EXERCISE 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Solution:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

As 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

As 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

It's seen that the same first element, i.e., 1, corresponds to two different images, i.e., 3 and 5; this relation cannot be called a function.

2. Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{(9 - x^2)}$

Solution:

(i) Given,

$f(x) = -|x|, x \in R$

We know that,

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

As $f(x)$ is defined for $x \in R$, the domain of f is R.

It is also seen that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of f is given by $(-\infty, 0]$.

(ii) $f(x) = \sqrt{(9 - x^2)}$

As $\sqrt{(9 - x^2)}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, for $9 - x^2 \geq 0$.

So, the domain of $f(x)$ is $\{x: -3 \leq x \leq 3\}$ or $[-3, 3]$.

Now,

For any value of x in the range $[-3, 3]$, the value of $f(x)$ will lie between 0 and 3.

Therefore, the range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

3. A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

Solution:

Given,

Function, $f(x) = 2x - 5$

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

4. The function 't', which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$

Solution:

Given function, $t(C) = \frac{9C}{5} + 32$

So,

(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) Given that, $t(C) = 212$

$\therefore 212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow 9C = 180 \times 5$

$\Rightarrow C = \frac{180 \times 5}{9} = 100$

Therefore, the value of t when $t(C) = 212$, is 100.

5. Find the range of each of the following functions:

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

(ii) $f(x) = x^2 + 2$, x is a real number

(iii) $f(x) = x$, x is a real number

Solution:

(i) Given,

$f(x) = 2 - 3x$, $x \in R$, $x > 0$

Here the values of f(x) for various values of real numbers x > 0 can be given as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
f(x)	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

It can be observed that the range of f is the set of all real numbers less than 2.

Range of f = $(-\infty, 2)$

We have,

$x > 0$

So,

$3x > 0$

$-3x < 0$ [Multiplying by -1 on both sides, the inequality sign changes]

$2 – 3x < 2$

Therefore, the value of $2 – 3x$ is less than 2.

Hence, Range = $(–\infty, 2)$

(ii) Given,

$f(x) = x^2 + 2$, x is a real number

Here the values of f(x) for various values of real numbers x can be given as

x	0	±0.3	±0.8	±1	±2	±3	...
f(x)	2	2.09	2.64	3	6	11	...

It can be oberserved that the range of f is the set of all real numbers greater than 2.
Range of f = $[2, \infty)$

We know that,

$x^2 \geq 0$

So,

$x^2 + 2 \geq 2$ [Adding 2 on both sides]

Therefore, the value of $x^2 + 2$ is always greater or equal to 2, for x is a real number.

Hence, Range = $[2, \infty)$

(iii) Given,

$f(x) = x$, x is a real number

Clearly, the range of f is the set of all real numbers.

Thus,

Range of f = R