

EXERCISE 1.5

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1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Solution:

It is given that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{3, 4, 5, 6\}$$

$$(i) A' = \{5, 6, 7, 8, 9\}$$

$$(ii) B' = \{1, 3, 5, 7, 9\}$$

$$(iii) A \cup C = \{1, 2, 3, 4, 5, 6\}$$

So we get

$$(A \cup C)' = \{7, 8, 9\}$$

$$(iv) A \cup B = \{1, 2, 3, 4, 6, 8\}$$

So we get

$$(A \cup B)' = \{5, 7, 9\}$$

$$(v) (A')' = A = \{1, 2, 3, 4\}$$

$$(vi) B - C = \{2, 8\}$$

So we get

$$(B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$$

2. If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets:

(i) $A = \{a, b, c\}$

(ii) $B = \{d, e, f, g\}$

(iii) $C = \{a, c, e, g\}$

(iv) $D = \{f, g, h, a\}$

Solution:

(i) $A = \{a, b, c\}$

So we get

$$A' = \{d, e, f, g, h\}$$

(ii) $B = \{d, e, f, g\}$

So we get

$$B' = \{a, b, c, h\}$$

(iii) $C = \{a, c, e, g\}$

So we get

$$C' = \{b, d, f, h\}$$

(iv) $D = \{f, g, h, a\}$

So we get

$$D' = \{b, c, d, e\}$$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) $\{x: x \text{ is an even natural number}\}$

(ii) $\{x: x \text{ is an odd natural number}\}$

(iii) $\{x: x \text{ is a positive multiple of 3}\}$

(iv) $\{x: x \text{ is a prime number}\}$

(v) $\{x: x \text{ is a natural number divisible by 3 and 5}\}$

(vi) $\{x: x \text{ is a perfect square}\}$

(vii) $\{x: x \text{ is perfect cube}\}$

(viii) $\{x: x + 5 = 8\}$

(ix) $\{x: 2x + 5 = 9\}$

(x) $\{x: x \geq 7\}$

(xi) $\{x: x \in \mathbb{N} \text{ and } 2x + 1 > 10\}$

Solution:

We know that

$U = \mathbb{N}$: Set of natural numbers

(i) $\{x: x \text{ is an even natural number}\}' = \{x: x \text{ is an odd natural number}\}$

(ii) $\{x: x \text{ is an odd natural number}\}' = \{x: x \text{ is an even natural number}\}$

(iii) $\{x: x \text{ is a positive multiple of } 3\}' = \{x: x \in \mathbb{N} \text{ and } x \text{ is not a multiple of } 3\}$

(iv) $\{x: x \text{ is a prime number}\}' = \{x: x \text{ is a positive composite number and } x \neq 1\}$

(v) $\{x: x \text{ is a natural number divisible by } 3 \text{ and } 5\}' = \{x: x \text{ is a natural number that is not divisible by } 3 \text{ or } 5\}$

(vi) $\{x: x \text{ is a perfect square}\}' = \{x: x \in \mathbb{N} \text{ and } x \text{ is not a perfect square}\}$

(vii) $\{x: x \text{ is a perfect cube}\}' = \{x: x \in \mathbb{N} \text{ and } x \text{ is not a perfect cube}\}$

(viii) $\{x: x + 5 = 8\}' = \{x: x \in \mathbb{N} \text{ and } x \neq 3\}$

(ix) $\{x: 2x + 5 = 9\}' = \{x: x \in \mathbb{N} \text{ and } x \neq 2\}$

(x) $\{x: x \geq 7\}' = \{x: x \in \mathbb{N} \text{ and } x < 7\}$

(xi) $\{x: x \in \mathbb{N} \text{ and } 2x + 1 > 10\}' = \{x: x \in \mathbb{N} \text{ and } x \leq 9/2\}$

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Solution:

It is given that

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$

$B = \{2, 3, 5, 7\}$

(i) $(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$

Therefore, $(A \cup B)' = A' \cap B'$.

(ii) $(A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$

Therefore, $(A \cap B)' = A' \cup B'$.

5. Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$

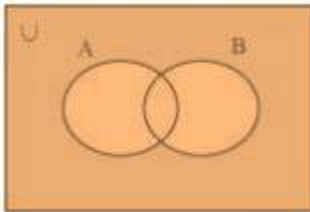
(ii) $A' \cap B'$

(iii) $(A \cap B)'$

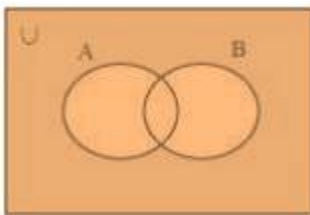
(iv) $A' \cup B'$

Solution:

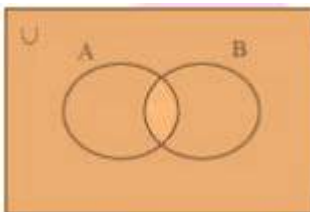
(i) $(A \cup B)'$



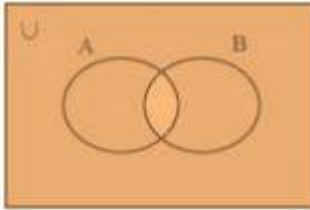
(ii) $A' \cap B'$



(iii) $(A \cap B)'$



(iv) $A' \cup B'$



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60° , what is A' ?

Solution:

A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \dots\dots\dots$

(ii) $\Phi' \cap A = \dots\dots\dots$

(iii) $A \cap A' = \dots\dots\dots$

(iv) $U' \cap A = \dots\dots\dots$

Solution:

(i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

So we get

$\Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$

So we get

$U' \cap A = \Phi$