EXERCISE 6.4 PAGE: 143

1. Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

Solution: Given, ΔABC ~ ΔDEF,

Area of ΔABC = 64 cm²

Area of ΔDEF = 121 cm²

EF = 15.4 cm

$$\therefore \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{AB^2}{DE^2}$$

As we know, if two triangles are similar, ratio of their areas are equal to the square of the ratio of their corresponding sides,

$= AC^2/DF^2 = BC^2/EF^2$

$\therefore 64/121 = BC^2/EF^2$

$\Rightarrow (8/11)^2 = (BC/15.4)^2$

$\Rightarrow 8/11 = BC/15.4$

$\Rightarrow BC = 8\times15.4/11$

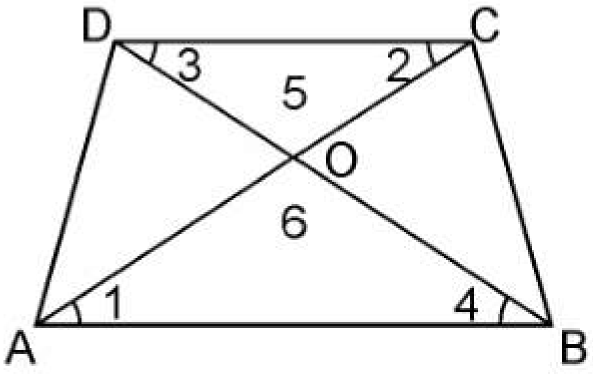
$\Rightarrow BC = 8 \times 1.4$

$\Rightarrow BC = 11.2$ cm

2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Solution:

Given, ABCD is a trapezium with AB || DC. Diagonals AC and BD intersect each other at point O.



In ΔAOB and ΔCOD, we have

∠1 = ∠2 (Alternate angles)

∠3 = ∠4 (Alternate angles)

∠5 = ∠6 (Vertically opposite angle)

∴ $\Delta AOB \sim \Delta COD$ [AAA similarity criterion]

As we know, If two triangles are similar then the ratio of their areas are equal to the square of the ratio of their corresponding sides. Therefore,

Area of (ΔAOB)/Area of (ΔCOD) = AB^2/CD^2

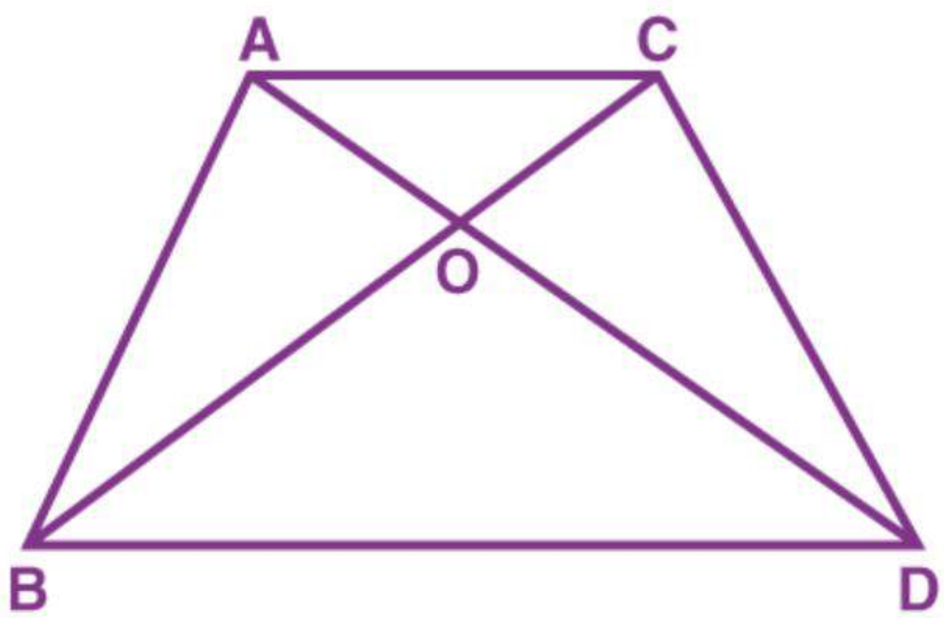
= $(2CD)^2/CD^2$ [∴ AB = 2CD]

∴ Area of (ΔAOB)/Area of (ΔCOD)

= $4CD^2/CD^2 = 4/1$

Hence, the required ratio of the area of ΔAOB and ΔCOD = 4:1

3. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that area (ΔABC)/area (ΔDBC) = AO/DO.

