## EXERCISE 10.1

1. Draw a quadrilateral in the Cartesian plane whose vertices are $(-4,5),(0,7),(5,-5)$ and $(-4,-2)$. Also, find its area.

## Solution:

Let ABCD be the given quadrilateral with vertices $\mathrm{A}(-4,5), \mathrm{B}(0,7), \mathrm{C}(5 .-5)$ and $\mathrm{D}(-4,-2)$.
Now, let us plot the points on the Cartesian plane by joining the points $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$, and AD , which give us the required quadrilateral.


To find the area, draw diagonal AC.
So, area $(\mathrm{ABCD})=\operatorname{area}(\triangle \mathrm{ABC})+\operatorname{area}(\triangle \mathrm{ADC})$
Then, area of triangle with vertices $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ and $\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$ is
Are of $\Delta A B C=1 / 2\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$
$=1 / 2[-4(7+5)+0(-5-5)+5(5-7)]$ unit $^{2}$
$=1 / 2[-4(12)+5(-2)]$ unit $^{2}$
$=1 / 2(58)$ unit $^{2}$
$=29$ unit $^{2}$
Are of $\Delta A C D=1 / 2\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$
$=1 / 2[-4(-5+2)+5(-2-5)+(-4)(5-(-5))]$ unit $^{2}$
$=1 / 2[-4(-3)+5(-7)-4(10)]$ unit $^{2}$
$=1 / 2(-63)$ unit $^{2}$
$=-63 / 2$ unit $^{2}$
Since area cannot be negative, area $\triangle \mathrm{ACD}=63 / 2$ unit $^{2}$
Area $(A B C D)=29+63 / 2$
$=121 / 2$ unit $^{2}$
2. The base of an equilateral triangle with side 2 a lies along the y -axis such that the mid-point of the base is at the origin. Find the vertices of the triangle.

Solution:



Let us consider ABC , the given equilateral triangle with side 2 a .
Where, $\mathrm{AB}=\mathrm{BC}=\mathrm{AC}=2 \mathrm{a}$
In the above figure, assuming that the base BC lies on the x -axis such that the mid-point of BC is at the origin, i.e., BO $=\mathrm{OC}=\mathrm{a}$, where O is the origin.

The coordinates of point $C$ are $(0, a)$ and that of $B$ are $(0,-a)$.
The line joining a vertex of an equilateral $\Delta$ with the mid-point of its opposite side is perpendicular.

So, vertex A lies on the y -axis.
By applying Pythagoras' theorem,
$(\mathrm{AC})^{2}=\mathrm{OA}^{2}+\mathrm{OC}^{2}$
$(2 \mathrm{a})^{2}=\mathrm{a}^{2}+\mathrm{OC}^{2}$
$4 \mathrm{a}^{2}-\mathrm{a}^{2}=\mathrm{OC}^{2}$
$3 a^{2}=\mathrm{OC}^{2}$
$O C=\sqrt{ } 3 a$
Co-ordinates of point $C= \pm \sqrt{3} \mathrm{a}, 0$
$\therefore$ The vertices of the given equilateral triangle are $(0, a),(0,-a),(\sqrt{ } 3 a, 0)$
Or $(0, a),(0,-a)$ and $(-\sqrt{3} a, 0)$
3. Find the distance between $P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$ when: (i) $P Q$ is parallel to the $y$-axis, (ii) $P Q$ is parallel to the x -axis.

## Solution:

Given:
Points $\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\mathrm{Q}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$
(i) When PQ is parallel to the $y$-axis, then $x_{1}=x_{2}$

So, the distance between P and Q is given by
$=\sqrt{\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)^{2}+\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2}}$
$=\sqrt{\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2}}$
$=\left|y_{2}-y_{1}\right|$
(ii) When PQ is parallel to the $x$-axis, then $y_{1}=y_{2}$

So, the distance between P and Q is given by $=$
$\sqrt{\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)^{2}+\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2}}$
$=\sqrt{\left(\mathrm{X}_{2}-\mathrm{X}_{1}\right)^{2}}$
$=\left|\mathrm{x}_{2}-\mathrm{x}_{1}\right|$
4. Find a point on the $x$-axis which is equidistant from points $(7,6)$ and $(3,4)$.

## Solution:

Let us consider $(a, 0)$ to be the point on the x -axis that is equidistant from the point $(7,6)$ and $(3,4)$.
So,

$$
\begin{aligned}
& \sqrt{(7-a)^{2}+(6-0)^{2}}=\sqrt{(3-a)^{2}+(4-0)^{2}} \\
& \sqrt{49+a^{2}-14 a+36}=\sqrt{9+a^{2}-6 a+16} \\
& \sqrt{a^{2}-14 a+85}=\sqrt{a^{2}-6 a+25}
\end{aligned}
$$

Now, let us square on both sides; we get,
$a^{2}-14 a+85=a^{2}-6 a+25$
$-8 a=-60$
$a=60 / 8$
$=15 / 2$
$\therefore$ The required point is $(15 / 2,0)$
5. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P(0,-4)$ and $B(8,0)$.

## Solution:

The co-ordinates of the mid-point of the line segment joining the points $\mathrm{P}(0,-4)$ and $\mathrm{B}(8,0)$ are $(0+8) / 2,(-4+0) / 2=$ (4, -2)

The slope ' $m$ ' of the line non-vertical line passing through the point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and
$\left(x_{2}, y_{2}\right)$ is given by $m=\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$ where, $x \neq x_{1}$
The slope of the line passing through $(0,0)$ and $(4,-2)$ is $(-2-0) /(4-0)=-1 / 2$
$\therefore$ The required slope is $-1 / 2$.
6. Without using Pythagoras' theorem, show that the points $(4,4),(3,5)$ and $(-1,-1)$ are the vertices of a rightangled triangle.

## Solution:

The vertices of the given triangle are $(4,4),(3,5)$ and $(-1,-1)$.
The slope (m) of the line non-vertical line passing through the point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and
$\left(x_{2}, y_{2}\right)$ is given by $m=\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$ where, $x \neq x_{1}$
So, the slope of the line $\mathrm{AB}\left(\mathrm{m}_{1}\right)=(5-4) /(3-4)=1 /-1=-1$
The slope of the line BC $\left(m_{2}\right)=(-1-5) /(-1-3)=-6 /-4=3 / 2$
The slope of the line CA $\left(\mathrm{m}_{3}\right)=(4+1) /(4+1)=5 / 5=1$

It is observed that $m_{1} \cdot m_{3}=-1.1=-1$
Hence, the lines AB and CA are perpendicular to each other.
$\therefore$ given triangle is right-angled at $\mathrm{A}(4,4)$
And the vertices of the right-angled $\Delta$ are $(4,4),(3,5)$ and $(-1,-1)$
7. Find the slope of the line, which makes an angle of $30^{\circ}$ with the positive direction of the $y$-axis measured anticlockwise.

## Solution:

We know that if a line makes an angle of $30^{\circ}$ with the positive direction of the $y$-axis measured anti-clock-wise, then the angle made by the line with the positive direction of the x -axis measured anti-clock-wise is $90^{\circ}+30^{\circ}=120^{\circ}$
$\therefore$ The slope of the given line is $\tan 120^{\circ}=\tan \left(180^{\circ}-60^{\circ}\right)$
$=-\tan 60^{\circ}$
$=-\sqrt{ } 3$
8. Find the value of $x$ for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

## Solution:

If the points $(x,-1),(2,1)$ and $(4,5)$ are collinear, then the Slope of $A B=$ Slope of $B C$
Then, $(1+1) /(2-x)=(5-1) /(4-2)$
$2 /(2-x)=4 / 2$
$2 /(2-x)=2$
$2=2(2-x)$
$2=4-2 x$
$2 x=4-2$
$2 \mathrm{x}=2$
$x=2 / 2$
$=1$
$\therefore$ The required value of x is 1 .
9. Without using the distance formula, show that points $(-2,-1),(4,0),(3,3)$ and $(-3,2)$ are the vertices of a parallelogram.

Solution:


## A ( $-2,-1$ )

Let the given point be $\mathrm{A}(-2,-1), \mathrm{B}(4,0), \mathrm{C}(3,3)$ and $\mathrm{D}(-3,2)$
So now, the slope of $\mathrm{AB}=(0+1) /(4+2)=1 / 6$
The slope of $\mathrm{CD}=(3-2) /(3+3)=1 / 6$
Hence, the Slope of $A B=$ Slope of $C D$
$\therefore \mathrm{AB} \| \mathrm{CD}$
Now,
The slope of $\mathrm{BC}=(3-0) /(3-4)=3 /-1=-3$
The slope of $\mathrm{AD}=(2+1) /(-3+2)=3 /-1=-3$
Hence, the Slope of $\mathrm{BC}=$ Slope of AD
$\therefore \mathrm{BC} \| \mathrm{AD}$

Thus, the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.
Hence, the given vertices, $\mathrm{A}(-2,-1), \mathrm{B}(4,0), \mathrm{C}(3,3)$ and $\mathrm{D}(-3,2)$ are vertices of a parallelogram.
10. Find the angle between the $x$-axis and the line joining the points $(3,-1)$ and $(4,-2)$.

## Solution:

The Slope of the line joining the points $(3,-1)$ and $(4,-2)$ is given by
$\mathrm{m}=\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right) /\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)$ where, $\mathrm{x} \neq \mathrm{x}_{1}$
$\mathrm{m}=(-2-(-1)) /(4-3)$
$=(-2+1) /(4-3)$
$=-1 / 1$
$=-1$
The angle of inclination of the line joining the points $(3,-1)$ and $(4,-2)$ is given by
$\tan \theta=-1$
$\theta=\left(90^{\circ}+45^{\circ}\right)=135^{\circ}$
$\therefore$ The angle between the x -axis and the line joining the points $(3,-1)$ and $(4,-2)$ is $135^{\circ}$.
11. The slope of a line is double the slope of another line. If the tangent of the angle between them is $1 / 3$, find the slopes of the lines.

## Solution:

Let us consider ' $m_{1}$ ' and ' $m$ ' be the slope of the two given lines such that $m_{1}=2 m$
We know that if $\theta$ is the angle between the lines 11 and 12 with slope $m_{1}$ and $m_{2}$, then

$$
\tan \theta=\left|\frac{\left(m_{2}-m_{1}\right.}{\left(1+m_{1} m_{2}\right)}\right|
$$

Given here that the tangent of the angle between the two lines is $1 / 3$
So,

$$
\begin{aligned}
& \frac{1}{3}=\left|\frac{\mathrm{m}-2 \mathrm{~m}}{1+2 \mathrm{~m} \times \mathrm{m}}\right|=\left|\frac{-\mathrm{m}}{1+2 \mathrm{~m}^{2}}\right| \\
& \frac{1}{3}=\frac{\mathrm{m}}{1+2 \mathrm{~m}^{2}}
\end{aligned}
$$

Now, case 1:
$\frac{1}{3}=\frac{-\mathrm{m}}{1+2 \mathrm{~m}^{2}}$
$1+2 \mathrm{~m}^{2}=-3 \mathrm{~m}$
$2 \mathrm{~m}^{2}+1+3 \mathrm{~m}=0$
$2 \mathrm{~m}(\mathrm{~m}+1)+1(\mathrm{~m}+1)=0$
$(2 m+1)(m+1)=0$
$\mathrm{m}=-1$ or $-1 / 2$
If $\mathrm{m}=-1$, then the slope of the lines are -1 and -2

If $m=-1 / 2$, then the slope of the lines are $-1 / 2$ and -1
Case 2:
$\frac{1}{3}=\frac{-m}{1+2 \mathrm{~m}^{2}}$
$2 m^{2}-3 m+1=0$
$2 \mathrm{~m}^{2}-2 \mathrm{~m}-\mathrm{m}+1=0$
$2 m(m-1)-1(m-1)=0$
$\mathrm{m}=1$ or $1 / 2$
If $m=1$, then the slope of the lines are 1 and 2
If $m=1 / 2$, then the slope of the lines are $1 / 2$ and 1
$\therefore$ The slope of the lines are [-1 and -2] or [-1/2 and -1] or [1 and 2] or [1/2 and 1]
12. A line passes through $\left(x_{1}, y_{1}\right)$ and $(h, k)$. If the slope of the line is $m$, show that $k-y_{1}=m\left(h-x_{1}\right)$.

Solution:

Given: the slope of the line is ' $m$ '.
The slope of the line passing through $\left(x_{1}, y_{1}\right)$ and $(h, k)$ is $\left(k-y_{1}\right) /\left(h-x_{1}\right)$
So,
$\left(\mathrm{k}-\mathrm{y}_{\mathrm{y}}\right) /\left(\mathrm{h}-\mathrm{x}_{1}\right)=\mathrm{m}$
$\left(\mathrm{k}-\mathrm{y}_{1}\right)=\mathrm{m}\left(\mathrm{h}-\mathrm{x}_{1}\right)$
Hence, proved.
13. If three points $(h, 0),(a, b)$ and $(0, k)$ lie on a line, show that $a / h+b / k=1$

## Solution:

Let us consider if the given points $A(h, 0), B(a, b)$ and $C(0, k)$ lie on a line.
Then, the slope of $\mathrm{AB}=$ slope of BC
$(b-0) /(a-h)=(k-b) /(0-a)$
By simplifying, we get
$-\mathrm{ab}=(\mathrm{k}-\mathrm{b})(\mathrm{a}-\mathrm{h})$
$-\mathrm{ab}=\mathrm{ka}-\mathrm{kh}-\mathrm{ab}+\mathrm{bh}$
$\mathrm{ka}+\mathrm{bh}=\mathrm{kh}$
Divide both sides by kh; we get
$\mathrm{ka} / \mathrm{kh}+\mathrm{bh} / \mathrm{kh}=\mathrm{kh} / \mathrm{kh}$
$\mathrm{a} / \mathrm{h}+\mathrm{b} / \mathrm{k}=1$

Hence, proved.
14. Consider the following population and year graph (Fig 10.10), find the slope of the line $A B$ and using it, find what will be the population in the year 2010 ?


