EXERCISE 10.2 PAGE NO: 219

In Exercises 1 to 8, find the equation of the line which satisfies the given conditions.

1. Write the equations for the x-and y-axes.

Solution:

The y-coordinate of every point on the x-axis is 0.

∴ The equation of the x-axis is y = 0.

The x-coordinate of every point on the y-axis is 0.

∴ The equation of the y-axis is y = 0.

2. Passing through the point (– 4, 3) with slope 1/2

Solution:

Given:

Point (-4, 3) and slope, m = 1/2

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

So, y – 3 = 1/2 (x – (-4))

y – 3 = 1/2 (x + 4)

2(y – 3) = x + 4

2y – 6 = x + 4

x + 4 – (2y – 6) = 0

x + 4 – 2y + 6 = 0

x – 2y + 10 = 0

∴ The equation of the line is x – 2y + 10 = 0

3. Passing through (0, 0) with slope m.

Solution:

Given:

Point (0, 0) and slope, m = m

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

So, y – 0 = m (x – 0)

y = mx

y – mx = 0

∴ The equation of the line is y – mx = 0

4. Passing through (2, 2√3) and inclined with the x-axis at an angle of 75°.

Solution:

Given: point (2, 2√3) and θ = 75°

Equation of line: $(y - y_1) = m (x - x_1)$

where, m = slope of line = tan θ and (x_1, y_1) are the points through which line passes

∴ m = tan 75°

75° = 45° + 30°

Applying the formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ . \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Let us rationalizing we get,

$$\tan 75^\circ = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_1, y1), only if its coordinates satisfy the equation $y - y_1 = m (x - x_1)$

Then, y – 2√3 = (2 + √3) (x – 2)

$y - 2\sqrt{3} = 2x - 4 + \sqrt{3}x - 2\sqrt{3}$

$y = 2x - 4 + \sqrt{3}x$

$(2 + \sqrt{3})x - y - 4 = 0$

∴ The equation of the line is $(2 + \sqrt{3})x - y - 4 = 0$

5. Intersecting the x-axis at a distance of 3 units to the left of origin with slope –2.

Solution:

Given:

Slope, m = -2

We know that if a line L with slope m makes x-intercept d, then the equation of L is

$y = m(x - d)$.

If the distance is 3 units to the left of the origin, then d = -3

So, $y = (-2)(x - (-3))$

$y = (-2)(x + 3)$

$y = -2x - 6$

$2x + y + 6 = 0$

∴ The equation of the line is $2x + y + 6 = 0$

6. Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30° with the positive direction of the x-axis.

Solution:

Given: $\theta = 30^\circ$

We know that slope, $m = \tan\theta$

$m = \tan 30^\circ = (1/\sqrt{3})$

We know that the point (x, y) on the line with slope m and y-intercept c lies on the line only if $y = mx + c$

If the distance is 2 units above the origin, c = +2

So, $y = (1/\sqrt{3})x + 2$

$y = (x + 2\sqrt{3}) / \sqrt{3}$

$\sqrt{3}y = x + 2\sqrt{3}$

$x - \sqrt{3}y + 2\sqrt{3} = 0$

∴ The equation of the line is $x - \sqrt{3}y + 2\sqrt{3} = 0$

7. Passing through the points (–1, 1) and (2, – 4).

Solution:

Given:

Points (-1, 1) and (2, -4)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-4 - 1}{2 - (-1)}(x - (-1))$$

$y - 1 = -5/3\ (x + 1)$

$3(y - 1) = (-5)(x + 1)$

$3y - 3 = -5x - 5$

$3y - 3 + 5x + 5 = 0$

$5x + 3y + 2 = 0$

∴ The equation of the line is $5x + 3y + 2 = 0$

8. Perpendicular distance from the origin is 5 units, and the angle made by the perpendicular with the positive x-axis is 30°.

Solution:

Given: $p = 5$ and $\omega = 30°$

We know that the equation of the line having normal distance p from the origin and angle ω, which the normal makes with the positive direction of the x-axis, is given by $x \cos \omega + y \sin \omega = p$.

