

EXERCISE 1.4

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1. Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$

$B = \{x: x \text{ is a natural number less than } 6\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

Solution:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

So the union of the pairs of set can be written as

$$X \cup Y = \{1, 2, 3, 5\}$$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

So the union of the pairs of set can be written as

$$A \cup B = \{a, b, c, e, i, o, u\}$$

(iii) $A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \dots\}$

$B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5, 6\}$

So the union of the pairs of set can be written as

$$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12 \dots\}$$

Hence, $A \cup B = \{x: x = 1, 2, 3, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

So the union of the pairs of set can be written as

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

Hence, $A \cup B = \{x: x \in \mathbb{N} \text{ and } 1 < x < 10\}$

$$(v) A = \{1, 2, 3\}, B = \Phi$$

So the union of the pairs of set can be written as

$$A \cup B = \{1, 2, 3\}$$

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution:

It is given that

$$A = \{a, b\} \text{ and } B = \{a, b, c\}$$

Yes, $A \subset B$

So the union of the pairs of set can be written as

$$A \cup B = \{a, b, c\} = B$$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution:

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Solution:

It is given that

$$A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\} \text{ and } D = \{7, 8, 9, 10\}$$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$(iii) B \cup C = \{3, 4, 5, 6, 7, 8\}$$

$$(iv) B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(v) A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$(vi) A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(vii) B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

5. Find the intersection of each pair of sets:

$$(i) X = \{1, 3, 5\} \quad Y = \{1, 2, 3\}$$

$$(ii) A = \{a, e, i, o, u\} \quad B = \{a, b, c\}$$

$$(iii) A = \{x: x \text{ is a natural number and multiple of } 3\}$$

$$B = \{x: x \text{ is a natural number less than } 6\}$$

$$(iv) A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$$

$$(v) A = \{1, 2, 3\}, B = \Phi$$

Solution:

$$(i) X = \{1, 3, 5\}, Y = \{1, 2, 3\}$$

So the intersection of the given set can be written as

$$X \cap Y = \{1, 3\}$$

$$(ii) A = \{a, e, i, o, u\}, B = \{a, b, c\}$$

So the intersection of the given set can be written as

$$A \cap B = \{a\}$$

$$(iii) A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9, \dots\}$$

$$B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5\}$$

So the intersection of the given set can be written as

$$A \cap B = \{3\}$$

$$(iv) A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$$

So the intersection of the given set can be written as

$$A \cap B = \Phi$$

$$(v) A = \{1, 2, 3\}, B = \Phi$$

So the intersection of the given set can be written as

$$A \cap B = \Phi$$

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$

(ii) $B \cap C$

(iii) $A \cap C \cap D$

(iv) $A \cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$

(vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

(x) $(A \cup D) \cap (B \cup C)$

Solution:

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \{A \cap C\} \cap D$

$$= \{11\} \cap \{15, 17\}$$

$$= \Phi$$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \Phi$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$= \{7, 9, 11\} \cup \{11\}$$

$$= \{7, 9, 11\}$$

(vii) $A \cap D = \Phi$

$$(viii) A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$$

$$= \{7, 9, 11\} \cup \Phi$$

$$= \{7, 9, 11\}$$

$$(ix) (A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$$

$$= \{7, 9, 11\}$$

$$(x) (A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$$

$$= \{7, 9, 11, 15\}$$

7. If $A = \{x: x \text{ is a natural number}\}$, $B = \{x: x \text{ is an even natural number}\}$

$C = \{x: x \text{ is an odd natural number}\}$ and $D = \{x: x \text{ is a prime number}\}$, find

(i) $A \cap B$

(ii) $A \cap C$

(iii) $A \cap D$

(iv) $B \cap C$

(v) $B \cap D$

(vi) $C \cap D$

Solution:

It can be written as

$$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4, 5 \dots\}$$

$$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8 \dots\}$$

$$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9 \dots\}$$

$$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7 \dots\}$$

(i) $A \cap B = \{x: x \text{ is an even natural number}\} = B$

(ii) $A \cap C = \{x: x \text{ is an odd natural number}\} = C$

(iii) $A \cap D = \{x: x \text{ is a prime number}\} = D$

(iv) $B \cap C = \Phi$

(v) $B \cap D = \{2\}$

(vi) $C \cap D = \{x: x \text{ is odd prime number}\}$

8. Which of the following pairs of sets are disjoint?

(i) $\{1, 2, 3, 4\}$ and $\{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x: x \text{ is an even integer}\}$ and $\{x: x \text{ is an odd integer}\}$

Solution:

(i) $\{1, 2, 3, 4\}$

$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$

So we get

$$\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$$

Hence, this pair of sets is not disjoint.

(ii) $\{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\}$

Hence, $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ are not disjoint.

(iii) $\{x: x \text{ is an even integer}\} \cap \{x: x \text{ is an odd integer}\} = \Phi$

Hence, this pair of sets is disjoint.

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$,

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$; find

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Solution:

(i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

(ix) $C - B = \{2, 6, 10, 14\}$

(x) $D - B = \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 15, 20\}$

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$

(ii) $Y - X$

(iii) $X \cap Y$

Solution:

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution:

We know that

R – Set of real numbers

Q – Set of rational numbers

Hence, $R - Q$ is a set of irrational numbers.

12. State whether each of the following statement is true or false. Justify your answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Solution:

(i) False

If $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

So we get $\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

If $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

So we get $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

Here $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$

(iv) True

Here $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$