Solution:
We know that line $A B$ passes through points $A(1985,92)$ and $B(1995,97)$.
Its slope will be $(97-92) /(1995-1985)=5 / 10=1 / 2$
Let ' y ' be the population in the year 2010 . Then, according to the given graph, AB must pass through point $\mathrm{C}(2010, \mathrm{y})$
So now, slope of $\mathrm{AB}=$ slope of BC
$\frac{1}{2}=\frac{y-97}{2010-1995}$
$15 / 2=y-97$
$y=7.5+97=104.5$
$\therefore$ The slope of line AB is $1 / 2$, while in the year 2010 , the population will be 104.5 crores.

## EXERCISE 10.2

In Exercises 1 to 8, find the equation of the line which satisfies the given conditions.

1. Write the equations for the $x$-and $y$-axes.

## Solution:

The $y$-coordinate of every point on the $x$-axis is 0 .
$\therefore$ The equation of the x -axis is $\mathrm{y}=0$.
The x -coordinate of every point on the y -axis is 0 .
$\therefore$ The equation of the y -axis is $\mathrm{y}=0$.
2. Passing through the point $(-4,3)$ with slope $1 / 2$

## Solution:

Given:
Point $(-4,3)$ and slope, $m=1 / 2$
We know that the point $(x, y)$ lies on the line with slope $m$ through the fixed point $\left(\mathrm{X}_{0}, \mathrm{y}_{0}\right)$ only if its coordinates satisfy the equation $y-y_{0}=m\left(x-x_{0}\right)$

So, $y-3=1 / 2(x-(-4))$
$y-3=1 / 2(x+4)$
$2(y-3)=x+4$
$2 y-6=x+4$
$x+4-(2 y-6)=0$
$x+4-2 y+6=0$
$x-2 y+10=0$
$\therefore$ The equation of the line is $\mathrm{x}-2 \mathrm{y}+10=0$
3. Passing through $(0,0)$ with slope $m$.

## Solution:

Given:

Point $(0,0)$ and slope, $\mathrm{m}=\mathrm{m}$

We know that the point $(x, y)$ lies on the line with slope $m$ through the fixed point $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ only if its coordinates satisfy the equation $y-y_{0}=m\left(x-x_{0}\right)$

So, $\mathrm{y}-0=\mathrm{m}(\mathrm{x}-0)$
$y=m x$
$y-m x=0$
$\therefore$ The equation of the line is $\mathrm{y}-\mathrm{mx}=0$
4. Passing through $(2,2 \sqrt{ } 3)$ and inclined with the $x$-axis at an angle of $75^{\circ}$.

## Solution:

Given: point $(2,2 \sqrt{ } 3)$ and $\theta=75^{\circ}$
Equation of line: $\left(y-y_{1}\right)=m\left(x-x_{1}\right)$
where, $\mathrm{m}=$ slope of line $=\tan \theta$ and $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ are the points through which line passes
$\therefore \mathrm{m}=\tan 75^{\circ}$
$75^{\circ}=45^{\circ}+30^{\circ}$
Applying the formula:

$$
\tan (A+B)=\frac{\tan A+\tan B}{1-\tan A \cdot \tan B}
$$

$$
\tan \left(45^{\circ}+30^{\circ}\right)=\frac{\tan 45^{\circ}+\tan 30^{\circ}}{1-\tan 45^{\circ} \cdot \tan 30^{\circ}}=\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}
$$

$$
\tan 75^{\circ}=\frac{\sqrt{3}+1}{\sqrt{3}-1}
$$

Let us rationalizing we get,
$\tan 75^{\circ}=\frac{3+1+2 \sqrt{3}}{3-1}=2+\sqrt{3}$
We know that the point $(x, y)$ lies on the line with slope $m$ through the fixed point $\left(x_{1}, y 1\right)$, only if its coordinates satisfy the equation $y-y_{1}=m\left(x-x_{1}\right)$

Then, $y-2 \sqrt{3}=(2+\sqrt{3})(x-2)$
$y-2 \sqrt{ } 3=2 x-4+\sqrt{ } 3 x-2 \sqrt{ } 3$
$y=2 x-4+\sqrt{3} x$
$(2+\sqrt{ } 3) x-y-4=0$
$\therefore$ The equation of the line is $(2+\sqrt{ } 3) x-y-4=0$
5. Intersecting the $\mathbf{x}$-axis at a distance of 3 units to the left of origin with slope $\mathbf{- 2}$.

## Solution:

Given:
Slope, $m=-2$
We know that if a line $L$ with slope $m$ makes $x$-intercept $d$, then the equation of $L$ is
$y=m(x-d)$.
If the distance is 3 units to the left of the origin, then $d=-3$
So, $\mathrm{y}=(-2)(\mathrm{x}-(-3))$
$y=(-2)(x+3)$
$y=-2 x-6$
$2 x+y+6=0$
$\therefore$ The equation of the line is $2 x+y+6=0$
6. Intersecting the $y$-axis at a distance of 2 units above the origin and making an angle of $30^{\circ}$ with the positive direction of the $x$-axis.

## Solution:

Given: $\theta=30^{\circ}$
We know that slope, $\mathrm{m}=\tan \theta$
$\mathrm{m}=\tan 30^{\circ}=(1 / \sqrt{ } 3)$
We know that the point $(x, y)$ on the line with slope $m$ and $y$-intercept $c$ lies on the line only if $y=m x+c$
If the distance is 2 units above the origin, $\mathrm{c}=+2$
So, $y=(1 / \sqrt{ } 3) x+2$
$y=(x+2 \sqrt{3}) / \sqrt{3}$
$\sqrt{ } 3 y=x+2 \sqrt{ } 3$
$x-\sqrt{ } 3 y+2 \sqrt{3}=0$
$\therefore$ The equation of the line is $x-\sqrt{3} y+2 \sqrt{3}=0$
7. Passing through the points $(-1,1)$ and $(2,-4)$.

## Solution:

Given:
Points ( $-1,1$ ) and (2, -4)
We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by

$$
\begin{aligned}
& \mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}\left(\mathrm{x}-\mathrm{x}_{1}\right) \\
& \mathrm{y}-1=\frac{-4-1}{2-(-1)}(\mathrm{x}-(-1))
\end{aligned}
$$

$y-1=-5 / 3(x+1)$
$3(y-1)=(-5)(x+1)$
$3 y-3=-5 x-5$
$3 y-3+5 x+5=0$
$5 x+3 y+2=0$
$\therefore$ The equation of the line is $5 x+3 y+2=0$
8. Perpendicular distance from the origin is 5 units, and the angle made by the perpendicular with the positive $x$ axis is $30^{\circ}$.

## Solution:

Given: $\mathrm{p}=5$ and $\omega=30^{\circ}$
We know that the equation of the line having normal distance $p$ from the origin and angle $\omega$, which the normal makes with the positive direction of the x -axis, is given by $\mathrm{x} \cos \omega+\mathrm{y} \sin \omega=\mathrm{p}$.

Substituting the values in the equation, we get
$\mathrm{x} \cos 30^{\circ}+\mathrm{y} \sin 30^{\circ}=5$
$x(\sqrt{3} / 2)+y(1 / 2)=5$
$\sqrt{3} x+y=5(2)=10$
$\sqrt{3} x+y-10=0$
$\therefore$ The equation of the line is $\sqrt{3} \mathrm{x}+\mathrm{y}-10=0$
9. The vertices of $\triangle P Q R$ are $P(2,1), Q(-2,3)$ and $R(4,5)$. Find the equation of the median through the vertex R.

Solution:
Given:
Vertices of $\triangle P Q R$, i.e., $P(2,1), Q(-2,3)$ and $R(4,5)$
Let RL be the median of vertex R .
So, $L$ is a midpoint of $P Q$.
We know that the midpoint formula is given by
$\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$.
$\therefore \mathrm{L}=\left(\frac{2-2}{2}, \frac{1+3}{2}\right)=(0,2)$
We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by
$y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right)$
$\therefore y-5=\frac{2-5}{0-4}(x-4)$
$y-5=-3 /-4(x-4)$
$(-4)(y-5)=(-3)(x-4)$
$-4 y+20=-3 x+12$
$-4 y+20+3 x-12=0$
$3 x-4 y+8=0$
$\therefore$ The equation of median through the vertex R is $3 \mathrm{x}-4 \mathrm{y}+8=0$
10. Find the equation of the line passing through $(-3,5)$ and perpendicular to the line through the points $(2,5)$ and $(-3,6)$.

## Solution:

Given:
Points are $(2,5)$ and $(-3,6)$.
We know that slope, $m=\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$
$=(6-5) /(-3-2)$
$=1 /-5=-1 / 5$
We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, $m=(-1 / m)$
$=-1 /(-1 / 5)$
$=5$
We know that the point ( $\mathrm{x}, \mathrm{y}$ ) lies on the line with slope m through the fixed point $\left(\mathrm{X}_{0}, \mathrm{y}_{0}\right)$, only if its coordinates satisfy the equation $\mathrm{y}-\mathrm{y}_{0}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{0}\right)$

Then, $\mathrm{y}-5=5(\mathrm{x}-(-3))$
$y-5=5 x+15$
$5 \mathrm{x}+15-\mathrm{y}+5=0$
$5 x-y+20=0$
$\therefore$ The equation of the line is $5 \mathrm{x}-\mathrm{y}+20=0$
11. A line perpendicular to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1: n$. Find the equation of the line.

## Solution:

We know that the coordinates of a point dividing the line segment joining the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ internally in the ratio m : n are
$\left(\frac{\mathrm{mx}_{2}+\mathrm{nx}_{1}}{\mathrm{~m}+\mathrm{n}}, \frac{\mathrm{my}_{2}+\mathrm{ny}_{1}}{\mathrm{~m}+\mathrm{n}}\right)$
$\left(\frac{1(2)+\mathrm{n}(1)}{1+\mathrm{n}}, \frac{1(3)+\mathrm{n}(0)}{1+\mathrm{n}}\right)=\left(\frac{2+\mathrm{n}}{1+\mathrm{n}}, \frac{3}{1+\mathrm{n}}\right)$
We know that slope, $m=\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$
$=(3-0) /(2-1)$
$=3 / 1$
$=3$
We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, $m=(-1 / \mathrm{m})=-1 / 3$

We know that the point ( $\mathrm{x}, \mathrm{y}$ ) lies on the line with slope m through the fixed point $\left(\mathrm{x}_{0}, \mathrm{y}_{\mathrm{y}}\right)$, only if its coordinates satisfy the equation $\mathrm{y}-\mathrm{y}_{0}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{0}\right)$

Here, the point is
$\left(\frac{2+\mathrm{n}}{1+\mathrm{n}}, \frac{3}{1+\mathrm{n}}\right)$
$\left(\mathrm{y}-\frac{3}{1+\mathrm{n}}\right)=\frac{-1}{3}\left(\mathrm{x}-\frac{2+\mathrm{n}}{1+\mathrm{n}}\right)$
$3((1+n) y-3)=(-(1+n) x+2+n)$
$3(1+\mathrm{n}) \mathrm{y}-9=-(1+\mathrm{n}) \mathrm{x}+2+\mathrm{n}$
$(1+n) x+3(1+n) y-n-9-2=0$
$(1+n) x+3(1+n) y-n-11=0$
$\therefore$ The equation of the line is $(1+\mathrm{n}) \mathrm{x}+3(1+\mathrm{n}) \mathrm{y}-\mathrm{n}-11=0$
12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2,3)$.

## Solution:

Given: the line cuts off equal intercepts on the coordinate axes, i.e., $\mathrm{a}=\mathrm{b}$
We know that equation of the line intercepts a and b on the $x$-and the $y$-axis, respectively, which is
$x / a+y / b=1$
So, $\mathrm{x} / \mathrm{a}+\mathrm{y} / \mathrm{a}=1$
$x+y=a$
Given: point $(2,3)$
$2+3=\mathrm{a}$
$a=5$
Substitute value of 'a' in (1), we get
$x+y=5$
$x+y-5=0$
$\therefore$ The equation of the line is $\mathrm{x}+\mathrm{y}-5=0$
13. Find the equation of the line passing through the point $(2,2)$ and cutting off intercepts on the axes whose sum is 9 .

## Solution:

We know that equation of the line-making intercepts $a$ and $b$ on the $x$-and the $y$-axis, respectively, is $x / a+y / b=1 \ldots$ (1)

Given: sum of intercepts $=9$
$a+b=9$
$\mathrm{b}=9-\mathrm{a}$
Now, substitute the value of $b$ in the above equation, and we get
$x / a+y /(9-a)=1$
Given: the line passes through point $(2,2)$
So, $2 / \mathrm{a}+2 /(9-\mathrm{a})=1$
$[2(9-a)+2 a] / a(9-a)=1[18-2 a+2 a] / a(9-a)=1$ $18 / a(9-a)=1$
$18=a(9-a)$
$18=9 a-a^{2}$
$a^{2}-9 a+18=0$
Upon factorising, we get
$a^{2}-3 a-6 a+18=0$
$a(a-3)-6(a-3)=0$
$(a-3)(a-6)=0$
$\mathrm{a}=3$ or $\mathrm{a}=6$
Let us substitute in (1)
Case $1(\mathrm{a}=3)$ :
Then $\mathrm{b}=9-3=6$
$x / 3+y / 6=1$
$2 x+y=6$
$2 x+y-6=0$
Case $2(a=6)$ :
Then $\mathrm{b}=9-6=3$
$x / 6+y / 3=1$
$x+2 y=6$
$x+2 y-6=0$
$\therefore$ The equation of the line is $2 x+y-6=0$ or $x+2 y-6=0$
14. Find the equation of the line through the point $(0,2)$, making an angle $2 \pi / 3$ with the positive $x$-axis. Also, find the equation of the line parallel to it and crossing the $y$-axis at a distance of $\mathbf{2}$ units below the origin.

