25 |
| 65-70 | 3 | 28 |
| 70-75 | 2 | 30 |

Given: $N = 30$ and $N/2 = 15$

Median class = 55-60

$l = 55$, $C_r = 13$, $f = 6$ & $h = 5$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\text{Median} = 55 + [(15 - 13)/6] \times 5$$

$$= 55 + (10/6)$$

$$= 55 + 1.666$$

$$= 56.67$$

Therefore, the median weight of the students = 56.67

EXERCISE 14.4

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1. The following distribution gives the daily income of 50 workers in a factory.

| Daily income (in Rs.) | 100-120 | 120-140 | 140-160 | 160-180 | 180-200 |
|-----------------------|---------|---------|---------|---------|---------|
| Number of workers | 12 | 14 | 8 | 6 | 10 |

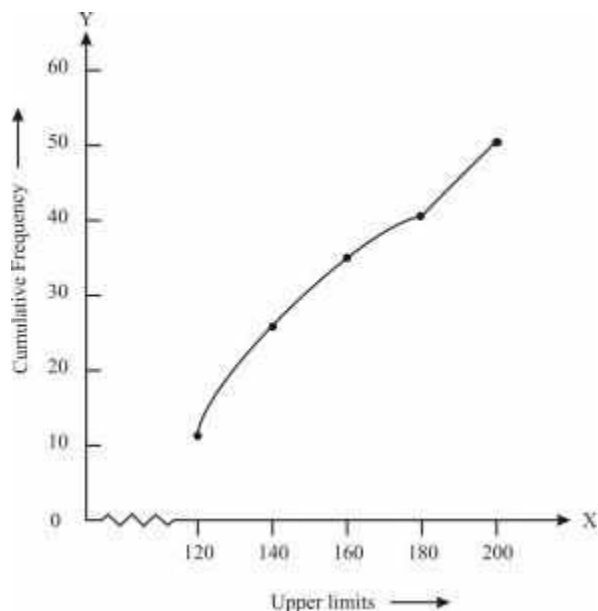
Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Solution

Convert the given distribution table to a less than type cumulative frequency distribution, and we get

| Daily Income | Cumulative Frequency (or) Number of workers |
|---------------|---|
| Less than 120 | 12 |
| Less than 140 | 26 |
| Less than 160 | 34 |
| Less than 180 | 40 |
| Less than 200 | 50 |

From the table plot the points corresponding to the ordered pairs such as (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50) on graph paper and the plotted points are joined to get a smooth curve and the obtained curve is known as less than type ogive curve



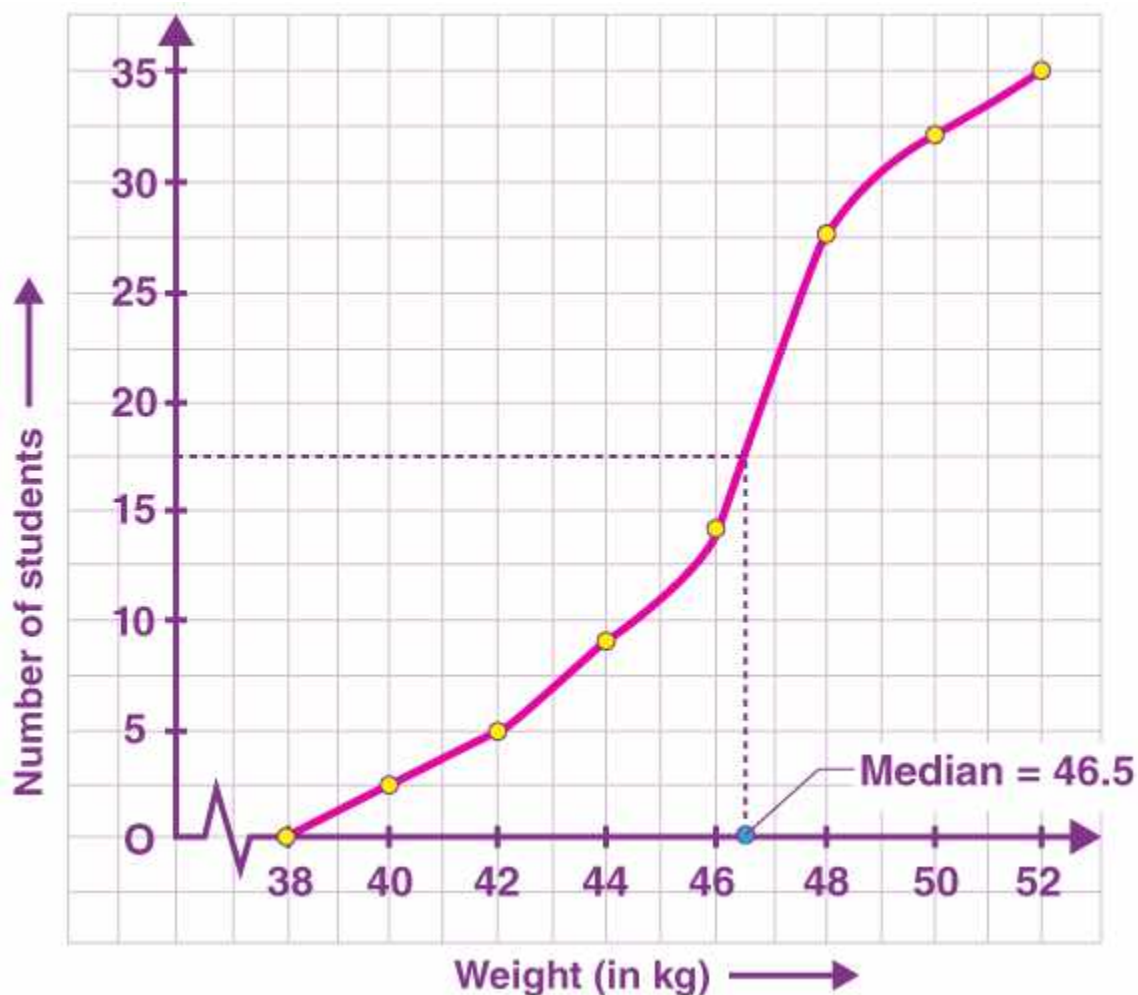
2. During the medical check-up of 35 students of a class, their weights were recorded as follows:

| Weight in kg | Number of students |
|--------------|--------------------|
| Less than 38 | 0 |
| Less than 40 | 3 |
| Less than 42 | 5 |
| Less than 44 | 9 |
| Less than 46 | 14 |
| Less than 48 | 28 |
| Less than 50 | 32 |
| Less than 52 | 35 |

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

Solution:

From the given data, to represent the table in the form of graph, choose the upper limits of the class intervals are in x-axis and frequencies on y-axis by choosing the convenient scale. Now plot the points corresponding to the ordered pairs given by (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) on a graph paper and join them to get a smooth curve. The curve obtained is known as less than type ogive.



Locate the point 17.5 on the y-axis and draw a line parallel to the x-axis cutting the curve at a point. From the point, draw a perpendicular line to the x-axis. The intersection point perpendicular to x-axis is the median of the given data. Now, to find the median by making a table.

| | Class interval | Number of students(Frequency) | Cumulative Frequency |
|--------------|----------------|-------------------------------|----------------------|
| Less than 38 | 0 – 38 | 0 | 0 |
| Less than 40 | 38 – 40 | $3 - 0 = 3$ | 3 |
| Less than 42 | 40 – 42 | $5 - 3 = 2$ | 5 |

| | | | |
|--------------|---------|----------------|----|
| Less than 44 | 42 – 44 | $9 - 5 = 4$ | 9 |
| Less than 46 | 44 – 46 | $14 - 9 = 5$ | 14 |
| Less than 48 | 46 – 48 | $28 - 14 = 14$ | 28 |
| Less than 50 | 48 – 50 | $32 - 28 = 4$ | 32 |
| Less than 52 | 50 – 52 | $35 - 22 = 3$ | 35 |

Here, $N = 35$ and $N/2 = 35/2 = 17.5$

Median class = 46 – 48

Here, $l = 46$, $h = 2$, $cf = 14$, $f = 14$

The mode formula is given as:

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 46 + [(17.5 - 14)/ 14] \times 2$$

$$= 46 + 0.5$$

$$= 46 + 0.5 = 46.5$$

Thus, median is verified.

3. The following table gives production yield per hectare of wheat of 100 farms of a village.

| Production Yield (in kg/ha) | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 | 75-80 |
|-----------------------------|-------|-------|-------|-------|-------|-------|
| Number of Farms | 2 | 8 | 12 | 24 | 38 | 16 |

Change the distribution to a more than type distribution and draw its ogive.

Solution:

Converting the given distribution to a more than type distribution, we get

| Production Yield (kg/ha) | Number of farms |
|--------------------------|-----------------|
|--------------------------|-----------------|

| | |
|--------------------------|----------------|
| More than or equal to 50 | 100 |
| More than or equal to 55 | $100 - 2 = 98$ |
| More than or equal to 60 | $98 - 8 = 90$ |
| More than or equal to 65 | $90 - 12 = 78$ |
| More than or equal to 70 | $78 - 24 = 54$ |
| More than or equal to 75 | $54 - 38 = 16$ |

From the table obtained draw the ogive by plotting the corresponding points where the upper limits in x-axis and the frequencies obtained in the y-axis are (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16) on the graph paper. The graph obtained is known as more than type ogive curve.

