EXERCISE 14.3   **PAGE: 287**

1. The following frequency distribution gives the monthly consumption of an electricity of 68 consumers in a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption(in units)	No. of customers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

Solution:

Find the cumulative frequency of the given data as follows:

Class Interval	Frequency	Cumulative frequency
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68
	N = 68	

From the table, it is observed that, N = 68 and hence N/2=34

Hence, the median class is 125-145 with cumulative frequency = 42

Where, $l = 125$, $N = 68$, $cf = 22$, $f = 20$, $h = 20$

Median is calculated as follows:

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$= 125 + [(34 - 22)/20] \times 20$

$= 125 + 12$

$= 137$

Therefore, median = 137

To calculate the mode:

Modal class = 125-145,

f_m or $f_1 = 20$, $f_0 = 13$, $f_2 = 14$ & $h = 20$

Mode formula:

Mode $= l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$

Mode $= 125 + [(20 - 13)/(40 - 13 - 14)] \times 20$

$= 125 + (140/13)$

$= 125 + 10.77$

$= 135.77$

Therefore, mode = 135.77

Calculate the Mean:

Class Interval	f_i	x_i	$d_i = x_i - a$	$u_i = d_i/h$	$f_i u_i$
65-85	4	75	-60	-3	-12
85-105	5	95	-40	-2	-10
105-125	13	115	-20	-1	-13
125-145	20	135 = a	0	0	0
145-165	14	155	20	1	14

165-185	8	175	40	2	16
185-205	4	195	60	3	12
	Sum $f_i = 68$				Sum $f_iu_i = 7$

$\bar{x} = a + h \left(\sum f_iu_i / \sum f_i \right) = 135 + 20 (7/68)$

Mean = 137.05

In this case, mean, median and mode are more/less equal in this distribution.

2. If the median of a distribution given below is 28.5, find the value of x & y.

Class Interval	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Solution:

Given data, n = 60

Median of the given data = 28.5

CI	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	20	15	y	5
Cumulative frequency	5	5+x	25+x	40+x	40+x+y	45+x+y

Where, N/2 = 30

Median class is 20 – 30 with a cumulative frequency = 25 + x

Lower limit of median class, $l = 20$,

cf = 5 + x,

f = 20 & h = 10

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Substitute the values

28.5 = 20 + [(30 − 5 − x)/20] × 10

8.5 = (25 − x)/2

17 = 25 − x

Therefore, x = 8.

Now, from cumulative frequency, we can identify the value of x + y as follows:

Since,

60 = 45 + x + y

Now, substitute the value of x, to find y

60 = 45 + 8 + y

y = 60 − 53

y = 7

Therefore, the value of x = 8 and y = 7.

3. The life insurance agent found the following data for the distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

Age (in years)	Number of policy holder
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89

Below 50	92
Below 55	98
Below 60	100

Solution:

Class interval	Frequency	Cumulative frequency
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

Given data: N = 100 and N/2 = 50

Median class = 35-40

Then, l = 35, cf = 45, f = 33 & h = 5

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Median = 35 + [(50 – 45)/33] × 5

= 35 + (25/33)

= 35.76

Therefore, the median age = 35.76 years.

4. The lengths of 40 leaves in a plant are measured correctly to the nearest millimeter, and the data obtained is represented as in the following table:

Length (in mm)	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Find the median length of the leaves.

(Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5, . . ., 171.5 – 180.5.)

Solution:

Since the data are not continuous reduce 0.5 in the lower limit and add 0.5 in the upper limit.

Class Interval	Frequency	Cumulative frequency
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	29
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40

So, the data obtained are:

N = 40 and N/2 = 20

Median class = 144.5-153.5

then, l = 144.5,

cf = 17, f = 12 & h = 9

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Median = 144.5 + [(20 – 17)/ 12] × 9

= 144.5 + (9/4)

= 146.75 mm

Therefore, the median length of the leaves = 146.75 mm.

5. The following table gives the distribution of a lifetime of 400 neon lamps.

Lifetime (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median lifetime of a lamp.

Solution:

Class Interval	Frequency	Cumulative
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290

4000-4500	62	352
4500-5000	48	400

Data:

N = 400 & N/2 = 200

Median class = 3000 – 3500

Therefore, l = 3000, cf = 130,

f = 86 & h = 500

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Median = 3000 + [(200 – 130)/86] × 500

= 3000 + (35000/86)

= 3000 + 406.98

= 3406.98

Therefore, the median lifetime of the lamps = 3406.98 hours

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Solution:

To calculate median:

Class Interval	Frequency	Cumulative Frequency
1-4	6	6
4-7	30	36
7-10	40	76

10-13	16	92
13-16	4	96
16-19	4	100

Given:

N = 100 & N/2 = 50

Median class = 7-10

Therefore, $l = 7$, cf = 36, f = 40 & h = 3

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Median = 7 + [(50 – 36)/40] × 3

Median = 7 + (42/40)

Median = 8.05

Calculate the Mode:

Modal class = 7-10,

Where, $l = 7$, $f_1 = 40$, $f_0 = 30$, $f_2 = 16$ & h = 3

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Mode = 7 + [(40 – 30)/(2 × 40 – 30 – 16)] × 3

= 7 + (30/34)

= 7.88

Therefore mode = 7.88

Calculate the Mean:

Class Interval	f_i	x_i	$f_i x_i$
1-4	6	2.5	15
4-7	30	5.5	165

7-10	40	8.5	340
10-13	16	11.5	184
13-16	4	14.5	58
16-19	4	17.5	70
	Sum f_i = 100		Sum f_ix_i = 832

Mean = $\bar{x}$ = $\sum f_i x_i / \sum f_i$

Mean = 832/100 = 8.32

Therefore, mean = 8.32

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight(in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Solution:

Class Interval	Frequency	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Given: N = 30 and N/2= 15

Median class = 55-60

l = 55, C_f = 13, f = 6 & h = 5

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Median = 55 + [(15 – 13)/6] × 5

= 55 + (10/6)

= 55 + 1.666

= 56.67

Therefore, the median weight of the students = 56.67