## Solution:

Given:
$\operatorname{Point}(0,2)$ and $\theta=2 \pi / 3$
We know that $\mathrm{m}=\tan \theta$
$\mathrm{m}=\tan (2 \pi / 3)=-\sqrt{ } 3$
We know that the point $(\mathrm{x}, \mathrm{y})$ lies on the line with slope m through the fixed point $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$, only if its coordinates satisfy the equation $\mathrm{y}-\mathrm{y}_{0}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{0}\right)$
$y-2=-\sqrt{3}(x-0)$
$y-2=-\sqrt{3} x$
$\sqrt{3} x+y-2=0$
Given, the equation of the line parallel to the above-obtained equation crosses the $y$-axis at a distance of 2 units below the origin.

So, the point $=(0,-2)$ and $m=-\sqrt{ } 3$
From point slope form equation,
$y-(-2)=-\sqrt{3}(x-0)$
$y+2=-\sqrt{3} x$
$\sqrt{3} x+y+2=0$
$\therefore$ The equation of the line is $\sqrt{ } 3 x+y-2=0$, and the line parallel to it is $\sqrt{3} x+y+2=0$
15. The perpendicular from the origin to a line meets it at the point $(-2,9)$. Find the equation of the line.

## Solution:

Given:
Points are origin $(0,0)$ and $(-2,9)$.
We know that slope, $m=\left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right)$
$=(9-0) /(-2-0)$
$=-9 / 2$
We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.
$m=(-1 / m)=-1 /(-9 / 2)=2 / 9$
We know that the point $(x, y)$ lies on the line with slope $m$ through the fixed point $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ only if its coordinates satisfy the equation $y-y_{0}=m\left(x-x_{0}\right)$
$y-9=(2 / 9)(x-(-2))$
$9(y-9)=2(x+2)$
$9 y-81=2 x+4$
$2 x+4-9 y+81=0$
$2 x-9 y+85=0$
$\therefore$ The equation of the line is $2 x-9 y+85=0$
16. The length $L$ (in centimetres) of a copper rod is a linear function of its Celsius temperature $C$. In an experiment, if $L=124.942$ when $C=20$ and $L=125.134$ when $C=110$, express $L$ in terms of $C$.

## Solution:

Let us assume 'L' along X-axis and 'C' along Y-axis; we have two points $(124.942,20)$ and $(125.134,110)$ in XYplane.

We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by
$\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
$\mathrm{C}-20=\frac{110-20}{125.134-124.942}(\mathrm{~L}-124.942)$
$C-20=\frac{90}{0.192}(\mathrm{~L}-124.942)$
$0.192(\mathrm{C}-20)=90(\mathrm{~L}-124.942)$
$\mathrm{L}=\frac{0.192}{90}(\mathrm{C}-20)+124.942$
$\therefore$ The required relation is $\mathrm{L}=\frac{0.192}{90}(\mathrm{C}-20)+124.942$
17. The owner of a milk store finds that he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs. 16/litre. Assuming a linear relationship between the selling price and demand, how many litres could he sell weekly at Rs. 17/litre?

## Solution:

Assuming the relationship between the selling price and demand is linear.
Let us assume the selling price per litre along X-axis and demand along Y-axis, we have two points $(14,980)$ and $(16$, 1220) in XY-plane.

We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by
$\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
$y-980=\frac{1220-980}{16-14}(x-14)$
$y-980=\frac{240}{2}(x-14)$
$y-980=120(x-14)$
$y=120(x-14)+980$
When $x=$ Rs 17/litre,
$y=120(17-14)+980$
$y=120(3)+980$
$y=360+980=1340$
$\therefore$ The owner can sell 1340 litres weekly at Rs. 17/litre.
18. $P(a, b)$ is the mid-point of a line segment between axes. Show that the equation of the line is $x / a+y / b=2$

## Solution:

Let AB be a line segment whose midpoint is $\mathrm{P}(\mathrm{a}, \mathrm{b})$.
Let the coordinates of A and B be $(0, y)$ and $(x, 0)$, respectively.


We know that the midpoint is given by $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
Since $P$ is the midpoint of $(a, b)$,
$\left(\frac{0+x}{2}, \frac{y+0}{2}\right)=(a, b)$
$\left(\frac{\mathrm{x}}{2}, \frac{\mathrm{y}}{2}\right)=(\mathrm{a}, \mathrm{b})$
$a=x / 2$ and $b=y / 2$
$\mathrm{x}=2 \mathrm{a}$ and $\mathrm{y}=2 \mathrm{~b}$
$\mathrm{A}=(0,2 \mathrm{~b})$ and $\mathrm{B}=(2 \mathrm{a}, 0)$
We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and ( $\mathrm{x}_{2}, \mathrm{y}_{2}$ ) is given by
$\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
$y-2 b=\frac{0-2 b}{2 a-0}(x-0)$
$y-2 b=\frac{-2 b}{2 a}(x)$
$y-2 b=\frac{-b}{a}(x)$
$a(y-2 b)=-b x$
$a y-2 a b=-b x$
$b x+a y=2 a b$

Divide both sides with ab, then

$$
\begin{aligned}
& \frac{b x}{a b}+\frac{a y}{a b}=\frac{2 a b}{a b} \\
& \frac{x}{a}+\frac{y}{b}=2
\end{aligned}
$$

Hence, proved.
19. Point $R(h, k)$ divides a line segment between the axes in the ratio $1: 2$. Find the equation of the line.

## Solution:

Let us consider AB to be the line segment, such that $\mathrm{r}(\mathrm{h}, \mathrm{k})$ divides it in the ratio $1: 2$.
So, the coordinates of A and B be $(0, y)$ and ( $x, 0)$, respectively.


We know that the coordinates of a point dividing the line segment join the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ internally in the ratio $\mathrm{m}: \mathrm{n}$ is
$\left(\frac{\mathrm{mx}_{2}+\mathrm{nx}_{1}}{\mathrm{~m}+\mathrm{n}}, \frac{\mathrm{my}_{2}+\mathrm{ny}_{1}}{\mathrm{~m}+\mathrm{n}}\right)$
$\left(\frac{1(0)+2(\mathrm{x})}{1+2}, \frac{1(\mathrm{y})+2(0)}{1+2}\right)=(\mathrm{h}, \mathrm{k})$
$\left(\frac{2 \mathrm{x}}{3}, \frac{\mathrm{y}}{3}\right)=(\mathrm{h}, \mathrm{k})$
$\mathrm{h}=2 \mathrm{x} / 3$ and $\mathrm{k}=\mathrm{y} / 3$
$\mathrm{x}=3 \mathrm{~h} / 2$ and $\mathrm{y}=3 \mathrm{k}$
$\therefore \mathrm{A}=(0,3 \mathrm{k})$ and $\mathrm{B}=(3 \mathrm{~h} / 2,0)$
We know that the equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by

$$
\begin{aligned}
& y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right) \\
& y-3 k=\frac{0-3 k}{\frac{3 h}{2}-0}(x-0)
\end{aligned}
$$

$3 h(y-3 k)=-6 k x$
$3 h y-9 h k=-6 k x$
$6 \mathrm{kx}+3 \mathrm{hy}=9 \mathrm{hk}$
Let us divide both sides by 9 hk , and we get,
$2 \mathrm{x} / 3 \mathrm{~h}+\mathrm{y} / 3 \mathrm{k}=1$
$\therefore$ The equation of the line is given by $2 \mathrm{x} / 3 \mathrm{~h}+\mathrm{y} / 3 \mathrm{k}=1$
20. By using the concept of the equation of a line, prove that the three points $(3,0),(-2,-2)$ and $(8,2)$ are collinear.

## Solution:

According to the question,
If we have to prove that the given three points $(3,0),(-2,-2)$ and $(8,2)$ are collinear, then we have to also prove that the line passing through the points $(3,0)$ and $(-2,-2)$ also passes through the point $(8,2)$.

By using the formula,
The equation of the line passing through the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by

$$
\begin{aligned}
& y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right) \\
& y-0=\frac{-2-0}{-2-3}(x-3) \\
& y=\frac{-2}{-5}(x-3) \\
& -5 y=-2(x-3) \\
& -5 y=-2 x+6 \\
& 2 x-5 y=6 \\
& \text { If } 2 x-5 y=6 \text { passes through }(8,2), \\
& 2 x-5 y=2(8)-5(2) \\
& =16-10
\end{aligned}
$$

$=6$
$=$ RHS
The line passing through points $(3,0)$ and $(-2,-2)$ also passes through the point $(8,2)$.
Hence, proved. The given three points are collinear.

## EXERCISE 10.3

1. Reduce the following equations into slope-intercept form and find their slopes and the $y$-intercepts.
(i) $x+7 y=0$
(ii) $6 x+3 y-5=0$
(iii) $\mathbf{y}=\mathbf{0}$

Solution:
(i) $x+7 y=0$

Given:
The equation is $\mathrm{x}+7 \mathrm{y}=0$
The slope-intercept form is represented in the form ' $y=m x+c$ ', where $m$ is the slope and $c$ is the $y$-intercept.
So, the above equation can be expressed as
$y=-1 / 7 x+0$
$\therefore$ The above equation is of the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$, where $\mathrm{m}=-1 / 7$ and $\mathrm{c}=0$
(ii) $6 x+3 y-5=0$

Given:
The equation is $6 x+3 y-5=0$
The slope-intercept form is represented in the form ' $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ ', where m is the slope and c is the y -intercept.
So, the above equation can be expressed as
$3 y=-6 x+5$
$y=-6 / 3 x+5 / 3$
$=-2 x+5 / 3$
$\therefore$ The above equation is of the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$, where $\mathrm{m}=-2$ and $\mathrm{c}=5 / 3$
(iii) $\mathrm{y}=0$

Given:
The equation is $\mathrm{y}=0$
The slope-intercept form is given by ' $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ ', where m is the slope and c is the y -intercept.
$y=0 \times x+0$
$\therefore$ The above equation is of the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$, where $\mathrm{m}=0$ and $\mathrm{c}=0$
2. Reduce the following equations into intercept form and find their intercepts on the axes.
(i) $3 x+2 y-12=0$
(ii) $4 x-3 y=6$
(iii) $3 y+2=0$

## Solution:

(i) $3 x+2 y-12=0$

Given:
The equation is $3 x+2 y-12=0$
The equation of the line in intercept form is given by $x / a+y / b=1$, where ' $a$ ' and ' $b$ ' are intercepted on the $x$-axis and the y-axis, respectively.

So, $3 \mathrm{x}+2 \mathrm{y}=12$
Now, let us divide both sides by 12 ; we get
$3 \mathrm{x} / 12+2 \mathrm{y} / 12=12 / 12$
$x / 4+y / 6=1$
$\therefore$ The above equation is of the form $x / a+y / b=1$, where $a=4, b=6$
The intercept on the x -axis is 4 .
The intercept on the $y$-axis is 6 .
(ii) $4 x-3 y=6$

Given:
The equation is $4 x-3 y=6$
The equation of the line in intercept form is given by $x / a+y / b=1$, where ' $a$ ' and ' $b$ ' are intercepted on the $x$-axis and the $y$-axis, respectively.

So, $4 x-3 y=6$
Now, let us divide both sides by 6 ; we get
$4 x / 6-3 y / 6=6 / 6$
$2 x / 3-y / 2=1$
$x /(3 / 2)+y /(-2)=1$
$\therefore$ The above equation is of the form $\mathrm{x} / \mathrm{a}+\mathrm{y} / \mathrm{b}=1$, where $\mathrm{a}=3 / 2, \mathrm{~b}=-2$
The intercept on the $x$-axis is $3 / 2$.
The intercept on the $y$-axis is -2 .
(iii) $3 y+2=0$

Given:

The equation is $3 y+2=0$
The equation of the line in intercept form is given by $x / a+y / b=1$, where ' $a$ ' and ' $b$ ' are intercepted on the $x$-axis and the $y$-axis, respectively.

So, $3 y=-2$
Now, let us divide both sides by -2 ; we get
$3 y /-2=-2 /-2$
$3 y /-2=1$
$y /(-2 / 3)=1$
$\therefore$ The above equation is of the form $\mathrm{x} / \mathrm{a}+\mathrm{y} / \mathrm{b}=1$, where $\mathrm{a}=0, \mathrm{~b}=-2 / 3$
The intercept on the x -axis is 0 .
The intercept on the $y$-axis is $-2 / 3$.
3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and the angle between the perpendicular and the positive $x$-axis.
(i) $x-\sqrt{ } 3 y+8=0$
(ii) $y-2=0$
(iii) $x-y=4$

## Solution:

(i) $x-\sqrt{ } 3 y+8=0$

Given:
The equation is $x-\sqrt{ } 3 y+8=0$
The equation of the line in normal form is given by $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$ where ' $\theta$ ' is the angle between the perpendicular and the positive $x$-axis and ' p ' is the perpendicular distance from the origin.

So now, $x-\sqrt{ } 3 y+8=0$
$x-\sqrt{ } 3 y=-8$
Divide both the sides by $\sqrt{ }\left(1^{2}+(\sqrt{ } 3)^{2}\right)=\sqrt{ }(1+3)=\sqrt{ } 4=2$
$x / 2-\sqrt{ } 3 y / 2=-8 / 2$
$(-1 / 2) x+\sqrt{ } 3 / 2 y=4$
This is in the form of: $\mathrm{x} \cos 120^{\circ}+\mathrm{y} \sin 120^{\circ}=4$
$\therefore$ The above equation is of the form $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$, where $\theta=120^{\circ}$ and $\mathrm{p}=4$.
Perpendicular distance of the line from origin $=4$
The angle between the perpendicular and positive x -axis $=120^{\circ}$
(ii) $y-2=0$

Given:
The equation is $y-2=0$
The equation of the line in normal form is given by $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$ where ' $\theta$ ' is the angle between the perpendicular and the positive x -axis and ' p ' is the perpendicular distance from the origin.

