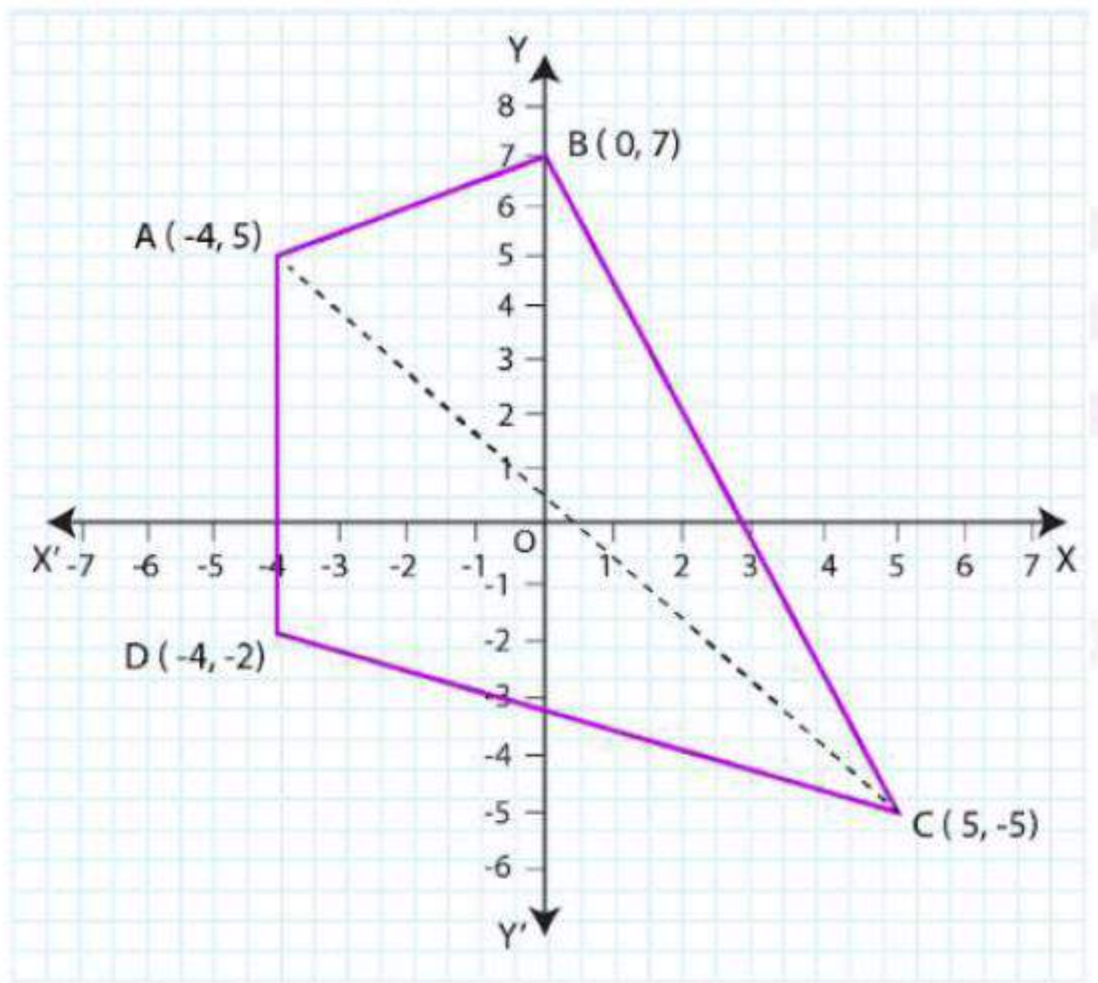
EXERCISE 10.1 PAGE NO: 211

1. Draw a quadrilateral in the Cartesian plane whose vertices are (– 4, 5), (0, 7), (5, – 5) and (– 4, –2). Also, find its area.

Solution:

Let ABCD be the given quadrilateral with vertices A (-4,5), B (0,7), C (5.-5) and D (-4,-2).

Now, let us plot the points on the Cartesian plane by joining the points AB, BC, CD, and AD, which give us the required quadrilateral.



To find the area, draw diagonal AC.

