## MISCELLANEOUS EXERCISE



**1. The relation *f* is defined by** $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

**The relation *g* is defined by** $g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$

**Show that *f* is a function and *g* is not a function.**

**Solution:**

The given relation $f$ is defined as:

$$f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$$

It is seen that for $0 \le x < 3$,

$f(x) = x^2$ and for $3 < x \le 10$,

$f(x) = 3x$

Also, at $x = 3$

$f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$ [Single image]

Hence, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Therefore, the given relation is a function.

Now,

In the given relation, $g$ is defined as

$$g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$$

It is seen that, for $x = 2$

$g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Thus, element 2 of the domain of the relation $g$ corresponds to two different images, i.e., 4 and 6.

Therefore, this relation is not a function.

**2. If $f(x) = x^2$, find**

$$\frac{f(1.1)-f(1)}{(1.1-1)}$$

**Solution:**

Given,

$f(x) = x^2$

Hence,

$$\frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

**3. Find the domain of the function**

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}$$

**Solution:**

Given function,

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}.$$

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It's clearly seen that the function $f$ is defined for all real numbers except at $x = 6$ and $x = 2$, as the denominator becomes zero otherwise.

Therefore, the domain of $f$ is R – {2, 6}.

**4. Find the domain and the range of the real function $f$ defined by $f(x) = \sqrt{(x - 1)}$.**

**Solution:**

Given real function,

$f(x) = \sqrt{(x - 1)}$

Clearly, $\sqrt{(x - 1)}$ is defined for $(x - 1) \geq 0$

So, the function $f(x) = \sqrt{(x - 1)}$ is defined for $x \geq 1$

Thus, the domain of $f$ is the set of all real numbers greater than or equal to 1.

Domain of $f = [1, \infty)$

Now,

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{(x - 1)} \geq 0$

Thus, the range of $f$ is the set of all real numbers greater than or equal to 0.

Range of $f = [0, \infty)$

**5. Find the domain and the range of the real function $f$ defined by $f(x) = |x - 1|$.**

**Solution:**

Given a real function,

$f(x) = |x - 1|$

Clearly, the function $|x - 1|$ is defined for all real numbers.

Hence,

Domain of $f$ = R

Also, for $x \in$ R, $|x - 1|$ assumes all real numbers.

Therefore, the range of $f$ is the set of all non-negative real numbers.

**6. Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$ be a function from R into R. Determine the range of $f$.**

**Solution:**

Given function,

$$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$$

Substituting values and determining the images, we have

$$= \left\{(0, 0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\right\}$$

The range of $f$ is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[As the denominator is greater than the numerator.]

Or,

We know that, for $x \in R$,

$x^2 \geq 0$

Then,

$x^2 + 1 \geq x^2$

$1 \geq x^2 / (x^2 + 1)$

Therefore, the range of $f = [0, 1)$

**7. Let $f, g$: R → R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x – 3$. Find $f + g$, $f – g$ and $f/g$.**

**Solution:**

Given the functions $f, g$: R → R is defined as

$f(x) = x + 1$, $g(x) = 2x – 3$

Now,

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x – 3) = 3x – 2$

Thus, $(f + g)(x) = 3x – 2$

$(f – g)(x) = f(x) – g(x) = (x + 1) – (2x – 3) = x + 1 – 2x + 3 = – x + 4$

Thus, $(f – g)(x) = –x + 4$

$f/g(x) = f(x)/g(x)$, $g(x) \neq 0$, $x \in R$

$f/g(x) = x + 1/ 2x – 3$, $2x – 3 \neq 0$

Thus, $f/g(x) = x + 1/ 2x – 3$, $x \neq 3/2$

**8. Let $f$ = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a function from Z to Z defined by $f(x) = ax + b$, for some integers $a, b$. Determine $a, b$.**

**Solution:**

Given, $f$ = {(1, 1), (2, 3), (0, –1), (–1, –3)}

And the function defined as, $f(x) = ax + b$

For $(1, 1) \in f$

We have, $f(1) = 1$

So, $a \times 1 + b = 1$

$a + b = 1$ …. (i)

And for $(0, –1) \in f$

We have $f(0) = –1$

$a \times 0 + b = –1$

$b = –1$

On substituting $b = –1$ in (i), we get

$a + (–1) = 1 \Rightarrow a = 1 + 1 = 2$.