So now, $0 \times x+1 \times y=2$
Divide both sides by $\sqrt{ }\left(0^{2}+1^{2}\right)=\sqrt{ } 1=1$
$0(x)+1(y)=2$
This is in the form of: $\mathrm{x} \cos 90^{\circ}+\mathrm{y} \sin 90^{\circ}=2$
$\therefore$ The above equation is of the form $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$, where $\theta=90^{\circ}$ and $\mathrm{p}=2$.
Perpendicular distance of the line from origin $=2$
The angle between the perpendicular and positive x -axis $=90^{\circ}$
(iii) $x-y=4$

Given:
The equation is $x-y+4=0$
The equation of the line in normal form is given by $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$ where ' $\theta$ ' is the angle between the perpendicular and the positive $x$-axis and ' $p$ ' is the perpendicular distance from the origin.

So now, $x-y=4$
Divide both the sides by $\sqrt{ }\left(1^{2}+1^{2}\right)=\sqrt{ }(1+1)=\sqrt{ } 2$
$x / \sqrt{ } 2-y / \sqrt{ } 2=4 / \sqrt{ } 2$
$(1 / \sqrt{ } 2) x+(-1 / \sqrt{ } 2) y=2 \sqrt{ } 2$
This is in the form: $\mathrm{x} \cos 315^{\circ}+\mathrm{y} \sin 315^{\circ}=2 \sqrt{ } 2$
$\therefore$ The above equation is of the form $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$, where $\theta=315^{\circ}$ and $\mathrm{p}=2 \sqrt{ }$.
Perpendicular distance of the line from origin $=2 \sqrt{ } 2$
The angle between the perpendicular and the positive x -axis $=315^{\circ}$
4. Find the distance of the point $(-1,1)$ from the line $\mathbf{1 2}(x+6)=5(y-2)$.

## Solution:

Given:
The equation of the line is $12(\mathrm{x}+6)=5(\mathrm{y}-2)$.
$12 \mathrm{x}+72=5 \mathrm{y}-10$
$12 x-5 y+82=0$.
Now, compare equation (1) with the general equation of line $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$, where $\mathrm{A}=12, \mathrm{~B}=-5$, and $\mathrm{C}=82$
Perpendicular distance (d) of a line $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$ from a point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ is given by
$\mathrm{d}=\frac{\left|\mathrm{Ax}_{1}+\mathrm{By}_{1}+\mathrm{C}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
Given point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(-1,1)$
$\therefore$ Distance of point $(-1,1)$ from the given line is

$$
\begin{aligned}
\mathrm{d} & =\frac{|12 \times(-1)+(-5) \times 1+82|}{\sqrt{12^{2}+(-5)^{2}}}=\frac{|-12-5+82|}{\sqrt{144+25}}=\frac{|65|}{\sqrt{169}}=\frac{65}{13} \text { units } \\
& =5 \text { units }
\end{aligned}
$$

$\therefore$ The distance is 5 units.
5. Find the points on the $x$-axis whose distances from the line $x / 3+y / 4=1$ are 4 units.

## Solution:

Given:
The equation of the line is $x / 3+y / 4=1$
$4 x+3 y=12$
$4 x+3 y-12=0$
Now, compare equation (1) with the general equation of line $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$, where $\mathrm{A}=4, \mathrm{~B}=3$, and $\mathrm{C}=-12$

Let $(a, 0)$ be the point on the $x$-axis whose distance from the given line is 4 units.
So, the perpendicular distance $(d)$ of a line $A x+B y+C=0$ from a point $\left(x_{1}, y_{1}\right)$ is given by
$\mathrm{d}=\frac{\left|\mathrm{Ax}_{1}+\mathrm{By}_{1}+\mathrm{C}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
$4=\frac{|4 a+3 \times 0-12|}{\sqrt{4^{2}+3^{2}}}$
$4=\frac{|4 a-12|}{\sqrt{16+9}}=\frac{|4 a-12|}{5}$
$|4 a-12|=4 \times 5$
$\pm(4 a-12)=20$
$4 a-12=20$ or $-(4 a-12)=20$
$4 \mathrm{a}=20+12$ or $4 \mathrm{a}=-20+12$
$\mathrm{a}=32 / 4$ or $\mathrm{a}=-8 / 4$
$\mathrm{a}=8$ or $\mathrm{a}=-2$
$\therefore$ The required points on the x -axis are $(-2,0)$ and $(8,0)$
6. Find the distance between parallel lines.
(i) $15 x+8 y-34=0$ and $15 x+8 y+31=0$
(ii) $l(x+y)+p=0$ and $l(x+y)-r=0$

## Solution:

(i) $15 x+8 y-34=0$ and $15 x+8 y+31=0$

Given:
The parallel lines are $15 x+8 y-34=0$ and $15 x+8 y+31=0$.
By using the formula,
The distance (d) between parallel lines $A x+B y+C_{1}=0$ and $A x+B y+C_{2}=0$ is given by
$\mathrm{d}=\frac{\left|\mathrm{C}_{1}-\mathrm{C}_{2}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
Where, $\mathrm{A}=15, \mathrm{~B}=8, \mathrm{C}_{1}=-34, \mathrm{C}_{2}=31$
Distance between parallel lines is

$$
\begin{aligned}
\mathrm{d} & =\frac{|-34-31|}{\sqrt{15^{2}+8^{2}}} \\
& =\frac{|-65|}{\sqrt{225+64}} \\
& =\frac{65}{\sqrt{289}} \\
& =65 / 17
\end{aligned}
$$

$\therefore$ The distance between parallel lines is $65 / 17$
(ii) $1(x+y)+p=0$ and $1(x+y)-r=0$

Given:
The parallel lines are $1(x+y)+p=0$ and $1(x+y)-r=0$
$l x+l y+p=0$ and $1 x+l y-r=0$
By using the formula,
The distance (d) between parallel lines $\mathrm{Ax}+\mathrm{By}+\mathrm{C}_{1}=0$ and $\mathrm{Ax}+\mathrm{By}+\mathrm{C}_{2}=0$ is given by
$\mathrm{d}=\frac{\left|\mathrm{C}_{1}-\mathrm{C}_{2}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
Where, $\mathrm{A}=1, \mathrm{~B}=1, \mathrm{C}_{1}=\mathrm{p}, \mathrm{C}_{2}=-\mathrm{r}$
Distance between parallel lines is

$$
\begin{aligned}
d & =\frac{|p-(-r)|}{\sqrt{1^{2}+1^{2}}} \\
& =\frac{|p+r|}{\sqrt{2} l} \\
& =\frac{|p+r|}{1 \sqrt{2}}
\end{aligned}
$$

$\therefore$ The distance between parallel lines is $|\mathrm{p}+\mathrm{r}| / 1 \sqrt{ } 2$
7. Find the equation of the line parallel to the line $3 x-4 y+2=0$ and passing through the point ( $-2,3$ ).

## Solution:

Given:

The line is $3 x-4 y+2=0$
So, $y=3 x / 4+2 / 4$
$=3 \mathrm{x} / 4+1 / 2$
Which is of the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$, where m is the slope of the given line.
The slope of the given line is $3 / 4$
We know that parallel lines have the same slope.
$\therefore$ Slope of other line $=\mathrm{m}=3 / 4$
The equation of line having slope $m$ and passing through $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ is given by
$y-y_{1}=m\left(x-x_{1}\right)$
$\therefore$ The equation of the line having slope $3 / 4$ and passing through $(-2,3)$ is
$y-3=3 / 4(x-(-2))$
$4 y-3 \times 4=3 x+3 \times 2$
$3 x-4 y=18$
$\therefore$ The equation is $3 x-4 y=18$
8. Find equation of the line perpendicular to the line $x-7 y+5=0$ and having $x$ intercept 3 .

## Solution:

Given:
The equation of line is $x-7 y+5=0$
So, $y=1 / 7 x+5 / 7$ [which is of the form $y=m x+c$, where $m$ is the slope of the given line.]
The slope of the given line is $1 / 7$
The slope of the line perpendicular to the line having slope $m$ is $-1 / \mathrm{m}$
The slope of the line perpendicular to the line having a slope of $1 / 7$ is $-1 /(1 / 7)=-7$
So, the equation of the line with slope -7 and the $x$-intercept 3 is given by $y=m(x-d)$
$y=-7(x-3)$
$y=-7 x+21$
$7 x+y=21$
$\therefore$ The equation is $7 \mathrm{x}+\mathrm{y}=21$
9. Find angles between the lines $\sqrt{ } 3 x+y=1$ and $x+\sqrt{ } 3 y=1$.

## Solution:

Given:
The lines are $\sqrt{3} x+y=1$ and $x+\sqrt{3} y=1$
So, $\mathrm{y}=-\sqrt{3} \mathrm{x}+1 \ldots$ (1) and
$y=-1 / \sqrt{3} x+1 / \sqrt{3}$
The slope of the line (1) is $m_{1}=-\sqrt{3}$, while the slope of the line (2) is $m_{2}=-1 / \sqrt{3}$
Let $\theta$ be the angle between two lines.
So,

$$
\begin{aligned}
\tan \theta & =\left|\frac{m_{1}-m_{2}}{1+m_{1} m_{2}}\right| \\
& =\left|\frac{-\sqrt{3}-\left(-\frac{1}{\sqrt{3}}\right)}{1+(-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)}\right|=\left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right|=\left|\frac{-2}{2 \times \sqrt{3}}\right| \\
& =1 / \sqrt{ } 3
\end{aligned}
$$

$\theta=30^{\circ}$
$\therefore$ The angle between the given lines is either $30^{\circ}$ or $180^{\circ}-30^{\circ}=150^{\circ}$
10. The line through the points $(h, 3)$ and $(4,1)$ intersects the line $7 x-9 y-19=0$. At the right angle. Find the value of $h$.

## Solution:

Let the slope of the line passing through $(h, 3)$ and $(4,1)$ be $m_{1}$
Then, $\mathrm{m}_{1}=(1-3) /(4-\mathrm{h})=-2 /(4-\mathrm{h})$
Let the slope of line $7 \mathrm{x}-9 \mathrm{y}-19=0$ be $\mathrm{m}_{2}$
$7 x-9 y-19=0$
So, $y=7 / 9 x-19 / 9$
$\mathrm{m}_{2}=7 / 9$
Since the given lines are perpendicular,
$\mathrm{m}_{1} \times \mathrm{m}_{2}=-1$
$-2 /(4-\mathrm{h}) \times 7 / 9=-1$
$-14 /(36-9 h)=-1$
$-14=-1 \times(36-9 h)$
$36-9 h=14$
$9 h=36-14$
$\mathrm{h}=22 / 9$
$\therefore$ The value of h is $22 / 9$
11. Prove that the line through the point $\left(x_{1}, y_{1}\right)$ and parallel to the line $A x+B y+C=0$ is $A\left(x-x_{1}\right)+B\left(y-y_{1}\right)=$ 0.

Solution:
Let the slope of line $A x+B y+C=0$ be $m$
$A x+B y+C=0$
So, $y=-A / B x-C / B$
$\mathrm{m}=-\mathrm{A} / \mathrm{B}$
By using the formula,
Equation of the line passing through point $\left(x_{1}, y_{1}\right)$ and having slope $m=-A / B$ is
$y-y_{1}=m\left(x-x_{1}\right)$
$y-y_{1}=-A / B\left(x-x_{1}\right)$
$B\left(y-y_{1}\right)=-A\left(x-x_{1}\right)$
$\therefore \mathrm{A}\left(\mathrm{x}-\mathrm{x}_{1}\right)+\mathrm{B}\left(\mathrm{y}-\mathrm{y}_{1}\right)=0$
So, the line through point $\left(x_{1}, y_{1}\right)$ and parallel to the line $A x+B y+C=0$ is $A\left(x-x_{1}\right)+B\left(y-y_{1}\right)=0$
Hence, proved.
12. Two lines passing through point $(2,3)$ intersects each other at an angle of $60^{\circ}$. If the slope of one line is 2 , find the equation of the other line.

## Solution:

Given: $\mathrm{m}_{1}=2$
Let the slope of the first line be $\mathrm{m}_{1}$
And let the slope of the other line be $m_{2}$.