So, area (ABCD) = area (ΔABC) + area (ΔADC)

Then, area of triangle with vertices (x_1,y_1) , (x_2, y_2) and (x_3,y_3) is

Are of Δ ABC = ½ $[x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]$

= ½ [-4 (7 + 5) + 0 (-5 – 5) + 5 (5 – 7)] $unit^2$

= ½ [-4 (12) + 5 (-2)] $unit^2$

= ½ (58) $unit^2$

= 29 $unit^2$

Are of Δ ACD = ½ [x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]

= ½ [-4 (-5 + 2) + 5 (-2 – 5) + (-4) (5 – (-5))] $unit^2$

= ½ [-4 (-3) + 5 (-7) – 4 (10)] $unit^2$

= ½ (-63) $unit^2$

= -63/2 $unit^2$

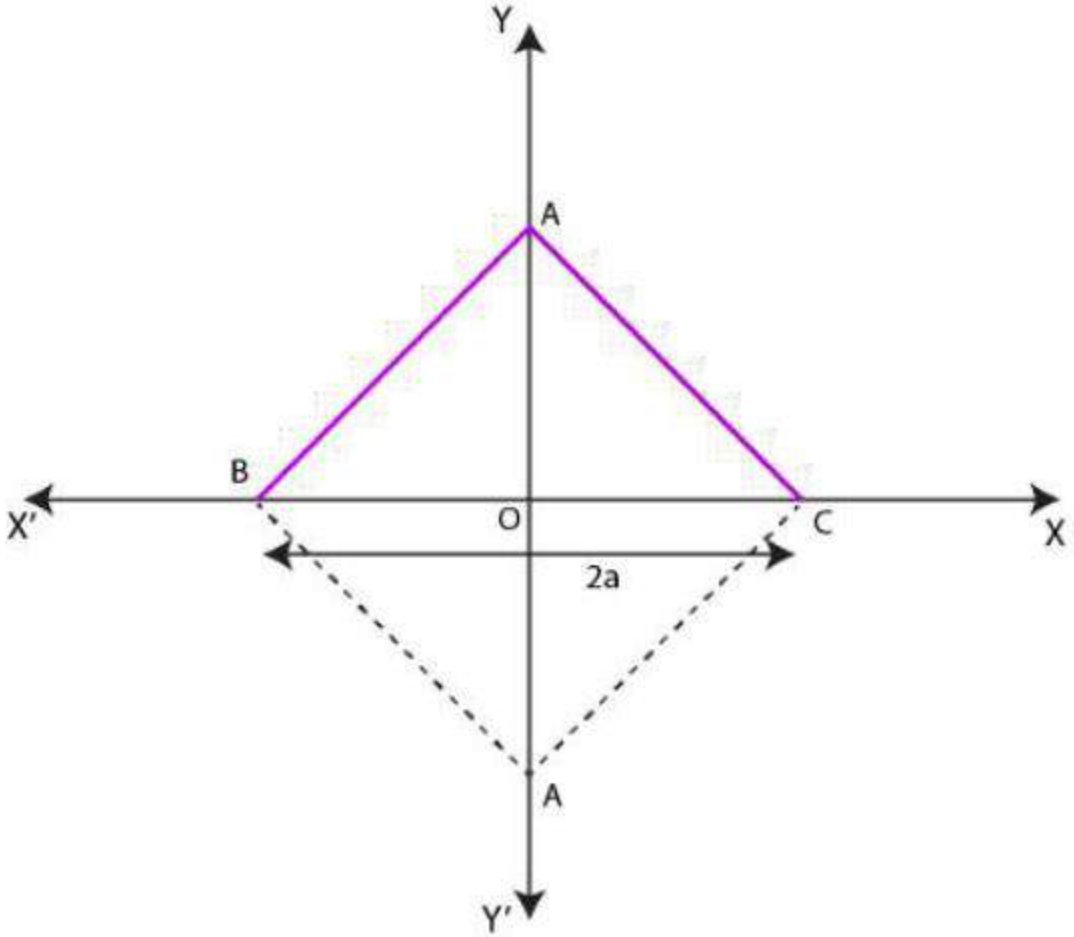
Since area cannot be negative, area Δ ACD = 63/2 $unit^2$

Area (ABCD) = 29 + 63/2

= 121/2 $unit^2$

2. The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find the vertices of the triangle.

Solution:



Let us consider ABC, the given equilateral triangle with side 2a.

Where, AB = BC = AC = 2a

In the above figure, assuming that the base BC lies on the x-axis such that the mid-point of BC is at the origin, i.e., BO = OC = a, where O is the origin.

The coordinates of point C are (0, a) and that of B are (0,-a).

The line joining a vertex of an equilateral Δ with the mid-point of its opposite side is perpendicular.