Therefore, the values of $a$ and $b$ are 2 and –1, respectively.

**9. Let R be a relation from N to N defined by R = {($a$, $b$): $a$, $b \in$ N and $a = b^2$}. Are the following true?**

**(i) ($a$, $a$) $\in$ R, for all $a \in$ N**
**(ii) ($a$, $b$) $\in$ R, implies ($b$, $a$) $\in$ R**

**(iii) ($a$, $b$) $\in$ R, ($b$, $c$) $\in$ R implies ($a$, $c$) $\in$ R**

**Justify your answer in each case.**

**Solution:**

Given relation R = {($a$, $b$): $a$, $b \in$ N and $a = b^2$}

(i) It can be seen that $2 \in$ N; however, $2 \neq 2^2 = 4$.

Thus, the statement "($a$, $a$) $\in$ R, for all $a \in$ N" is not true.

(ii) Its clearly seen that $(9, 3) \in$ N because $9, 3 \in$ N and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin$ N

Thus, the statement "($a$, $b$) $\in$ R, implies ($b$, $a$) $\in$ R" is not true.

(iii) It's clearly seen that $(16, 4) \in$ R, $(4, 2) \in$ R because $16, 4, 2 \in$ N and $16 = 4^2$ and $4 = 2^2$.

Now, $16 \neq 2^2 = 4$; therefore, $(16, 2) \notin$ N

Thus, the statement "($a$, $b$) $\in$ R, ($b$, $c$) $\in$ R implies ($a$, $c$) $\in$ R" is not true.

**10. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and $f$ = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?**

**(i) $f$ is a relation from A to B (ii) $f$ is a function from A to B**

**Justify your answer in each case.**

**Solution:**

Given,

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

So,

A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

Also, given that,

$f$ = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.

It's clearly seen that $f$ is a subset of A × B.

Therefore, $f$ is a relation from A to B.

(ii) As the same first element, i.e., 2 corresponds to two different images (9 and 11), relation $f$ is not a function.

**11. Let $f$ be the subset of Z × Z defined by $f = \{(ab, a + b): a, b \in Z\}$. Is $f$ a function from Z to Z: justify your answer.**

**Solution:**

Given relation, $f$ is defined as

$f = \{(ab, a + b): a, b \in Z\}$

We know that a relation $f$ from a set A to a set B is said to be a function if every element of set A has unique images in set B.

As $2, 6, –2, –6 \in Z$, $(2 \times 6, 2 + 6), (–2 \times –6, –2 + (–6)) \in f$

i.e., $(12, 8), (12, –8) \in f$

It's clearly seen that the same first element, 12, corresponds to two different images (8 and –8).

Therefore, the relation $f$ is not a function.

**12. Let A = {9, 10, 11, 12, 13} and let $f$: A → N be defined by $f(n)$ = the highest prime factor of $n$. Find the range of $f$.**

**Solution:**

Given,

A = {9, 10, 11, 12, 13}

Now, $f$: A → **N** is defined as

$f(n)$ = The highest prime factor of $n$

So,

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

Thus, it can be expressed as

$f(9)$ = The highest prime factor of 9 = 3

$f(10)$ = The highest prime factor of 10 = 5

$f(11)$ = The highest prime factor of 11 = 11

$f(12)$ = The highest prime factor of 12 = 3

$f(13)$ = The highest prime factor of 13 = 13

The range of $f$ is the set of all $f(n)$, where $n \in$ A.

Therefore,

Range of $f$ = {3, 5, 11, 13}