The angle between the two lines is $60^{\circ}$.
So,

$$
\begin{aligned}
& \tan \theta=\left|\frac{m_{1}-m_{2}}{1+m_{1} m_{2}}\right| \\
& \tan 60^{\circ}=\left|\frac{2-m_{2}}{1+2 m_{2}}\right| \\
& \sqrt{3}= \pm\left(\frac{2-m_{2}}{1+2 m_{2}}\right) \\
& \text { i.e } e_{\text {ur }} \\
& \sqrt{3}=\frac{2-m_{2}}{1+2 m_{2}} \text { or } \sqrt{3}=-\left(\frac{2-m_{2}}{1+2 m_{2}}\right) \\
& \sqrt{3}\left(1+2 m_{2}\right)=2-m_{2} \text { or } \sqrt{3}\left(1+2 m_{2}\right)=-\left(2-m_{2}\right) \\
& \sqrt{3}+2 \sqrt{3} m_{2}+m_{2}=2 \text { or } \sqrt{3}+2 \sqrt{3} m_{2}-m_{2}=-2 \\
& m_{2}(2 \sqrt{3}+1)=2-\sqrt{3} \text { or } m_{2}(2 \sqrt{3}-1)=-(2+\sqrt{3}) \\
& m_{2}=\frac{2-\sqrt{3}}{(2 \sqrt{3}+1)} \text { or } m_{2}=\frac{-(2+\sqrt{3})}{(2 \sqrt{3}-1)}
\end{aligned}
$$

So now let us consider

## Case 1: When

$$
\mathrm{m}_{2}=\frac{2-\sqrt{3}}{(2 \sqrt{3}+1)}
$$

The equation of the line passing through point $(2,3)$ and having a slope $m_{2}$ is

$$
\begin{aligned}
& y-3=\left(\frac{2-\sqrt{3}}{2 \sqrt{3}+1}\right)(x-2) \\
& (2 \sqrt{3}+1) y-3(2 \sqrt{3}+1)=(2-\sqrt{3}) x-2(2-\sqrt{3}) \\
& (\sqrt{3}-2) x+(2 \sqrt{3}+1) y=-4+2 \sqrt{3}+6 \sqrt{3}+3 \\
& (\sqrt{3}-2) x+(2 \sqrt{3}+1) y=8 \sqrt{3}-1
\end{aligned}
$$

$\therefore$ Equation of the other line is $(\sqrt{3}-2) \mathrm{x}+(2 \sqrt{3}+1) \mathrm{y}=8 \sqrt{3}-1$

## Case 2: When

$\mathrm{m}_{2}=\frac{-(2+\sqrt{3})}{(2 \sqrt{3}-1)}$
The equation of the line passing through point $(2,3)$ and having a slope $m_{2}$ is

$$
\begin{aligned}
& y-3=\frac{-(2+\sqrt{3})}{(2 \sqrt{3}-1)}(x-2) \\
& y-3=\frac{-(2+\sqrt{3})}{(2 \sqrt{3}-1)}(x-2) \\
& (2 \sqrt{3}-1) y-3(2 \sqrt{3}-1)=-(2+\sqrt{3}) x+2(2+\sqrt{3}) \\
& (2 \sqrt{3}-1) y+(2+\sqrt{3}) x=4+2 \sqrt{3}+6 \sqrt{3}-3 \\
& (2 \sqrt{3}-1) y+(2+\sqrt{3}) x=8 \sqrt{3}+1
\end{aligned}
$$

$\therefore$ Equation of the other line is $(2 \sqrt{3}-1) \mathrm{y}+(2+\sqrt{3}) \mathrm{x}=8 \sqrt{3}+1$
13. Find the equation of the right bisector of the line segment joining the points $(3,4)$ and $(-1,2)$.

## Solution:

Given:
The right bisector of a line segment bisects the line segment at $90^{\circ}$.
End-points of the line segment AB are given as $\mathrm{A}(3,4)$ and $\mathrm{B}(-1,2)$.
Let the mid-point of AB be (x, y).
$\mathrm{x}=(3-1) / 2=2 / 2=1$
$y=(4+2) / 2=6 / 2=3$
$(x, y)=(1,3)$
Let the slope of line $A B$ be $m_{1}$
$m_{1}=(2-4) /(-1-3)$
$=-2 /(-4)$
$=1 / 2$
And let the slope of the line perpendicular to AB be $\mathrm{m}_{2}$
$m_{2}=-1 /(1 / 2)$
$=-2$

The equation of the line passing through $(1,3)$ and having a slope of -2 is
$(y-3)=-2(x-1)$
$y-3=-2 x+2$
$2 x+y=5$
$\therefore$ The required equation of the line is $2 x+y=5$
14. Find the coordinates of the foot of the perpendicular from the point $(-1,3)$ to the line $3 x-4 y-16=0$.

## Solution:

Let us consider the coordinates of the foot of the perpendicular from $(-1,3)$ to the line $3 x-4 y-16=0$ be $(a, b)$
So, let the slope of the line joining $(-1,3)$ and $(a, b)$ be $m_{1}$
$\mathrm{m}_{1}=(\mathrm{b}-3) /(\mathrm{a}+1)$
And let the slope of the line $3 x-4 y-16=0$ be $m_{2}$
$y=3 / 4 x-4$
$\mathrm{m}_{2}=3 / 4$
Since these two lines are perpendicular, $\mathrm{m}_{1} \times \mathrm{m}_{2}=-1$
$(b-3) /(a+1) \times(3 / 4)=-1$
$(3 b-9) /(4 a+4)=-1$
$3 b-9=-4 a-4$
$4 a+3 b=5$ $\qquad$
Point $(a, b)$ lies on the line $3 x-4 y=16$
$3 a-4 b=16$ $\qquad$
Solving equations (1) and (2), we get
$\mathrm{a}=68 / 25$ and $\mathrm{b}=-49 / 25$
$\therefore$ The coordinates of the foot of perpendicular are $(68 / 25,-49 / 25)$
15. The perpendicular from the origin to the line $y=m x+c$ meets it at the point $(-1,2)$. Find the values of $m$ and c.

## Solution:

Given:
The perpendicular from the origin meets the given line at $(-1,2)$.
The equation of the line is $y=m x+c$
The line joining the points $(0,0)$ and $(-1,2)$ is perpendicular to the given line.
So, the slope of the line joining $(0,0)$ and $(-1,2)=2 /(-1)=-2$
The slope of the given line is $m$.
$\mathrm{m} \times(-2)=-1$
$\mathrm{m}=1 / 2$
Since point $(-1,2)$ lies on the given line,
$y=m x+c$
$2=1 / 2 \times(-1)+c$
$\mathrm{c}=2+1 / 2=5 / 2$
$\therefore$ The values of m and c are $1 / 2$ and $5 / 2$, respectively.
16. If $p$ and $q$ are the lengths of perpendiculars from the origin to the lines $x \cos \theta-y \sin \theta=k \cos 2 \theta$ and $x \sec \theta$ $+y \operatorname{cosec} \theta=k$, respectively, prove that $p^{2}+4 q^{2}=k^{2}$

## Solution:

Given:
The equations of the given lines are
$\mathrm{x} \cos \theta-\mathrm{y} \sin \theta=\mathrm{k} \cos 2 \theta$
$\mathrm{x} \sec \theta+\mathrm{y} \operatorname{cosec} \theta=\mathrm{k}$ $\qquad$
Perpendicular distance (d) of a line $A x+B y+C=0$ from a point $\left(x_{1}, y_{1}\right)$ is given by
$\mathrm{d}=\frac{\left|\mathrm{Ax}_{1}+\mathrm{By}_{1}+\mathrm{C}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
So now, compare equation (1) to the general equation of line i.e., $\mathrm{Ax}+\mathrm{By}+\mathrm{C}$ $=0$, we get
$\mathrm{A}=\cos \theta, \mathrm{B}=-\sin \theta$, and $\mathrm{C}=-\mathrm{k} \cos 2 \theta$
It is given that p is the length of the perpendicular from $(0,0)$ to line (1).
$p=\frac{|A \times 0+B \times 0+C|}{\sqrt{A^{2}+B^{2}}}=\frac{|C|}{\sqrt{A^{2}+\mathrm{B}^{2}}}=\frac{|-k \cos 2 \theta|}{\sqrt{\cos ^{2} \theta+\sin ^{2} \theta}}=k \cos 2 \theta$
$\mathrm{p}=\mathrm{k} \cos 2 \theta$
Let us square on both sides we get,
$\mathbf{P}^{2}=k^{2} \cos ^{2} 2 \theta \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ (3)
Now, compare equation (2) to the general equation of line i.e., $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$, we get
$\mathrm{A}=\sec \theta, \mathrm{B}=\operatorname{cosec} \theta$, and $\mathrm{C}=-\mathrm{k}$
It is given that q is the length of the perpendicular from $(0,0)$ to line (2)

$$
\begin{aligned}
q & =\frac{|A \times 0+B \times 0+C|}{\sqrt{A^{2}+B^{2}}} \\
& =\frac{|C|}{\sqrt{A^{2}+\mathrm{B}^{2}}} \\
& =\frac{|-\mathrm{k}|}{\sqrt{\sec ^{2} \theta+\operatorname{cosec}^{2} \theta}} \\
& =\frac{\mathrm{k}}{\sqrt{\sec ^{2} \theta+\operatorname{cosec}^{2} \theta}} \\
& =\frac{\mathrm{k}}{\sqrt{\frac{1}{\cos ^{2} \theta}+\frac{1}{\sin ^{2} \theta}}}=\frac{\mathrm{k} \cos \theta \sin \theta}{\sqrt{\cos ^{2} \theta+\sin ^{2} \theta}}=\mathrm{k} \cos \theta \sin \theta
\end{aligned}
$$

$\mathrm{q}=\mathrm{k} \cos \theta \sin \theta$
Multiply both sides by 2 , and we get
$2 \mathrm{q}=2 \mathrm{k} \cos \theta \sin \theta=\mathrm{k} \times 2 \sin \theta \cos \theta$
$2 \mathrm{q}=\mathrm{k} \sin 2 \theta$
Squaring both sides, we get
$4 q^{2}=k^{2} \sin ^{2} 2 \theta$

Now add (3) and (4); we get
$p^{2}+4 q^{2}=k^{2} \cos ^{2} 2 \theta+k^{2} \sin ^{2} 2 \theta$
$\mathrm{p}^{2}+4 \mathrm{q}^{2}=\mathrm{k}^{2}\left(\cos ^{2} 2 \theta+\sin ^{2} 2 \theta\right)\left[\right.$ Since, $\left.\cos ^{2} 2 \theta+\sin ^{2} 2 \theta=1\right]$
$\therefore \mathrm{p}^{2}+4 \mathrm{q}^{2}=\mathrm{k}^{2}$
Hence proved.
17. In the triangle $A B C$ with vertices $A(2,3), B(4,-1)$ and $C(1,2)$, find the equation and length of altitude from vertex $A$.

## Solution:



Let $A D$ be the altitude of triangle $A B C$ from vertex $A$.
So, AD is perpendicular to BC .
Given:
Vertices A $(2,3), \mathrm{B}(4,-1)$ and $\mathrm{C}(1,2)$
Let the slope of the line $\mathrm{BC}=\mathrm{m}_{1}$
$m_{1}=(-1-2) /(4-1)$
$m_{1}=-1$
Let the slope of the line AD be $\mathrm{m}_{2}$
$A D$ is perpendicular to $B C$.
$m_{1} \times m_{2}=-1$
$-1 \times m_{2}=-1$
$\mathrm{m}_{2}=1$
The equation of the line passing through the point $(2,3)$ and having a slope of 1 is
$y-3=1 \times(x-2)$
$y-3=x-2$
$y-x=1$
Equation of the altitude from vertex $A=y-x=1$
Length of $\mathrm{AD}=$ Length of the perpendicular from $\mathrm{A}(2,3)$ to BC
The equation of BC is
$y+1=-1 \times(x-4)$
$y+1=-x+4$
$x+y-3=0$
Perpendicular distance $(d)$ of a line $A x+B y+C=0$ from a point $\left(x_{1}, y_{1}\right)$ is given by
$\mathrm{d}=\frac{\left|A \mathrm{Ax}_{1}+\mathrm{By}_{1}+\mathrm{C}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
Now, compare equation (1) to the general equation of the line, i.e., $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$; we get
Length of $\mathrm{AD}=$
$\frac{|1 \times 2+1 \times 3-3|}{\sqrt{1^{2}+1^{2}}}=\frac{|2|}{\sqrt{2}}=\sqrt{2}$ units
[where, $\mathrm{A}=1, \mathrm{~B}=1$ and $\mathrm{C}=-3$ ]
$\therefore$ The equation and the length of the altitude from vertex $A$ are $y-x=1$ and
$\sqrt{2}$ units, respectively.
18. If $p$ is the length of the perpendicular from the origin to the line whose intercepts on the axes are a and $b$, then show that $1 / p^{2}=1 / a^{2}+1 / b^{2}$

## Solution:

The equation of a line whose intercepts on the axes are $a$ and $b$ is $x / a+y / b=1$
$b x+a y=a b$
$b x+a y-a b=0$
Perpendicular distance $(d)$ of a line $A x+B y+C=0$ from a point $\left(x_{1}, y_{1}\right)$ is given by
$\mathrm{d}=\frac{\left|\mathrm{Ax}_{1}+\mathrm{By}_{1}+\mathrm{C}\right|}{\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}$
Now compare equation (1) to the general equation of line i.e., $\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$, we get
$\mathrm{A}=\mathrm{b}, \mathrm{B}=\mathrm{a}$ and $\mathrm{C}=-\mathrm{ab}$
If $p$ is the length of the perpendicular from point $\left(x_{1}, y_{1}\right)=(0,0)$ to line (1), we get
$\mathrm{p}=\frac{|\mathrm{A} \times 0+\mathrm{B} \times 0-\mathrm{ab}|}{\sqrt{\mathrm{a}^{2}+\mathrm{b}^{2}}}$

$$
=\frac{|-a b|}{\sqrt{a^{2}+b^{2}}}
$$

Now, square on both sides; we get
$\mathrm{p}^{2}=\frac{(-\mathrm{ab})^{2}}{\mathrm{a}^{2}+\mathrm{b}^{2}}$
$\frac{1}{\mathrm{p}^{2}}=\frac{\mathrm{a}^{2}+\mathrm{b}^{2}}{\mathrm{a}^{2} \mathrm{~b}^{2}}$
$\frac{1}{p^{2}}=\frac{a^{2}}{a^{2} b^{2}}+\frac{b^{2}}{a^{2} b^{2}}$
$\therefore \frac{1}{p^{2}}=\frac{1}{a^{2}}+\frac{1}{b^{2}}$
$\therefore 1 / \mathrm{p}^{2}=1 / \mathrm{a}^{2}+1 / \mathrm{b}^{2}$
Hence, proved.