So, vertex A lies on the y –axis.

By applying Pythagoras' theorem,

$(AC)^2 = OA^2 + OC^2$

$(2a)^2 = a^2 + OC^2$

$4a^2 – a^2 = OC^2$

$3a^2 = OC^2$

$OC = \sqrt{3}a$

Co-ordinates of point C = $\pm \sqrt{3}a, 0$

∴ The vertices of the given equilateral triangle are (0, a), (0, -a), ($\sqrt{3}$a, 0)

Or (0, a), (0, -a) and (-$\sqrt{3}$a, 0)

3. Find the distance between P (x_1, y_1) and Q (x_2, y_2) when: (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis.

Solution:

Given:

Points P (x_1, y_1) and Q(x_2, y_2)

(i) When PQ is parallel to the y-axis, then $x_1 = x_2$

So, the distance between P and Q is given by

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(y_2 - y_1)^2}$$

$= |y_2 - y_1|$

(ii) When PQ is parallel to the x-axis, then $y_1 = y_2$

So, the distance between P and Q is given by =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2}$$

$= |x_2 - x_1|$

4. Find a point on the x-axis which is equidistant from points (7, 6) and (3, 4).

Solution:

Let us consider (a, 0) to be the point on the x-axis that is equidistant from the point (7, 6) and (3, 4).

So,

$$\sqrt{(7-a)^2+(6-0)^2} = \sqrt{(3-a)^2+(4-0)^2}$$

$$\sqrt{49+a^2-14a+36} = \sqrt{9+a^2-6a+16}$$

$$\sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$$

Now, let us square on both sides; we get,

$a^2 – 14a + 85 = a^2 – 6a + 25$

-8a = -60

a = 60/8

= 15/2

∴ The required point is (15/2, 0)

5. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, – 4) and B (8, 0).

Solution:

The co-ordinates of the mid-point of the line segment joining the points P (0, – 4) and B (8, 0) are (0+8)/2, (-4+0)/2 = (4, -2)

The slope 'm' of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 – y_1)/(x_2 – x_1)$ where, $x \neq x_1$

The slope of the line passing through (0, 0) and (4, -2) is (-2-0)/(4-0) = -1/2

∴ The required slope is -1/2.

6. Without using Pythagoras' theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-angled triangle.

Solution:

The vertices of the given triangle are (4, 4), (3, 5) and (–1, –1).

The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 – y_1)/(x_2 – x_1)$ where, $x \neq x_1$

So, the slope of the line AB (m_1) = (5-4)/(3-4) = 1/-1 = -1

The slope of the line BC (m_2) = (-1-5)/(-1-3) = -6/-4 = 3/2

The slope of the line CA (m_3) = (4+1)/(4+1) = 5/5 = 1

It is observed that $m_1.m_3 = -1.1 = -1$

Hence, the lines AB and CA are perpendicular to each other.

∴ given triangle is right-angled at A (4, 4)

And the vertices of the right-angled Δ are (4, 4), (3, 5) and (-1, -1)

7. Find the slope of the line, which makes an angle of 30° with the positive direction of the y-axis measured anticlockwise.

Solution:

We know that if a line makes an angle of 30° with the positive direction of the y-axis measured anti-clock-wise, then the angle made by the line with the positive direction of the x-axis measured anti-clock-wise is 90° + 30° = 120°

∴ The slope of the given line is tan 120° = tan (180° – 60°)

= – tan 60°

= –√3

8. Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Solution:

If the points (x, – 1), (2, 1) and (4, 5) are collinear, then the Slope of AB = Slope of BC

Then, (1+1)/(2-x) = (5-1)/(4-2)

2/(2-x) = 4/2

2/(2-x) = 2

2 = 2(2-x)

