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1. Find the area of the triangle whose vertices are:

(i) (2, 3), (-1, 0), (2, -4)

(ii) (-5, -1), (3, -5), (5, 2)

Solution:

Area of a triangle formula = $1/2 \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

(i) Here,

$x_1 = 2$, $x_2 = -1$, $x_3 = 2$, $y_1 = 3$, $y_2 = 0$ and $y_3 = -4$

Substitute all the values in the above formula, we get

Area of triangle = 1/2 [2 {0- (-4)} + (-1) {(-4) – (3)} + 2 (3 – 0)]

= 1/2 {8 + 7 + 6}

= 21/2

So, the area of the triangle is 21/2 square units.

(ii) Here,

$x_1 = -5$, $x_2 = 3$, $x_3 = 5$, $y_1 = -1$, $y_2 = -5$ and $y_3 = 2$

Area of the triangle = 1/2 [-5 { (-5)- (2)} + 3(2-(-1)) + 5{-1 – (-5)}]

= 1/2{35 + 9 + 20} = 32

Therefore, the area of the triangle is 32 square units.

2. In each of the following, find the value of 'k', for which the points are collinear.

(i) (7, -2), (5, 1), (3, -k)

(ii) (8, 1), (k, -4), (2, -5)

Solution:

(i) For collinear points, the area of triangle formed by them is always zero.

Let points (7, -2), (5, 1), and (3, k) are vertices of a triangle.

Area of triangle = 1/2 [7 { 1- k} + 5(k-(-2)) + 3{(-2) – 1}] = 0

7 – 7k + 5k +10 -9 = 0

-2k + 8 = 0

k = 4

(ii) For collinear points, the area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, – 4), and (2, – 5), area = 0

1/2 [8 { -4- (-5)} + k{(-5)-(1)} + 2{1 -(-4)}] = 0

8 – 6k + 10 = 0

6k = 18

k = 3

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Solution:

Let the vertices of the triangle be A (0, -1), B (2, 1), and C (0, 3).

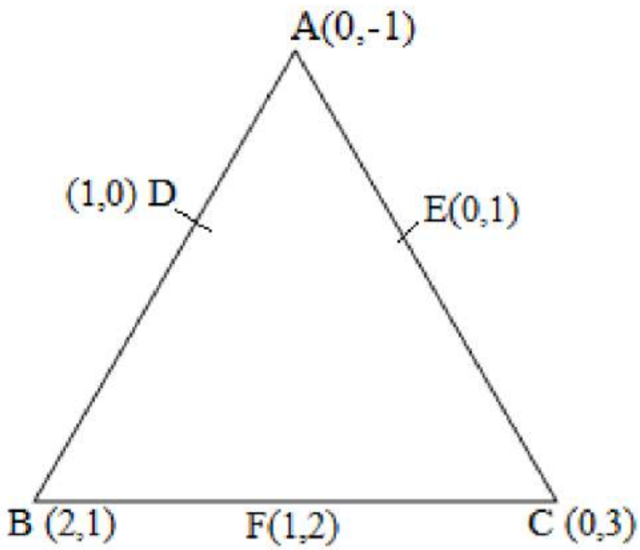
Let D, E, and F be the mid-points of the sides of this triangle.

Coordinates of D, E, and F are given by

D = (0+2/2, -1+1/2) = (1, 0)

E = (0+0/2, -1+3/2) = (0, 1)

F = (0+2/2, 3+1/2) = (1, 2)



Area of a triangle = 1/2 × $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔDEF = 1/2 {1(2-1) + 1(1-0) + 0(0-2)} = 1/2 (1+1) = 1

The area of ΔDEF is 1 square unit

Area of ΔABC = 1/2 [0(1-3) + 2{3-(-1)} + 0(-1-1)] = 1/2 {8} = 4

The area of ΔABC is 4 square units

Therefore, the required ratio is 1:4.