Substituting the values in the equation, we get

$x \cos 30° + y \sin 30° = 5$

$x(\sqrt{3}/2) + y(1/2) = 5$

$\sqrt{3}x + y = 5(2) = 10$

$\sqrt{3}x + y - 10 = 0$

∴ The equation of the line is $\sqrt{3}x + y - 10 = 0$

9. The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find the equation of the median through the vertex R.

Solution:

Given:

Vertices of ΔPQR, i.e., P (2, 1), Q (-2, 3) and R (4, 5)

Let RL be the median of vertex R.

So, L is a midpoint of PQ.

We know that the midpoint formula is given by
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

∴ L = $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$ = (0, 2)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by
$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$$

$$\therefore y - 5 = \frac{2-5}{0-4}(x-4)$$

y – 5 = -3/-4 (x-4)

(-4) (y – 5) = (-3) (x – 4)

-4y + 20 = -3x + 12

-4y + 20 + 3x – 12 = 0

3x – 4y + 8 = 0

∴ The equation of median through the vertex R is 3x – 4y + 8 = 0

10. Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

Solution:

Given:

Points are (2, 5) and (-3, 6).

We know that slope, m = $(y_2 - y_1)/(x_2 - x_1)$

= (6 – 5)/(-3 – 2)

= 1/-5 = -1/5

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, m = (-1/m)

= -1/(-1/5)

= 5

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

Then, y – 5 = 5(x – (-3))

y – 5 = 5x + 15

5x + 15 – y + 5 = 0

5x – y + 20 = 0

∴ The equation of the line is 5x – y + 20 = 0

11. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.

Solution:

We know that the coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\left(\frac{1(2)+n(1)}{1+n}, \frac{1(3)+n(0)}{1+n}\right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

We know that slope, $m = (y_2 – y_1)/(x_2 – x_1)$

= (3 – 0)/(2 – 1)

= 3/1

= 3

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, m = (-1/m) = -1/3

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

Here, the point is

$$\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

$$\left(y - \frac{3}{1+n}\right) = \frac{-1}{3}\left(x - \frac{2+n}{1+n}\right)$$

3((1 + n) y – 3) = (-(1 + n) x + 2 + n)

3(1 + n) y – 9 = – (1 + n) x + 2 + n

(1 + n) x + 3(1 + n) y – n – 9 – 2 = 0

(1 + n) x + 3(1 + n) y – n – 11 = 0

∴ The equation of the line is (1 + n) x + 3(1 + n) y – n – 11 = 0

12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Solution:

Given: the line cuts off equal intercepts on the coordinate axes, i.e., a = b

We know that equation of the line intercepts a and b on the x-and the y-axis, respectively, which is

x/a + y/b = 1

So, x/a + y/a = 1

x + y = a … (1)

Given: point (2, 3)

2 + 3 = a

a = 5

Substitute value of ‘a’ in (1), we get

x + y = 5

x + y – 5 = 0

∴ The equation of the line is x + y – 5 = 0

13. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Solution:

We know that equation of the line-making intercepts a and b on the x-and the y-axis, respectively, is x/a + y/b = 1 . …(1)

Given: sum of intercepts = 9

a + b = 9

b = 9 – a

Now, substitute the value of b in the above equation, and we get

x/a + y/(9 – a) = 1

Given: the line passes through point (2, 2)

So, 2/a + 2/(9 – a) = 1

[2(9 – a) + 2a] / a(9 – a) = 1 [18 – 2a + 2a] / a(9 – a) = 1
18/a(9 – a) = 1

18 = a (9 – a)

18 = 9a – a^2

a^2 – 9a + 18 = 0

Upon factorising, we get

a^2 – 3a – 6a + 18 = 0

a (a – 3) – 6 (a – 3) = 0

(a – 3) (a – 6) = 0

a = 3 or a = 6

Let us substitute in (1)

Case 1 (a = 3):

Then b = 9 – 3 = 6

x/3 + y/6 = 1

2x + y = 6

2x + y – 6 = 0

Case 2 (a = 6):

Then b = 9 – 6 = 3

x/6 + y/3 = 1

x + 2y = 6

x + 2y – 6 = 0

∴ The equation of the line is 2x + y – 6 = 0 or x + 2y – 6 = 0

14. Find the equation of the line through the point (0, 2), making an angle 2π/3 with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution:

Given:

Point (0, 2) and θ = 2π/3

We know that m = tan θ

m = tan (2π/3) = -√3

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

y – 2 = -√3 (x – 0)

y – 2 = -√3 x

√3 x + y – 2 = 0

Given, the equation of the line parallel to the above-obtained equation crosses the y-axis at a distance of 2 units below the origin.