2 = 4 – 2x

2x = 4 – 2

2x = 2

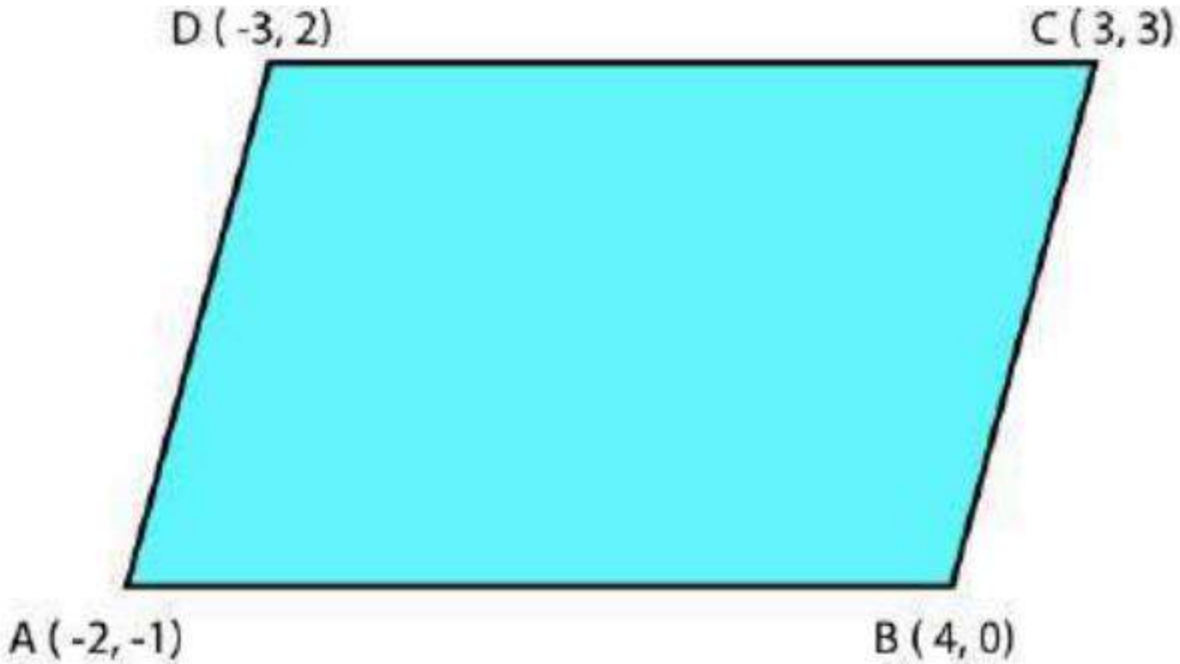
x = 2/2

= 1

∴ The required value of x is 1.

9. Without using the distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) are the vertices of a parallelogram.

Solution:



Let the given point be A (-2, -1) , B (4, 0) , C (3, 3) and D (-3, 2)

So now, the slope of AB = (0+1)/(4+2) = 1/6

The slope of CD = (3-2)/(3+3) = 1/6

Hence, the Slope of AB = Slope of CD

∴ AB ∥ CD

Now,

The slope of BC = (3-0)/(3-4) = 3/-1 = -3

The slope of AD = (2+1)/(-3+2) = 3/-1 = -3

Hence, the Slope of BC = Slope of AD

∴ BC ∥ AD

Thus, the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.

Hence, the given vertices, A (-2, -1), B (4, 0), C(3, 3) and D(-3, 2) are vertices of a parallelogram.

10. Find the angle between the x-axis and the line joining the points (3, –1) and (4, –2).

Solution:

The Slope of the line joining the points (3, -1) and (4, -2) is given by

$m = (y_2 – y_1)/(x_2 – x_1)$ where, $x \neq x_1$

m = (-2 –(-1))/(4-3)

= (-2+1)/(4-3)

= -1/1

= -1

The angle of inclination of the line joining the points (3, -1) and (4, -2) is given by

tan θ = -1

θ = (90° + 45°) = 135°

∴ The angle between the x-axis and the line joining the points (3, –1) and (4, –2) is 135°.

11. The slope of a line is double the slope of another line. If the tangent of the angle between them is 1/3, find the slopes of the lines.

Solution:

Let us consider 'm_1' and 'm' be the slope of the two given lines such that $m_1 = 2m$

We know that if θ is the angle between the lines l1 and l2 with slope m_1 and m_2, then

$$\tan\theta = \left|\frac{(m_2 - m_1)}{(1 + m_1 m_2)}\right|$$

Given here that the tangent of the angle between the two lines is 1/3

So,

$$\frac{1}{3} = \left|\frac{m-2m}{1+2m \times m}\right| = \left|\frac{-m}{1+2m^2}\right|$$

$$\frac{1}{3} = \frac{m}{1+2m^2}$$

Now, <u>case 1:</u>

$$\frac{1}{3} = \frac{-m}{1+2m^2}$$

$1+2m^2 = -3m$

$2m^2 +1 +3m = 0$

2m (m+1) + 1(m+1) = 0

(2m+1) (m+1)= 0

m = -1 or -1/2

If m = -1, then the slope of the lines are -1 and -2

If m = -1/2, then the slope of the lines are -1/2 and -1

Case 2:

$$\frac{1}{3} = \frac{-m}{1+2m^2}$$

$2m^2 – 3m + 1 = 0$

$2m^2 – 2m – m + 1 = 0$

$2m (m – 1) – 1(m – 1) = 0$

m = 1 or 1/2

If m = 1, then the slope of the lines are 1 and 2

If m = 1/2, then the slope of the lines are 1/2 and 1

∴ The slope of the lines are [-1 and -2] or [-1/2 and -1] or [1 and 2] or [1/2 and 1]

