EXERCISE 2.2 PAGE NO: 35

1. Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {(x, y): $3x – y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.

Solution:

The relation R from A to A is given as:

R = {(x, y): $3x – y = 0$, where $x, y \in$ A}

= {(x, y): $3x = y$, where $x, y \in$ A}

So,

R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3, 4}

The whole set A is the codomain of the relation R.

Hence, Codomain of R = A = {1, 2, 3, …, 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

Hence, Range of R = {3, 6, 9, 12}

2. Define a relation R on the set N of natural numbers by R = {(x, y): $y = x + 5$, x is a natural number less than 4; $x, y \in$ N}. Depict this relationship using roster form. Write down the domain and the range.

Solution:

The relation R is given by:

R = {(x, y): $y = x + 5$, x is a natural number less than 4, $x, y \in$ **N**}

The natural numbers less than 4 are 1, 2, and 3.

So,

R = {(1, 6), (2, 7), (3, 8)}

Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

Hence, Range of R = {6, 7, 8}

3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}. Write R in roster form.

Solution:

Given,

A = {1, 2, 3, 5} and B = {4, 6, 9}

The relation from A to B is given as

R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}

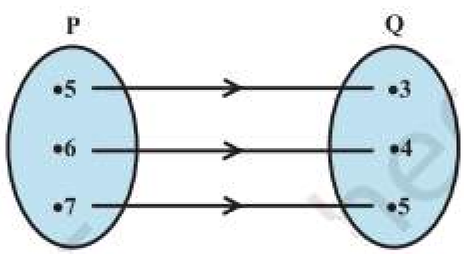
Thus,

R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. The figure shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form (ii) in roster form

What is its domain and range?



Solution:

From the given figure, it's seen that

P = {5, 6, 7}, Q = {3, 4, 5}

The relation between P and Q:

Set-builder form

(i) R = {(x, y): $y = x – 2$; $x \in$ P} or R = {(x, y): $y = x – 2$ for x = 5, 6, 7}

Roster form

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R

Solution:

Given,

A = {1, 2, 3, 4, 6} and relation R = {(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}

Hence,

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by R = {(*x*, *x* + 5): *x* ∈ {0, 1, 2, 3, 4, 5}}.

Solution:

Given,

Relation R = {(*x*, *x* + 5): *x* ∈ {0, 1, 2, 3, 4, 5}}

Thus,

R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

So,

Domain of R = {0, 1, 2, 3, 4, 5} and,

Range of R = {5, 6, 7, 8, 9, 10}

7. Write the relation R = {(x, x^3): *x* is a prime number less than 10} in roster form.

Solution:

Given,

Relation R = {(x, x^3): *x* is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

Therefore,

R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. Let A = {*x*, *y*, z} and B = {1, 2}. Find the number of relations from A to B.

Solution:

Given, A = {*x*, *y*, z} and B = {1, 2}

Now,

A × B = {(*x*, 1), (*x*, 2), (*y*, 1), (*y*, 2), (*z*, 1), (*z*, 2)}

As *n*(A × B) = 6, the number of subsets of A × B will be 2^6.

Thus, the number of relations from A to B is 2^6.

9. Let R be the relation on Z defined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}. Find the domain and range of R.

Solution:

Given,

Relation R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}

We know that the difference between any two integers is always an integer.

Therefore,

Domain of R = Z and Range of R = Z