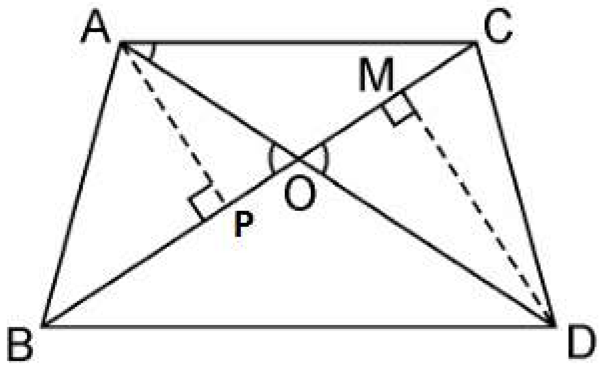


Solution:

Given, ABC and DBC are two triangles on the same base BC. AD intersects BC at O.

We have to prove: Area (ΔABC)/Area (ΔDBC) = AO/DO

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = 1/2 × Base × Height

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}BC \times AP}{\frac{1}{2}BC \times DM} = \frac{AP}{DM}$$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (AA similarity criterion)

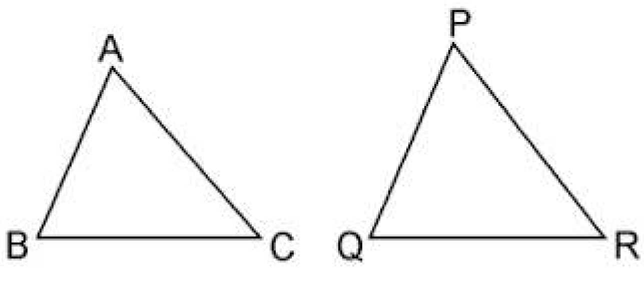
∴ AP/DM = AO/DO

⇒ Area (ΔABC)/Area (ΔDBC) = AO/DO.

4. If the areas of two similar triangles are equal, prove that they are congruent.

Solution:

Say ΔABC and ΔPQR are two similar triangles and equal in area



Now let us prove ΔABC ≅ ΔPQR.

Since, ΔABC ~ ΔPQR

∴ Area of (ΔABC)/Area of (ΔPQR) = BC^2/QR^2

⇒ BC^2/QR^2 =1 [Since, Area(ΔABC) = (ΔPQR)

⇒ BC^2/QR^2

⇒ BC = QR

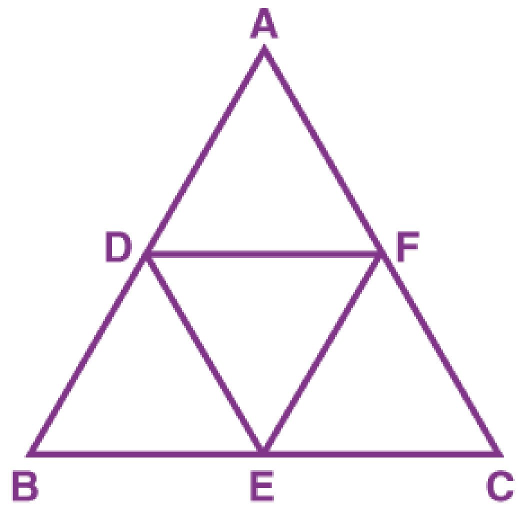
Similarly, we can prove that

AB = PQ and AC = PR

Thus, ΔABC ≅ ΔPQR [SSS criterion of congruence]