So, the point = (0, -2) and m = -√3

From point slope form equation,

y – (-2) = -√3 (x – 0)

y + 2 = -√3 x

√3 x + y + 2 = 0

∴ The equation of the line is √3 x + y – 2 = 0, and the line parallel to it is √3 x + y + 2 = 0

15. The perpendicular from the origin to a line meets it at the point (–2, 9). Find the equation of the line.

Solution:

Given:

Points are origin (0, 0) and (-2, 9).

We know that slope, $m = (y_2 – y_1)/(x_2 – x_1)$

= (9 – 0)/(-2-0)

= -9/2

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

m = (-1/m) = -1/(-9/2) = 2/9

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

y – 9 = (2/9) (x – (-2))

9(y – 9) = 2(x + 2)

9y – 81 = 2x + 4

2x + 4 – 9y + 81 = 0

2x – 9y + 85 = 0

∴ The equation of the line is 2x – 9y + 85 = 0

16. The length L (in centimetres) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L= 125.134 when C = 110, express L in terms of C.

Solution:

Let us assume 'L' along X-axis and 'C' along Y-axis; we have two points (124.942, 20) and (125.134, 110) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$C - 20 = \frac{110-20}{125.134-124.942}(L - 124.942)$$

$$C - 20 = \frac{90}{0.192}(L - 124.942)$$

$$0.192\,(C - 20) = 90\,(L - 124.942)$$

$$L = \frac{0.192}{90}(C - 20) + 124.942$$

∴ The required relation is $L = \frac{0.192}{90}(C - 20) + 124.942$

17. The owner of a milk store finds that he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs. 16/litre. Assuming a linear relationship between the selling price and demand, how many litres could he sell weekly at Rs. 17/litre?

Solution:

Assuming the relationship between the selling price and demand is linear.

Let us assume the selling price per litre along X-axis and demand along Y-axis, we have two points (14, 980) and (16, 1220) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = \frac{240}{2}(x - 14)$$

y – 980 = 120 (x – 14)

y = 120 (x – 14) + 980

When x = Rs 17/litre,

y = 120 (17 – 14) + 980

y = 120(3) + 980

y = 360 + 980 = 1340

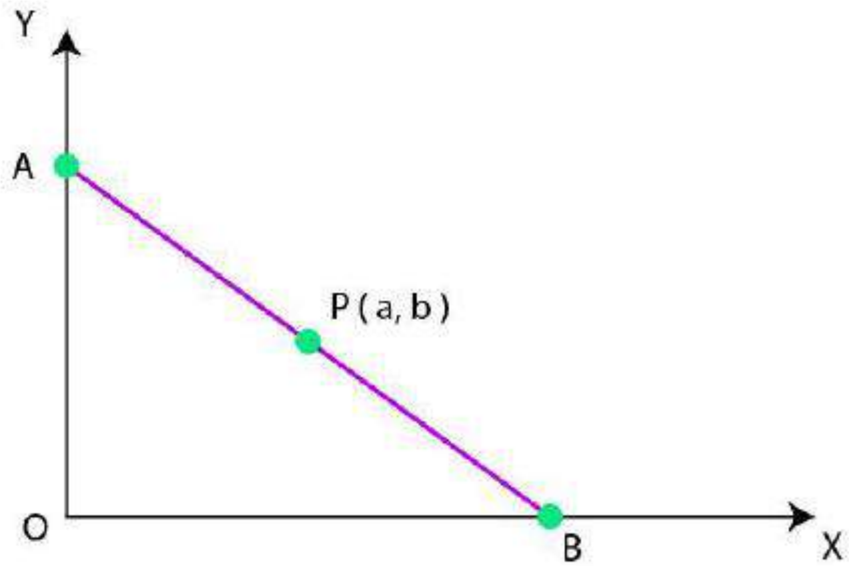
∴ The owner can sell 1340 litres weekly at Rs. 17/litre.