## MISCELLANEOUS EXERCISE

1. Find the values of $k$ for which the line $(k-3) x-\left(4-k^{2}\right) y+k^{2}-7 k+6=0$ is
(a) Parallel to the x-axis
(b) Parallel to the $y$-axis
(c) Passing through the origin

## Solution:

It is given that
$(k-3) x-\left(4-k^{2}\right) y+k^{2}-7 k+6=0$
(a) Here, if the line is parallel to the $x$-axis

Slope of the line $=$ Slope of the x -axis
It can be written as
$\left(4-k^{2}\right) y=(k-3) x+k^{2}-7 k+6=0$
We get
$y=\frac{(k-3)}{\left(4-k^{2}\right)} x+\frac{k^{2}-7 k+6}{\left(4-k^{2}\right)}$
Which is of the form $y=m x+c$
Here the slope of the given line
$=\frac{(k-3)}{\left(4-k^{2}\right)}$
Consider the slope of $x$-axis $=0$

$$
\frac{(k-3)}{\left(4-k^{2}\right)}=0
$$

By further calculation,
$\mathrm{k}-3=0$
$\mathrm{k}=3$
Hence, if the given line is parallel to the x -axis, then the value of k is 3 .
(b) Here, if the line is parallel to the $y$-axis, it is vertical, and the slope will be undefined.

So, the slope of the given line
$=\frac{(k-3)}{\left(4-k^{2}\right)}$
Here,
$\frac{(k-3)}{\left(4-k^{2}\right)}$ is undefined at $k^{2}=4$
$k^{2}=4$
$\mathrm{k}= \pm 2$
Hence, if the given line is parallel to the y -axis, then the value of k is $\pm 2$.
(c) Here, if the line is passing through $(0,0)$, which is the origin satisfies the given equation of the line.
$(\mathrm{k}-3)(0)-\left(4-\mathrm{k}^{2}\right)(0)+\mathrm{k}^{2}-7 \mathrm{k}+6=0$
By further calculation,
$\mathrm{k}^{2}-7 \mathrm{k}+6=0$
Separating the terms,
$\mathrm{k}^{2}-6 \mathrm{k}-\mathrm{k}+6=0$
We get
$(k-6)(k-1)=0$
$\mathrm{k}=1$ or 6
Hence, if the given line is passing through the origin, then the value of k is either 1 or 6 .
2. Find the values of $\theta$ and $p$, if the equation $x \cos \theta+y \sin \theta=p$ is the normal form of the line $\sqrt{ } 3 x+y+2=0$.

## Solution:

It is given that
$\sqrt{ } 3 x+y+2=0$
It can be reduced as
$\sqrt{3} x+y+2=0$
$-\sqrt{ } 3 x-y=2$
By dividing both sides by $\sqrt{(-\sqrt{3})^{2}+(-1)^{2}}=2$, we get
$-\frac{\sqrt{3}}{2} x-\frac{1}{2} y=\frac{2}{2}$
It can be written as
$\left(-\frac{\sqrt{3}}{2}\right) x+\left(-\frac{1}{2}\right) y=1$
By comparing equation (1) to $\mathrm{x} \cos \theta+\mathrm{y} \sin \theta=\mathrm{p}$, we get

$$
\cos \theta=-\frac{\sqrt{3}}{2}, \sin \theta=-\frac{1}{2}, \text { and } p=1
$$

Here the values of $\sin \theta$ and $\cos \theta$ are negative
$\theta=\pi+\frac{\pi}{6}=\frac{7 \pi}{6}$
Hence, the respective values of $\theta$ and p are $7 \pi / 6$ and 1 .
3. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are $\mathbf{1}$ and $\mathbf{- 6}$, respectively.

## Solution:

Consider the intercepts cut by the given lines on the a and b axes.
$a+b=1$ $\qquad$
$a b=-6$ $\qquad$
By solving both equations, we get
$\mathrm{a}=3$ and $\mathrm{b}=-2$ or $\mathrm{a}=-2$ and $\mathrm{b}=3$
We know that the equation of the line whose intercepts on the a and b axes is
$\frac{x}{a}+\frac{y}{b}=1$ or $b x+a y-a b=0$

Case $\mathrm{I}-\mathrm{a}=3$ and $\mathrm{b}=-2$
So, the equation of the line is $-2 x+3 y+6=0$, i.e. $2 x-3 y=6$
Case II $-\mathrm{a}=-2$ and $\mathrm{b}=3$
So, the equation of the line is $3 x-2 y+6=0$, i.e. $-3 x+2 y=6$
Hence, the required equation of the lines are $2 x-3 y=6$ and $-3 x+2 y=6$
4. What are the points on the $y$-axis whose distance from the line $x / 3+y / 4=1$ is 4 units?

## Solution:

Consider $(0, b)$ as the point on the $y$-axis whose distance from line $x / 3+y / 4=1$ is 4 units.
It can be written as $4 x+3 y-12=0$ $\qquad$
By comparing equation (1) to the general equation of line $A x+B y+C=0$, we get
$\mathrm{A}=4, \mathrm{~B}=3$ and $\mathrm{C}=-12$
We know that the perpendicular distance (d) of a line $A x+B y+C=0$ from $\left(x_{1}, y_{1}\right)$ is written as

$$
d=\frac{\left|A x_{1}+B y_{1}+C\right|}{\sqrt{A^{2}+B^{2}}}
$$

If $(0, b)$ is the point on the $y$-axis whose distance from line $x / 3+y / 4=1$ is 4 units, then
$4=\frac{|4(0)+3(b)-12|}{\sqrt{4^{2}+3^{2}}}$

## By further calculation

$$
4=\frac{|3 b-12|}{5}
$$

By cross multiplication,
$20=|3 b-12|$
We get
$20= \pm(3 b-12)$
Here, $20=(3 b-12)$ or $20=-(3 b-12)$
It can be written as
$3 b=20+12$ or $3 b=-20+12$

So, we get
$\mathrm{b}=32 / 3$ or $\mathrm{b}=-8 / 3$
Hence, the required points are $(0,32 / 3)$ and $(0,-8 / 3)$.
5. Find the perpendicular distance from the origin to the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$.

## Solution:

Here the equation of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is written as

$$
y-\sin \theta=\frac{\sin \phi-\sin \theta}{\cos \phi-\cos \theta}(x-\cos \theta)
$$

By cross multiplication

$$
y(\cos \phi-\cos \theta)-\sin \theta(\cos \phi-\cos \theta)=x(\sin \phi-\sin \theta)-\cos \theta(\sin \phi-\sin \theta)
$$

By multiplying the terms we get
$x(\sin \theta-\sin \phi)+y(\cos \phi-\cos \theta)+\cos \theta \sin \phi-\cos \theta \sin \theta-\sin \theta \cos \phi+\sin \theta \cos \theta=0$
On further simplification
$x(\sin \theta-\sin \phi)+y(\cos \phi-\cos \theta)+\sin (\phi-\theta)=0$
So we get
$A x+B y+C=0$, where $A=\sin \theta-\sin \phi, B=\cos \phi-\cos \theta$, and $C=\sin (\phi-\theta)$
We know that the perpendicular distance (d) of a line $A x+B y+C=0$ from $\left(x_{1}, y_{1}\right)$ is written as
$d=\frac{\left|A x_{1}+B y_{1}+C\right|}{\sqrt{A^{2}+B^{2}}}$
So the perpendicular distance $(\mathrm{d})$ of the given line from $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(0,0)$ is
$d=\frac{|(\sin \theta-\sin \phi)(0)+(\cos \phi-\cos \theta)(0)+\sin (\phi-\theta)|}{\sqrt{(\sin \theta-\sin \phi)^{2}+(\cos \phi-\cos \theta)^{2}}}$
By expanding using formula
$=\frac{|\sin (\phi-\theta)|}{\sqrt{\sin ^{2} \theta+\sin ^{2} \phi-2 \sin \theta \sin \phi+\cos ^{2} \phi+\cos ^{2} \theta-2 \cos \phi \cos \theta}}$
Grouping of terms
$=\frac{|\sin (\phi-\theta)|}{\sqrt{\left(\sin ^{2} \theta+\cos ^{2} \theta\right)+\left(\sin ^{2} \phi+\cos ^{2} \phi\right)-2(\sin \theta \sin \phi+\cos \theta \cos \phi)}}$
By further simplification

$$
=\frac{|\sin (\phi-\theta)|}{\sqrt{1+1-2(\cos (\phi-\theta))}}
$$

Taking out 2 as common
$=\frac{|\sin (\phi-\theta)|}{\sqrt{2(1-\cos (\phi-\theta))}}$
Using the formula
$=\frac{|\sin (\phi-\theta)|}{\sqrt{2\left(2 \sin ^{2}\left(\frac{\phi-\theta}{2}\right)\right)}}$
We get
$=\frac{|\sin (\phi-\theta)|}{\left|2 \sin \left(\frac{\phi-\theta}{2}\right)\right|}$
6. Find the equation of the line parallel to the $y$-axis and draw through the point of intersection of the lines $x$ $7 y+5=0$ and $3 x+y=0$.

## Solution:

Here, the equation of any line parallel to the $y$-axis is of the form
$\mathrm{x}=\mathrm{a}$.
Two given lines are
$x-7 y+5=0$
$3 x+y=0$
By solving equations (2) and (3), we get
$x=-5 / 22$ and $y=15 / 22$
$(-5 / 22,15 / 22)$ is the point of intersection of lines (2) and (3)
If the line $\mathrm{x}=$ a passes through point $(-5 / 22,15 / 22)$, we get $\mathrm{a}=-5 / 22$
Hence, the required equation of the line is $x=-5 / 22$
7. Find the equation of a line drawn perpendicular to the line $x / 4+y / 6=1$ through the point where it meets the y -axis.

## Solution:

It is given that
$x / 4+y / 6=1$
We can write it as
$3 x+2 y-12=0$
So, we get
$y=-3 / 2 x+6$, which is of the form $y=m x+c$
Here, the slope of the given line $=-3 / 2$
So, the slope of line perpendicular to the given line $=-1 /(-3 / 2)=2 / 3$
Consider the given line intersects, the $y$-axis at ( $0, \mathrm{y}$ )
By substituting x as zero in the equation of the given line,
$y / 6=1$
$y=6$
Hence, the given line intersects the $y$-axis at $(0,6)$.
We know that the equation of the line that has a slope of $2 / 3$ and passes through the point $(0,6)$ is
$(y-6)=2 / 3(x-0)$
By further calculation,
$3 y-18=2 x$
So, we get
$2 x-3 y+18=0$
Hence, the required equation of the line is $2 x-3 y+18=0$
8. Find the area of the triangle formed by the lines $y-x=0, x+y=0$ and $x-k=0$.

Solution:
It is given that
$y-x=0$ $\qquad$
$x+y=0$ $\qquad$
$\mathrm{x}-\mathrm{k}=0$ $\qquad$
Here, the point of intersection of
Lines (1) and (2) is
$\mathrm{x}=0$ and $\mathrm{y}=0$
Lines (2) and (3) is
$x=k$ and $y=-k$
Lines (3) and (1) is
$\mathrm{x}=\mathrm{k}$ and $\mathrm{y}=\mathrm{k}$
So, the vertices of the triangle formed by the three given lines are $(0,0),(k,-k)$ and $(k, k)$.
Here, the area of triangle whose vertices are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ and $\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$ is
$1 / 2\left|x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right|$
So, the area of the triangle formed by the three given lines is
$=1 / 2|0(-k-k)+k(k-0)+k(0+k)|$ square units
By further calculation,
$=1 / 2\left|k^{2}+\mathrm{k}^{2}\right|$ square units
So, we get
$=1 / 2\left|2 \mathrm{k}^{2}\right|$
$=\mathrm{k}^{2}$ square units
9. Find the value of $p$ so that the three lines $3 x+y-2=0, p x+2 y-3=0$ and $2 x-y-3=0$ may intersect at one point.

## Solution:

It is given that
$3 x+y-2=0$ $\qquad$
$p x+2 y-3=0$ $\qquad$
$2 x-y-3=0$ $\qquad$
By solving equations (1) and (3), we get
$x=1$ and $y=-1$
Here, the three lines intersect at one point, and the point of intersection of lines (1) and (3) will also satisfy line (2)
$\mathrm{p}(1)+2(-1)-3=0$
By further calculation,
$\mathrm{p}-2-3=0$
So we get
$p=5$
Hence, the required value of p is 5 .
10. If three lines whose equations are $y=m_{1} x+c_{1}, y=m_{2} x+c_{2}$ and $y=m_{3} x+c_{3}$ are concurrent, then show that $\mathbf{m}_{1}\left(\mathbf{c}_{2}-\mathbf{c}_{3}\right)+\mathbf{m}_{2}\left(\mathbf{c}_{3}-\mathbf{c}_{1}\right)+\mathbf{m}_{3}\left(\mathbf{c}_{1}-\mathbf{c}_{2}\right)=0$.