5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Solution:

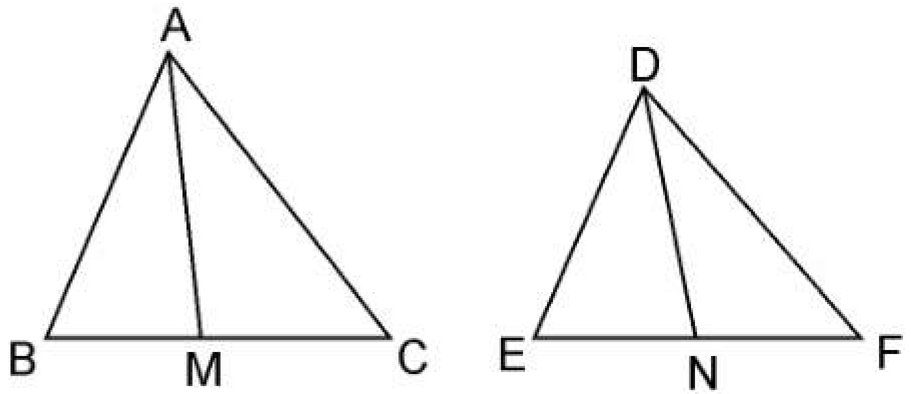


D, E, and F are the mid-points of ΔABC
∴ DE || AC and
DE = (1/2) AC (Midpoint theorem) …. (1)
In ΔBED and ΔBCA
∠BED = ∠BCA (Corresponding angles)
∠BDE = ∠BAC (Corresponding angles)
∠EBD = ∠CBA (Common angles)
∴ΔBED~ΔBCA (AAA similarity criterion)
ar (ΔBED) / ar (ΔBCA)=$(DE/AC)^2$
⇒ar (ΔBED) / ar (ΔBCA) = (1/4) [From (1)]
⇒ar (ΔBED) = (1/4) ar (ΔBCA)
Similarly,
ar (ΔCFE) = (1/4) ar (CBA) and ar (ΔADF) = (1/4) ar (ΔADF) = (1/4) ar (ΔABC)
Also,
ar (ΔDEF) = ar (ΔABC) − [ar (ΔBED) + ar (ΔCFE) + ar (ΔADF)]
⇒ar (ΔDEF) = ar (ΔABC) − (3/4) ar (ΔABC) = (1/4) ar (ΔABC)
⇒ar (ΔDEF) / ar (ΔABC) = (1/4)

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:

Given: AM and DN are the medians of triangles ABC and DEF respectively and ΔABC ~ ΔDEF.



We have to prove: Area(ΔABC)/Area(ΔDEF) = AM^2/DN^2

Since, ΔABC ~ ΔDEF (Given)

∴ Area(ΔABC)/Area(ΔDEF) = (AB^2/DE^2) ……………………………**(i)**

and, AB/DE = BC/EF = CA/FD ………………………………………**(ii)**

$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{CD}{FD}$$

In ΔABM and ΔDEN,

Since ΔABC ~ ΔDEF

∴ ∠B = ∠E

AB/DE = BM/EN [Already Proved in equation **(i)**]

∴ ΔABC ~ ΔDEF [SAS similarity criterion]

⇒ AB/DE = AM/DN …………………………………………………..**(iii)**

∴ ΔABM ~ ΔDEN

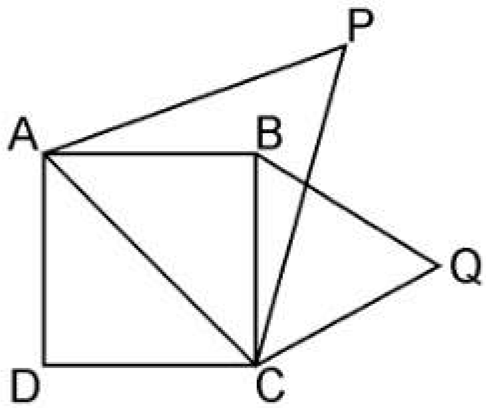
As the areas of two similar triangles are proportional to the squares of the corresponding sides.