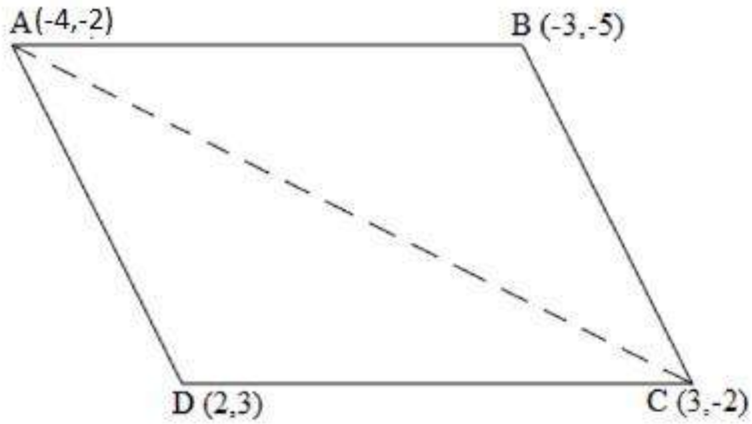
4. Find the area of the quadrilateral whose vertices, taken in order, are

(-4, -2), (-3, -5), (3, -2) and (2, 3).

Solution:

Let the vertices of the quadrilateral be A (- 4, – 2), B (– 3, – 5), C (3, – 2), and D (2, 3).

Join AC and divide the quadrilateral into two triangles.



We have two triangles, ΔABC and ΔACD.

Area of a triangle = $1/2 \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔABC = 1/2 [(-4) {(-5) – (-2)} + (-3) {(-2) – (-2)} + 3 {(-2) – (-5)}]

= 1/2 (12 + 0 + 9)

= 21/2 square units

Area of ΔACD = 1/2 [(-4) {(-2) – (3)} + 3{(3) – (-2)} + 2 {(-2) – (-2)}]

= 1/2 (20 + 15 + 0)

= 35/2 square units

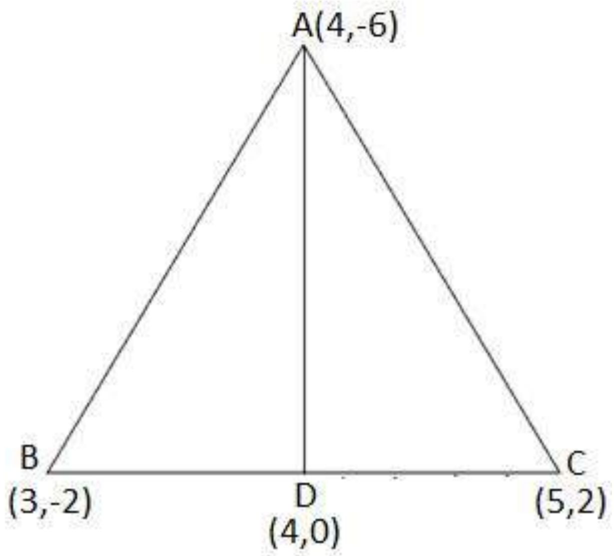
Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

= (21/2 + 35/2) square units = 28 square units

5. You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC, whose vertices are A (4, – 6), B (3, – 2) and C (5, 2).

Solution:

Let the vertices of the triangle be A (4, -6), B (3, -2), and C (5, 2).



Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

Coordinates of point D = Midpoint of BC = ((3+5)/2, (-2+2)/2) = (4, 0)

Formula, to find area of a triangle = $1/2 \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Now, a**rea of ΔABD** = 1/2 [(4) {(-2) – (0)} + 3{(0) – (-6)} + (4) {(-6) – (-2)}]

= 1/2 (-8 + 18 – 16)

= -3 square units

However, the area cannot be negative. Therefore, the area of ΔABD is 3 square units.

Area of ΔACD = 1/2 [(4) {0 – (2)} + 4{(2) – (-6)} + (5) {(-6) – (0)}]

= 1/2 (-8 + 32 – 30) = -3 square units

However, the area cannot be negative. Therefore, the area of ΔACD is 3 square units.

The area of both sides is the same. Thus, median AD has divided ΔABC into two triangles of equal areas.