## Solution:

It is given that
$y=m_{1} x+c_{1}$
$\mathrm{y}=\mathrm{m}_{2} \mathrm{x}+\mathrm{c}_{2}$
$y=m_{3} x+c_{3}$
By subtracting equation (1) from (2), we get
$0=\left(\mathrm{m}_{2}-\mathrm{m}_{1}\right) \mathrm{x}+\left(\mathrm{c}_{2}-\mathrm{c}_{1}\right)$
$\left(m_{1}-m_{2}\right) x=c_{2}-c_{1}$
So we get
$x=\frac{c_{2}-c_{1}}{m_{1}-m_{2}}$
By substituting this value in equation (1) we get
$y=m_{1}\left(\frac{c_{2}-c_{1}}{m_{1}-m_{2}}\right)+c_{1}$
By multiplying the terms
$y=\frac{m_{1} c_{2}-m_{1} c_{1}}{m_{1}-m_{2}}+c_{1}$
Taking LCM
$y=\frac{m_{1} c_{2}-m_{1} c_{1}+m_{1} c_{1}-m_{2} c_{1}}{m_{1}-m_{2}}$
On further simplification
$y=\frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}}$
Here
$\left(\frac{c_{2}-c_{1}}{m_{1}-m_{2}}, \frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}}\right)$ is the point of intersection of lines (1) and (2)
Lines (1), (2) and (3) are concurrent. So the point of intersection of lines (1) and (2) will satisfy equation (3)

$$
\frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}}=m_{3}\left(\frac{c_{2}-c_{1}}{m_{1}-m_{2}}\right)+c_{3}
$$

By multiplying the terms and taking LCM

$$
\frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}}=\frac{m_{3} c_{2}-m_{3} c_{1}+c_{3} m_{1}-c_{3} m_{2}}{m_{1}-m_{2}}
$$

By cross multiplication
$m_{1} c_{2}-m_{2} c_{1}-m_{3} c_{2}+m_{3} c_{1}-c_{3} m_{1}+c_{3} m_{2}=0$
Taking out the common terms,
$\mathrm{m}_{1}\left(\mathrm{c}_{2}-\mathrm{c}_{3}\right)+\mathrm{m}_{2}\left(\mathrm{c}_{3}-\mathrm{c}_{1}\right)+\mathrm{m}_{3}\left(\mathrm{c}_{1}-\mathrm{c}_{2}\right)=0$
Therefore, $m_{1}\left(c_{2}-c_{3}\right)+m_{2}\left(c_{3}-c_{1}\right)+m_{3}\left(c_{1}-c_{2}\right)=0$
11. Find the equation of the lines through the point $(3,2)$, which makes an angle of $45^{\circ}$ with the line $x-2 y=3$.

## Solution:

Consider $\mathrm{m}_{1}$ as the slope of the required line
It can be written as
$y=1 / 2 x-3 / 2$ which is of the form $y=m x+c$
So, the slope of the given line $m_{2}=1 / 2$
We know that the angle between the required line and line $x-2 y=3$ is $45^{\circ}$
If $\theta$ is the acute angle between lines $1_{1}$ and $l_{2}$ with slopes $m_{1}$ and $m_{2}$.
$\tan \theta=\left|\frac{m_{2}-m_{1}}{1+m_{1} m_{2}}\right|$

## We get

$\tan 45^{\circ}=\frac{\left|m_{1}-m_{2}\right|}{1+m_{1} m_{2}}$
Substituting the values
$I=\left|\frac{\frac{1}{2}-m_{1}}{1+\frac{m_{1}}{2}}\right|$
By taking LCM
$1=\left|\frac{\left(\frac{1-2 m_{1}}{2}\right)}{\frac{2+m_{1}}{2}}\right|$

On further calculation
$1=\left|\frac{1-2 m_{1}}{2+m_{1}}\right|$
We get
$1= \pm\left(\frac{1-2 m_{1}}{2+m_{1}}\right)$
Here

$$
1=\frac{1-2 m_{1}}{2+m_{1}} \text { or } 1=-\left(\frac{1-2 m_{1}}{2+m_{1}}\right)
$$

It can be written as
$2+m_{1}=1-2 m_{1}$ or $2+m_{1}=-1+2 m_{1}$
$m_{1}=-1 / 3$ or $m_{1}=3$
Case I $-\mathrm{m}_{1}=3$
Here, the equation of the line passing through $(3,2)$ and having a slope 3 is
$\mathrm{y}-2=3(\mathrm{x}-3)$
By further calculation,
$y-2=3 x-9$
So, we get
$3 x-y=7$
Case II $-\mathrm{m}_{1}=-1 / 3$
Here, the equation of the line passing through $(3,2)$ and having a slope $-1 / 3$ is
$y-2=-1 / 3(x-3)$
By further calculation,
$3 y-6=-x+3$
So, we get
$x+3 y=9$
Hence, the equations of the lines are $3 x-y=7$ and $x+3 y=9$
12. Find the equation of the line passing through the point of intersection of the lines $4 x+7 y-3=0$ and $2 x-$ $3 y+1=0$ that has equal intercepts on the axes.

## Solution:

Consider the equation of the line having equal intercepts on the axes as
$x / a+y / a=1$
It can be written as
$x+y=a$ $\qquad$
By solving equations $4 x+7 y-3=0$ and $2 x-3 y+1=0$, we get
$x=1 / 13$ and $y=5 / 13$
$(1 / 13,5 / 13)$ is the point of intersection of two given lines.
We know that equation (1) passes through the point $(1 / 13,5 / 13)$.
$1 / 13+5 / 13=\mathrm{a}$
$a=6 / 13$
So, equation (1) passes through ( $1 / 13,5 / 13$ ).
$1 / 13+5 / 13=a$
We get
$a=6 / 13$
Her, equation (1) becomes
$x+y=6 / 13$
$13 x+13 y=6$
Hence, the required equation of the line is $13 x+13 y=6$
13. Show that the equation of the line passing through the origin and making an angle $\theta$ with the line $\mathbf{y}=\mathrm{mx}+\mathrm{c}$ is $\frac{y}{x}=\frac{m \pm \tan \theta}{1 \mp m \tan \theta}$.

## Solution:

Consider $\mathrm{y}=\mathrm{m}_{1} \mathrm{x}$ as the equation of the line passing through the origin

It is given that the line makes an angle $\theta$ with line $y=m x+c$, then angle $\theta$ is written as
$\tan \theta=\left|\frac{\mathrm{m}_{1}-\mathrm{m}}{1+\mathrm{m}_{1} \mathrm{~m}}\right|$
By substituting the values
$\tan \theta=\left|\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}\right|$
We get
$\tan \theta= \pm\left(\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}\right)$
Here
$\tan \theta=\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}$ or $\tan \theta=-\left(\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}\right)$
Case I-
$\tan \theta=\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}$
We can write it as
$\tan \theta+\frac{y}{x} m \tan \theta=\frac{y}{x}-m$
By further simplification
$\mathrm{m}+\tan \theta=\frac{\mathrm{y}}{\mathrm{x}}(1-\mathrm{m} \tan \theta)$
So we get
$\frac{\mathrm{y}}{\mathrm{x}}=\frac{\mathrm{m}+\tan \theta}{1-\mathrm{m} \tan \theta}$
Case II -
$\tan \theta=-\left(\frac{\frac{y}{x}-m}{1+\frac{y}{x} m}\right)$
We can write it as
$\tan \theta+\frac{y}{x} m \tan \theta=-\frac{y}{x}+m$
By further simplification
$\frac{\mathrm{y}}{\mathrm{x}}(1+\mathrm{m} \tan \theta)=\mathrm{m}-\tan \theta$
So we get
$\frac{\mathrm{y}}{\mathrm{x}}=\frac{\mathrm{m}-\tan \theta}{1+\mathrm{m} \tan \theta}$
Hence, the required line is given by
$\frac{\mathrm{y}}{\mathrm{x}}=\frac{\mathrm{m} \pm \tan \theta}{1 \mp \mathrm{~m} \tan \theta}$
14. In what ratio, the line joining $(-1,1)$ and $(5,7)$ is divided by the line $x+y=4$ ?

Solution:

We know that the equation of the line joining the points $(-1,1)$ and $(5,7)$ is given by
$y-1=\frac{7-1}{5+1}(x+1)$
By further calculation
$y-1=\frac{6}{6}(x+1)$
So we get
$x-y+2=0$ $\qquad$
So the equation of the given line is
$x+y-4=0$ $\qquad$
Here the point of intersection of lines (1) and (2) is given by
$\mathrm{x}=1$ and $\mathrm{y}=3$
Consider $(1,3)$ divide the line segment joining $(-1,1)$ and $(5,7)$ in the ratio $1: k$.
Using the section formula

$$
(1,3)=\left(\frac{k(-1)+1(5)}{1+k}, \frac{k(1)+1(7)}{1+k}\right)
$$

By further calculation
$(1,3)=\left(\frac{-k+5}{1+k}, \frac{k+7}{1+k}\right)$
So we get
$\frac{-k+5}{1+k}=1, \frac{k+7}{1+k}=3$
We can write it as
$\frac{-k+5}{1+k}=1$
By cross multiplication,
$-\mathrm{k}+5=1+\mathrm{k}$
We get
$2 \mathrm{k}=4$
$\mathrm{k}=2$

Hence, the line joining the points $(-1,1)$ and $(5,7)$ is divided by the line $\mathrm{x}+\mathrm{y}=4$ in the ratio 1:2.
15. Find the distance of the line $4 x+7 y+5=0$ from the point $(1,2)$ along the line $2 x-y=0$.

## Solution:

It is given that
$2 x-y=0$
$4 x+7 y+5=0$
Here, $\mathrm{A}(1,2)$ is a point on the line (1).
Consider B as the point of intersection of lines (1) and (2).


By solving equations (1) and (2), we get $x=-5 / 18$ and $y=-5 / 9$
So, the coordinates of point B are $(-5 / 18,-5 / 9)$.
From the distance formula, the distance between A and B
$\mathrm{AB}=\sqrt{\left(1+\frac{5}{18}\right)^{2}+\left(2+\frac{5}{9}\right)^{2}}$ units
By taking LCM
$=\sqrt{\left(\frac{23}{18}\right)^{2}+\left(\frac{23}{9}\right)^{2}}$ units
It can be written as
$=\sqrt{\left(\frac{23}{2 \times 9}\right)^{2}+\left(\frac{23}{9}\right)^{2}}$ units
So we get
$=\sqrt{\left(\frac{23}{9}\right)^{2}\left(\frac{1}{2}\right)^{2}+\left(\frac{23}{9}\right)^{2}}$ units
By taking the common terms out
$=\sqrt{\left(\frac{23}{9}\right)^{2}\left(\frac{1}{4}+1\right)}$ units
We get
$=\frac{23}{9} \sqrt{\frac{5}{4}}$ units
$=\frac{23}{9} \times \frac{\sqrt{5}}{2}$ units
So we get
$=\frac{23 \sqrt{5}}{18}$ units
Hence, the required distance is
$\frac{23 \sqrt{5}}{18}$ units.
16. Find the direction in which a straight line must be drawn through the point $(-1,2)$ so that its point of intersection with the line $x+y=4$ may be at a distance of 3 units from this point.

Solution:
Consider $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ as the line passing through the point $(-1,2)$.

So, we get
$2=m(-1)+c$
By further calculation,
$2=-m+c$
$\mathrm{c}=\mathrm{m}+2$
Substituting the value of c
$y=m x+m+2$ $\qquad$
So the given line is
$x+y=4$ $\qquad$
By solving both equations, we get
$x=\frac{2-m}{m+1}$ and $y=\frac{5 m+2}{m+1}$
$\left(\frac{2-m}{m+1}, \frac{5 m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2)
Here the point is at a distance of 3 units from ( $-1,2$ )
From distance formula

$$
\sqrt{\left(\frac{2-m}{m+1}+1\right)^{2}+\left(\frac{5 m+2}{m+1}-2\right)^{2}}=3
$$

Squaring on both sides

$$
\left(\frac{2-m+m+1}{m+1}\right)^{2}+\left(\frac{5 m+2-2 m-2}{m+1}\right)^{2}=3^{2}
$$

By further calculation
$\frac{9}{(m+1)^{2}}+\frac{9 m^{2}}{(m+1)^{2}}=9$
Dividing the equation by 9

$$
\frac{1+m^{2}}{(m+1)^{2}}=1
$$

By cross multiplication,

$$
1+\mathrm{m}^{2}=\mathrm{m}^{2}+1+2 \mathrm{~m}
$$

So, we get
$2 \mathrm{~m}=0$
$\mathrm{m}=0$
Hence, the slope of the required line must be zero, i.e., the line must be parallel to the x -axis.
17. The hypotenuse of a right-angled triangle has its ends at points $(1,3)$ and $(-4,1)$. Find the equation of the legs (perpendicular sides) of the triangle.

## Solution:

Consider ABC as the right angles triangle where $\angle \mathrm{C}=90^{\circ}$
Here, infinity such lines are present.
m is the slope of AC
So, the slope of $B C=-1 / m$
Equation of AC -
$y-3=m(x-1)$
By cross multiplication,
$x-1=1 / m(y-3)$
Equation of BC -
$y-1=-1 / m(x+4)$
By cross multiplication,
$x+4=-m(y-1)$
By considering the values of $m$, we get
If $\mathrm{m}=0$,
So, we get
$y-3=0, x+4=0$
If $\mathrm{m}=\infty$,
So, we get
$x-1=0, y-1=0$ we get $x=1, y=1$
18. Find the image of the point $(3,8)$ with respect to the line $x+3 y=7$, assuming the line to be a plane mirror.

## Solution:

It is given that
$x+3 y=7$
Consider $\mathrm{B}(\mathrm{a}, \mathrm{b})$ as the image of point $\mathrm{A}(3,8)$.
So line (1) is the perpendicular bisector of AB .


Here
Slope of $\mathrm{AB}=\frac{b-8}{a-3}$
slope of line $(1)=-\frac{1}{3}$
Line (1) is perpendicular to $A B$
$\left(\frac{b-8}{a-3}\right) \times\left(-\frac{1}{3}\right)=-1$
By further calculation
$\frac{b-8}{3 a-9}=1$
By cross multiplication
$\mathrm{b}-8=3 \mathrm{a}-9$
$3 \mathrm{a}-\mathrm{b}=1$
We know that
Mid-point of $\mathrm{AB}=\left(\frac{a+3}{2}, \frac{b+8}{2}\right)$
So the mid-point of line segment $A B$ will satisfy line (1)
From equation (1)
$\left(\frac{a+3}{2}\right)+3\left(\frac{b+8}{2}\right)=7$
By further calculation
$a+3+3 b+24=14$
On further simplification,
$a+3 b=-13$
By solving equations (2) and (3), we get
$\mathrm{a}=-1$ and $\mathrm{b}=-4$
Hence, the image of the given point with respect to the given line is $(-1,-4)$.
19. If the lines $y=3 x+1$ and $2 y=x+3$ are equally inclined to the line $y=m x+4$, find the value of $m$.

