EXERCISE 1.6 PAGE: 24

1. If X and Y are two sets such that *n*(X) = 17, *n*(Y) = 23 and *n*(X ∪ Y) = 38, find *n*(X ∩ Y).

Solution:

Given

n (X) = 17

n (Y) = 23

n (X U Y) = 38

We can write it as

n (X U Y) = n (X) + n (Y) – n (X ∩ Y)

Substituting the values

38 = 17 + 23 – n (X ∩ Y)

By further calculation

n (X ∩ Y) = 40 – 38 = 2

So we get

n (X ∩ Y) = 2

2. If X and Y are two sets such that X ∪Y has 18 elements, X has 8 elements and Y has 15 elements; how many elements does X ∩ Y have?

Solution:

Given

n (X U Y) = 18

n (X) = 8

n (Y) = 15

We can write it as

n (X U Y) = n (X) + n (Y) – n (X ∩ Y)

Substituting the values

18 = 8 + 15 – n (X ∩ Y)

By further calculation

$n(X \cap Y) = 23 - 18 = 5$

So we get

$n(X \cap Y) = 5$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution:

Consider H as the set of people who speak Hindi

E as the set of people who speak English

We know that

$n(H \cup E) = 400$

$n(H) = 250$

$n(E) = 200$

It can be written as

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

By substituting the values

$400 = 250 + 200 - n(H \cap E)$

By further calculation

$400 = 450 - n(H \cap E)$

So we get

$n(H \cap E) = 450 - 400$

$n(H \cap E) = 50$

Therefore, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and S ∩ T has 11 elements, how many elements does S ∪ T have?

Solution:

We know that

$n(S) = 21$

$n(T) = 32$

$n(S \cap T) = 11$

It can be written as

$n(S \cup T) = n(S) + n(T) - n(S \cap T)$

Substituting the values

$n(S \cup T) = 21 + 32 - 11$

So we get

$n(S \cup T) = 42$

Therefore, the set $(S \cup T)$ has 42 elements.

5. If X and Y are two sets such that X has 40 elements, X ∪Y has 60 elements and X ∩Y has 10 elements, how many elements does Y have?

Solution:

We know that

$n(X) = 40$

$n(X \cup Y) = 60$

$n(X \cap Y) = 10$

It can be written as

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

By substituting the values

$60 = 40 + n(Y) - 10$

On further calculation

$n(Y) = 60 - (40 - 10) = 30$

Therefore, the set Y has 30 elements.

6. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution:

Consider C as the set of people who like coffee

T as the set of people who like tea

$n(C \cup T) = 70$

$n(C) = 37$

$n(T) = 52$

It is given that

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

Substituting the values

$70 = 37 + 52 - n(C \cap T)$

By further calculation

$70 = 89 - n(C \cap T)$

So we get

$n(C \cap T) = 89 - 70 = 19$

Therefore, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution:

Consider C as the set of people who like cricket

T as the set of people who like tennis

$n(C \cup T) = 65$

$n(C) = 40$

$n(C \cap T) = 10$

It can be written as

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

Substituting the values

$65 = 40 + n(T) - 10$

By further calculation

$65 = 30 + n(T)$

So we get

$n(T) = 65 – 30 = 35$

Hence, 35 people like tennis.

We know that,

$(T – C) \cup (T \cap C) = T$

So we get,

$(T – C) \cap (T \cap C) = \Phi$

Here

$n (T) = n (T – C) + n (T \cap C)$

Substituting the values

$35 = n (T – C) + 10$

By further calculation

$n (T – C) = 35 – 10 = 25$

Therefore, 25 people like only tennis.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution:

Consider F as the set of people in the committee who speak French

S as the set of people in the committee who speak Spanish

$n(F) = 50$

$n(S) = 20$

$n(S \cap F) = 10$

It can be written as

$n(S \cup F) = n(S) + n(F) – n(S \cap F)$

By substituting the values

$n(S \cup F) = 20 + 50 – 10$

By further calculation

$n(S \cup F) = 70 – 10$

$n(S \cup F) = 60$

Therefore, 60 people in the committee speak at least one of the two languages.