∴ area(ΔABC)/area(ΔDEF) = AB^2/DE^2 = AM^2/DN^2

Hence, proved.

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution:



Given, ABCD is a square whose one diagonal is AC. ΔAPC and ΔBQC are two equilateral triangles described on the diagonals AC and side BC of the square ABCD.

Area(ΔBQC) = ½ Area(ΔAPC)

Since, ΔAPC and ΔBQC are both equilateral triangles, as per given,

∴ ΔAPC ~ ΔBQC [AAA similarity criterion]

∴ area(ΔAPC)/area(ΔBQC) = $(AC^2/BC^2) = AC^2/BC^2$

Since, Diagonal = √2 side = √2 BC = AC

$$\left(\frac{\sqrt{2}BC}{BC}\right)^2 = 2$$

⇒ area(ΔAPC) = 2 × area(ΔBQC)

⇒ area(ΔBQC) = 1/2area(ΔAPC)

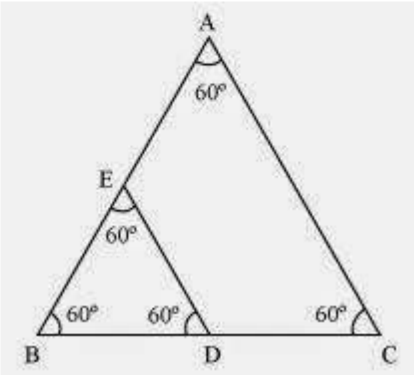
Hence, proved.

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is
(A) 2 : 1
(B) 1 : 2
(C) 4 : 1
(D) 1 : 4

Solution:

Given**,** ΔABC and ΔBDE are two equilateral triangle. D is the midpoint of BC.



∴ BD = DC = 1/2BC

Let each side of triangle is $2a$.

As, ΔABC ~ ΔBDE

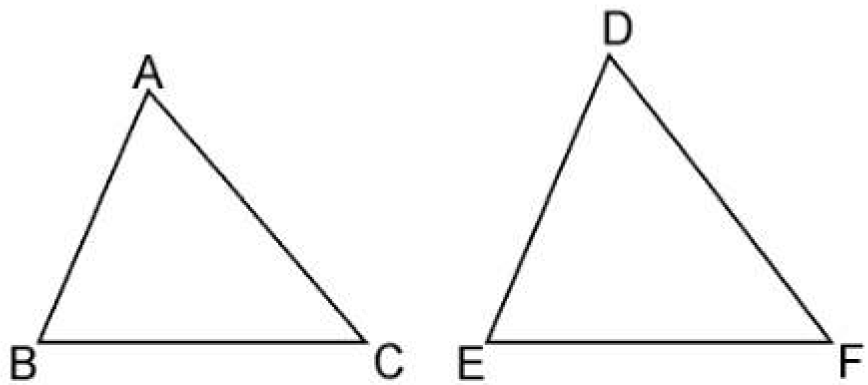
∴ Area(ΔABC)/Area(ΔBDE) = $AB^2/BD^2 = (2a)^2/(a)^2 = 4a^2/a^2 = 4/1 = 4:1$

Hence, the correct answer is (C).

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio
(A) 2 : 3
(B) 4 : 9
(C) 81 : 16
(D) 16 : 81

Solution:

Given, Sides of two similar triangles are in the ratio 4 : 9.



Let ABC and DEF are two similar triangles, such that,

ΔABC ~ ΔDEF

And AB/DE = AC/DF = BC/EF = 4/9

As, the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides,

∴ Area(ΔABC)/Area(ΔDEF) = AB^2/DE^2

∴ Area(ΔABC)/Area(ΔDEF) = $(4/9)^2 = 16/81 = 16:81$

Hence, the correct answer is (D).