18. P (a, b) is the mid-point of a line segment between axes. Show that the equation of the line is x/a + y/b = 2

Solution:

Let AB be a line segment whose midpoint is P (a, b).

Let the coordinates of A and B be (0, y) and (x, 0), respectively.



We know that the midpoint is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since P is the midpoint of (a, b),

$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (a, b)$

$\left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$

a = x/2 and b = y/2

x = 2a and y = 2b

A = (0, 2b) and B = (2a, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$

$y - 2b = \frac{0-2b}{2a-0}(x - 0)$

$y - 2b = \frac{-2b}{2a}(x)$

$y - 2b = \frac{-b}{a}(x)$

a (y – 2b) = -bx

ay – 2ab = -bx

bx + ay = 2ab

Divide both sides with ab, then

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$
$$\frac{x}{a} + \frac{y}{b} = 2$$

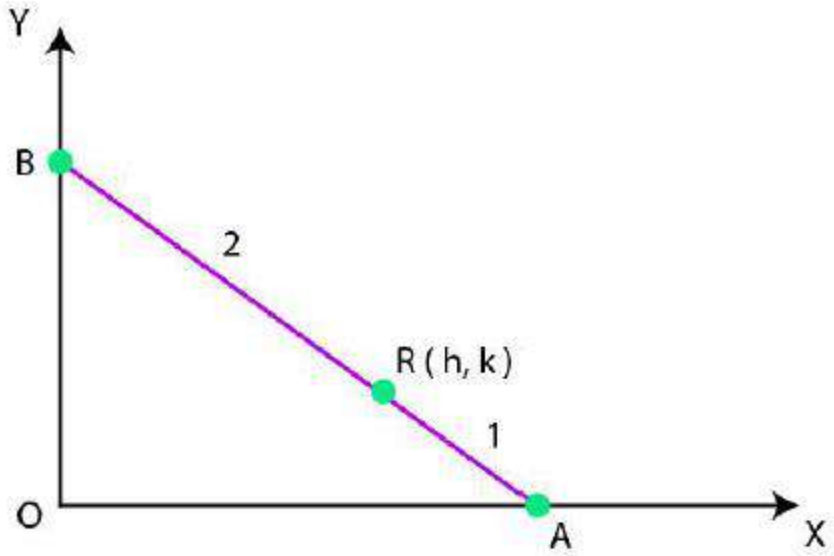
Hence, proved.

19. Point R (h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the line.

Solution:

Let us consider AB to be the line segment, such that r (h, k) divides it in the ratio 1: 2.

So, the coordinates of A and B be (0, y) and (x, 0), respectively.



We know that the coordinates of a point dividing the line segment join the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n is

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$
$$\left(\frac{1(0)+2(x)}{1+2}, \frac{1(y)+2(0)}{1+2}\right) = (h, k)$$
$$\left(\frac{2x}{3}, \frac{y}{3}\right) = (h, k)$$

h = 2x/3 and k = y/3

x = 3h/2 and y = 3k

∴ A = (0, 3k) and B = (3h/2, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3k = \frac{0 - 3k}{\frac{3h}{2} - 0}(x - 0)$$

3h(y – 3k) = -6kx

3hy – 9hk = -6kx

6kx + 3hy = 9hk

Let us divide both sides by 9hk, and we get,

2x/3h + y/3k = 1

∴ The equation of the line is given by 2x/3h + y/3k = 1

20. By using the concept of the equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.

Solution:

According to the question,

If we have to prove that the given three points (3, 0), (– 2, – 2) and (8, 2) are collinear, then we have to also prove that the line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

By using the formula,

The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-2 - 0}{-2 - 3}(x - 3)$$

$$y = \frac{-2}{-5}(x - 3)$$

-5y = -2 (x – 3)

-5y = -2x + 6

2x – 5y = 6

If 2x – 5y = 6 passes through (8, 2),

2x – 5y = 2(8) – 5(2)

= 16 – 10

= 6

= RHS

The line passing through points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

Hence, proved. The given three points are collinear.