12. A line passes through (x_1, y_1) and (h, k). If the slope of the line is m, show that $k – y_1 = m (h – x_1)$.

Solution:

Given: the slope of the line is 'm'.

The slope of the line passing through (x_1, y_1) and (h, k) is $(k – y_1)/(h – x_1)$

So,

$(k – y_1)/(h – x_1) = m$

$(k – y_1) = m (h – x_1)$

Hence, proved.

13. If three points (h, 0), (a, b) and (0, k) lie on a line, show that a/h + b/k = 1

Solution:

Let us consider if the given points A (h, 0), B (a, b) and C (0, k) lie on a line.

Then, the slope of AB = slope of BC

(b – 0)/(a – h) = (k – b)/(0 – a)

By simplifying, we get

-ab = (k-b) (a-h)

-ab = ka- kh –ab +bh

ka +bh = kh

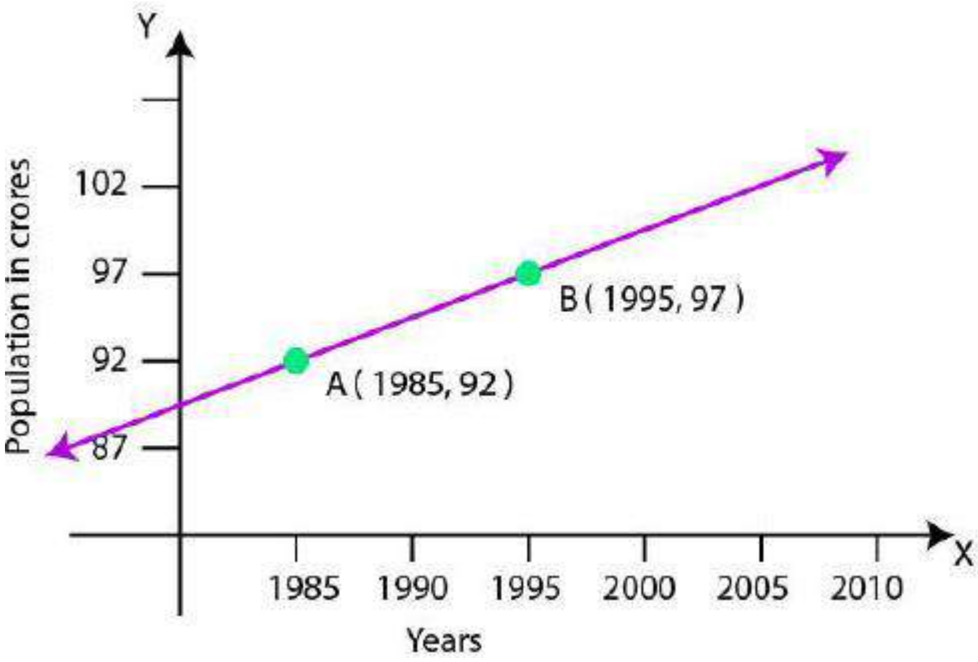
Divide both sides by kh; we get

ka/kh + bh/kh = kh/kh

a/h + b/k = 1

Hence, proved.

14. Consider the following population and year graph (Fig 10.10), find the slope of the line AB and using it, find what will be the population in the year 2010?



Solution:

We know that line AB passes through points A (1985, 92) and B (1995, 97).

Its slope will be (97 – 92)/(1995 – 1985) = 5/10 = 1/2

Let 'y' be the population in the year 2010. Then, according to the given graph, AB must pass through point C (2010, y)

So now, slope of AB = slope of BC

$$\frac{1}{2} = \frac{y - 97}{2010 - 1995}$$

15/2 = y – 97

y = 7.5 + 97 = 104.5

∴ The slope of line AB is 1/2, while in the year 2010, the population will be 104.5 crores.