Solution:

It is given that
$y=3 x+1$
$2 y=x+3$
$y=m x+4$
Here, the slopes of
Line (1), $\mathrm{m}_{1}=3$
Line (2), $\mathrm{m}_{2}=1 / 2$
Line (3), $\mathrm{m}_{3}=\mathrm{m}$
We know that lines (1) and (2) are equally inclined to line (3), which means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).
$\left|\frac{m_{1}-m_{3}}{1+m_{1} m_{3}}\right|=\left|\frac{m_{2}-m_{3}}{1+m_{2} m_{3}}\right|$
Substituting the values we get
$\left|\frac{3-m}{1+3 m}\right|=\left|\frac{\frac{1}{2}-m}{1+\frac{1}{2} m}\right|$
By taking LCM
$\left|\frac{3-m}{1+3 m}\right|=\left|\frac{1-2 m}{m+2}\right|$
It can be written as
$\frac{3-m}{1+3 m}= \pm\left(\frac{1-2 m}{m+2}\right)$

## Here

$\frac{3-m}{1+3 m}=\frac{1-2 m}{m+2}$ or $\frac{3-m}{1+3 m}=-\left(\frac{1-2 m}{m+2}\right)$
If
$\frac{3-m}{1+3 m}=\frac{1-2 m}{m+2}$
By cross multiplication
$(3-m)(m+2)=(1-2 m)(1+3 m)$

On further calculation,
$-m^{2}+m+6=1+m-6 m^{2}$
So, we get
$5 m^{2}+5=0$
Dividing the equation by 5 ,
$\mathrm{m}^{2}+1=0$
$\mathrm{m}=\sqrt{ }-1$, which is not real.
Therefore, this case is not possible.
If
$\frac{3-m}{1+3 m}=-\left(\frac{1-2 m}{m+2}\right)$
By cross multiplication
$(3-m)(m+2)=-(1-2 m)(1+3 m)$
On further calculation
$-m^{2}+m+6=-\left(1+m-6 m^{2}\right)$
So we get
$7 \mathrm{~m}^{2}-2 \mathrm{~m}-7=0$
Here we get

$$
m=\frac{2 \pm \sqrt{4-4(7)(-7)}}{2(7)}
$$

By further simplification

$$
m=\frac{2 \pm 2 \sqrt{1+49}}{14}
$$

We can write it as

$$
m=\frac{1 \pm 5 \sqrt{2}}{7}
$$

Hence, the required value of $m$ is

$$
\frac{1 \pm 5 \sqrt{2}}{7}
$$

20. If the sum of the perpendicular distances of a variable point $P(x, y)$ from the lines $x+y-5=0$ and $3 x-2 y+$ $7=0$ is always 10 . Show that $P$ must move on a line.

## Solution:

It is given that
$x+y-5=0$
$3 \mathrm{x}-2 \mathrm{y}+7=0$
Here the perpendicular distances of $P(x, y)$ from lines (1) and (2) are written as

$$
d_{1}=\frac{|x+y-5|}{\sqrt{(1)^{2}+(1)^{2}}} \text { and } d_{2}=\frac{|3 x-2 y+7|}{\sqrt{(3)^{2}+(-2)^{2}}}
$$

So we get

$$
d_{1}=\frac{|x+y-5|}{\sqrt{2}} \text { and } d_{2}=\frac{|3 x-2 y+7|}{\sqrt{13}}
$$

We know that $\mathrm{d}_{1}+\mathrm{d}_{2}=10$
Substituting the values

$$
\frac{|x+y-5|}{\sqrt{2}}+\frac{|3 x-2 y+7|}{\sqrt{13}}=10
$$

By further calculation

$$
\sqrt{13}|x+y-5|+\sqrt{2}|3 x-2 y+7|-10 \sqrt{26}=0
$$

It can be written as

$$
\sqrt{13}(x+y-5)+\sqrt{2}(3 x-2 y+7)-10 \sqrt{26}=0
$$

Now by assuming $(x+y-5)$ and ( $3 x-2 y+7$ ) are positive

$$
\sqrt{13} x+\sqrt{13} y-5 \sqrt{13}+3 \sqrt{2} x-2 \sqrt{2} y+7 \sqrt{2}-10 \sqrt{26}=0
$$

Taking out the common terms

$$
x(\sqrt{13}+3 \sqrt{2})+y(\sqrt{13}-2 \sqrt{2})+(7 \sqrt{2}-5 \sqrt{13}-10 \sqrt{26})=0 \text {, which is the equation of a line. }
$$

In the same way, we can find the equation of the line for any signs of $(x+y-5)$ and $(3 x-2 y+7)$
Hence, point P must move on a line.
21. Find the equation of the line which is equidistant from parallel lines $9 x+6 y-7=0$ and $3 x+2 y+6=0$.

Solution:

It is given that
$9 x+6 y-7=0$
$3 \mathrm{x}+2 \mathrm{y}+6=0$
Consider $P(h, k)$ be the arbitrary point that is equidistant from lines (1) and (2)
Here the perpendicular distance of $P(h, k)$ from line (1) is written as
$d_{1}=\frac{|9 h+6 k-7|}{(9)^{2}+(6)^{2}}=\frac{|9 h+6 k-7|}{\sqrt{117}}=\frac{|9 h+6 k-7|}{3 \sqrt{13}}$
Similarly the perpendicular distance of $P(h, k)$ from line (2) is written as
$d_{2}=\frac{|3 h+2 k+6|}{\sqrt{(3)^{2}+(2)^{2}}}=\frac{|3 h+2 k+6|}{\sqrt{13}}$
We know that $P(h, k)$ is equidistant from lines (1) and (2) $d_{l}=d_{2}$
Substituting the values

$$
\frac{|9 h+6 k-7|}{3 \sqrt{13}}=\frac{|3 h+2 k+6|}{\sqrt{13}}
$$

By further calculation
$|9 h+6 k-7|=3|3 h+2 k+6|$
It can be written as
$|9 h+6 k-7|= \pm 3(3 h+2 k+6)$
Here,
$9 \mathrm{~h}+6 \mathrm{k}-7=3(3 \mathrm{~h}+2 \mathrm{k}+6)$ or $9 \mathrm{~h}+6 \mathrm{k}-7=-3(3 \mathrm{~h}+2 \mathrm{k}+6)$
$9 \mathrm{~h}+6 \mathrm{k}-7=3(3 \mathrm{~h}+2 \mathrm{k}+6)$ is not possible as
$9 \mathrm{~h}+6 \mathrm{k}-7=3(3 \mathrm{~h}+2 \mathrm{k}+6)$
By further calculation,
$-7=18$ (which is wrong)
We know that
$9 \mathrm{~h}+6 \mathrm{k}-7=-3(3 \mathrm{~h}+2 \mathrm{k}+6)$
By multiplication,
$9 \mathrm{~h}+6 \mathrm{k}-7=-9 \mathrm{~h}-6 \mathrm{k}-18$

We get
$18 \mathrm{~h}+12 \mathrm{k}+11=0$

Hence, the required equation of the line is $18 x+12 y+11=0$
22. A ray of light passing through the point $(1,2)$ reflects on the $x$-axis at point $A$, and the reflected ray passes through the point (5, 3). Find the coordinates of $A$.

## Solution:



Consider the coordinates of point $A$ as $(a, 0)$.
Construct a line (AL) which is perpendicular to the x -axis.
Here, the angle of incidence is equal to the angle of reflection
$\angle \mathrm{BAL}=\angle \mathrm{CAL}=\Phi$
$\angle \mathrm{CAX}=\theta$

It can be written as
$\angle \mathrm{OAB}=180^{\circ}-(\theta+2 \Phi)=180^{\circ}-\left[\theta+2\left(90^{\circ}-\theta\right)\right]$
On further calculation,
$=180^{\circ}-\theta-180^{\circ}+2 \theta$
$=\theta$
So, we get
$\angle \mathrm{BAX}=180^{\circ}-\theta$
slope of line $\mathrm{AC}=\frac{3-0}{5-a}$
$\tan \theta=\frac{3}{5-a}$
Slope of line $\mathrm{AB}=\frac{2-0}{1-a}$
We get
$\tan \left(180^{\circ}-\theta\right)=\frac{2}{1-a}$
By further calculation

$$
\begin{align*}
& -\tan \theta=\frac{2}{1-a} \\
& \tan \theta=\frac{2}{a-1} \tag{2}
\end{align*}
$$

From equations (1) and (2) we get
$\frac{3}{5-a}=\frac{2}{a-1}$
By cross multiplication,
$3 a-3=10-2 a$
We get
$a=13 / 5$
Hence, the coordinates of point A are $(13 / 5,0)$.
23. Prove that the product of the lengths of the perpendiculars drawn from
points $\left(\sqrt{a^{2}-b^{2}}, 0\right)$ and $\left(-\sqrt{a^{2}-b^{2}}, 0\right)$ to the line $\frac{x}{a} \cos \theta+\frac{y}{b} \sin \theta=1$ is $b^{2}$.
Solution:
It is given that
$\frac{x}{a} \cos \theta+\frac{y}{b} \sin \theta=1$
We can write it as
$\mathrm{bx} \cos \theta+\mathrm{ay} \sin \theta-\mathrm{ab}=0$

Here the length of the perpendicular from point $\left(\sqrt{a^{2}-b^{2}}, 0\right)$ to line (1)

$$
\begin{equation*}
p_{1}=\frac{\left|b \cos \theta\left(\sqrt{a^{2}-b^{2}}\right)+a \sin \theta(0)-a b\right|}{\sqrt{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}}=\frac{\left|b \cos \theta \sqrt{a^{2}-b^{2}}-a b\right|}{\sqrt{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}} \tag{2}
\end{equation*}
$$

Similarly the length of the perpendicular from point $\left(-\sqrt{a^{2}-b^{2}}, 0\right)$ to line (2)
$p_{2}=\frac{\left|b \cos \theta\left(-\sqrt{a^{2}-b^{2}}\right)+a \sin \theta(0)-a b\right|}{\sqrt{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}}=\frac{\left|b \cos \theta \sqrt{a^{2}-b^{2}}+a b\right|}{\sqrt{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}}$
By multiplying equations (2) and (3) we get
$p_{1} p_{2}=\frac{\left|b \cos \theta \sqrt{a^{2}-b^{2}}-a b\right|\left(b \cos \theta \sqrt{a^{2}-b^{2}}+a b\right) \mid}{\left(\sqrt{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}\right)^{2}}$
We get

$$
=\frac{\left|\left(b \cos \theta \sqrt{a^{2}-b^{2}}-a b\right)\left(b \cos \theta \sqrt{a^{2}-b^{2}}+a b\right)\right|}{\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)}
$$

From the formula

$$
=\frac{\left|\left(b \cos \theta \sqrt{a^{2}-b^{2}}\right)^{2}-(a b)^{2}\right|}{\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)}
$$

By squaring the numerator we get

$$
=\frac{\left|b^{2} \cos ^{2} \theta\left(a^{2}-b^{2}\right)-a^{2} b^{2}\right|}{\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)}
$$

By expanding using formula

$$
=\frac{\left|a^{2} b^{2} \cos ^{2} \theta-b^{4} \cos ^{2} \theta-a^{2} b^{2}\right|}{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}
$$

Taking out the common terms
$=\frac{b^{2}\left|a^{2} \cos ^{2} \theta-b^{2} \cos ^{2} \theta-a^{2}\right|}{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}$

We get

$$
=\frac{b^{2}\left|a^{2} \cos ^{2} \theta-b^{2} \cos ^{2} \theta-a^{2} \sin ^{2} \theta-a^{2} \cos ^{2} \theta\right|}{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}
$$

Here $\sin ^{2} \theta+\cos ^{2} \theta=1$
$=\frac{b^{2}\left|-\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)\right|}{b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta}$
So we get
$=\frac{b^{2}\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)}{\left(b^{2} \cos ^{2} \theta+a^{2} \sin ^{2} \theta\right)}$
$=b^{2}$
Therefore, it is proved.
24. A person standing at the junction (crossing) of two straight paths represented by the equations $2 x-3 y+4=$ 0 and $3 x+4 y-5=0$ wants to reach the path whose equation is $6 x-7 y+8=0$ in the least time. Find the equation of the path that he should follow.

## Solution:

It is given that
$2 x-3 y+4=0 \ldots \ldots$ (1)
$3 x+4 y-5=0$
$6 x-7 y+8=0$
Here, the person is standing at the junction of the paths represented by lines (1) and (2).
By solving equations (1) and (2), we get
$x=-1 / 17$ and $y=22 / 17$
Hence, the person is standing at point ( $-1 / 17,22 / 17$ ).
We know that the person can reach path (3) in the least time if they walk along the perpendicular line to (3) from point (-1/17, 22/17)

Here, the slope of line (3) $=6 / 7$
We get the slope of the line perpendicular to the line $(3)=-1 /(6 / 7)=-7 / 6$
So, the equation of the line passing through $(-1 / 17,22 / 17)$ and having a slope of $-7 / 6$ is written as
$\left(y-\frac{22}{17}\right)=-\frac{7}{6}\left(x+\frac{1}{17}\right)$
By further calculation,
$6(17 y-22)=-7(17 x+1)$
By multiplication,
$102 \mathrm{y}-132=-119 \mathrm{x}-7$
We get
$1119 x+102 y=125$
Therefore, the path that the person should follow is $119 \mathrm{x}+102 \mathrm{y}=125$

