## EXERCISE 2.1

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of $x$ and $y$.

Solution:
Given,
$\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$
As the ordered pairs are equal, the corresponding elements should also be equal.
Thus,
$x / 3+1=5 / 3$ and $y-2 / 3=1 / 3$
Solving, we get
$x+3=5$ and $3 y-2=1$ [Taking L.C.M. and adding]
$x=2$ and $3 y=3$
Therefore,
$x=2$ and $y=1$
2. If set $A$ has $\mathbf{3}$ elements and set $B=\{3,4,5\}$, then find the number of elements in $(A \times B)$.

## Solution:

Given, set A has 3 elements, and the elements of set B are $\{3,4$, and 5$\}$.
So, the number of elements in set $\mathrm{B}=3$
Then, the number of elements in $(\mathrm{A} \times \mathrm{B})=($ Number of elements in A$) \times($ Number of elements in $B)$
$=3 \times 3=9$
Therefore, the number of elements in $(\mathrm{A} \times \mathrm{B})$ will be 9 .
3. If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G \times H$ and $H \times G$.

## Solution:

Given, $\mathrm{G}=\{7,8\}$ and $\mathrm{H}=\{5,4,2\}$
We know that,
The Cartesian product of two non-empty sets P and Q is given as
$\mathrm{P} \times \mathrm{Q}=\{(p, q): p \in \mathrm{P}, q \in \mathrm{Q}\}$
So,
$\mathrm{G} \times \mathrm{H}=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$
$\mathrm{H} \times \mathrm{G}=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$
4. State whether each of the following statements is true or false. If the statement is false, rewrite the given statement correctly.
(i) If $P=\{m, n\}$ and $Q=\{n, m\}$, then $P \times Q=\{(m, n),(n, m)\}$
(ii) If $A$ and $B$ are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs $(x, y)$ such that $x \in A$ and $y \in B$.
(iii) If $A=\{1,2\}, B=\{3,4\}$, then $A \times(B \cap \Phi)=\Phi$

## Solution:

(i) The statement is false. The correct statement is

If $\mathrm{P}=\{m, n\}$ and $\mathrm{Q}=\{n, m\}$, then
$\mathrm{P} \times \mathrm{Q}=\{(m, m),(m, n),(n, m),(n, n)\}$
(ii) True
(iii) True
5. If $A=\{-1,1\}$, find $A \times A \times A$.

## Solution:

The $\mathrm{A} \times \mathrm{A} \times \mathrm{A}$ for a non-empty set A is given by
$\mathrm{A} \times \mathrm{A} \times \mathrm{A}=\{(a, b, c): a, b, c \in \mathrm{~A}\}$
Here, it is given $\mathrm{A}=\{-1,1\}$
So,
$\mathrm{A} \times \mathrm{A} \times \mathrm{A}=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$
6. If $\mathbf{A} \times \mathbf{B}=\{(a, x),(a, y),(b, x),(b, y)\}$. Find $\mathbf{A}$ and $B$.

Solution:

Given,
$\mathrm{A} \times \mathrm{B}=\{(a, x),(a, y),(b, x),(b, y)\}$
We know that the Cartesian product of two non-empty sets, P and Q is given by:
$\mathrm{P} \times \mathrm{Q}=\{(p, q): p \in \mathrm{P}, q \in \mathrm{Q}\}$
Hence, A is the set of all first elements, and B is the set of all second elements.
Therefore, $\mathrm{A}=\{a, b\}$ and $\mathrm{B}=\{x, y\}$
7. Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that
(i) $\mathbf{A} \times(\mathbf{B} \cap \mathbf{C})=(\mathbf{A} \times \mathbf{B}) \cap(\mathbf{A} \times \mathbf{C})$
(ii) $A \times C$ is a subset of $B \times D$

## Solution:

Given,
$A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$
(i) To verify: $\mathrm{A} \times(\mathrm{B} \cap \mathrm{C})=(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})$

Now, $\mathrm{B} \cap \mathrm{C}=\{1,2,3,4\} \cap\{5,6\}=\Phi$
Thus,
L.H.S. $=\mathrm{A} \times(\mathrm{B} \cap \mathrm{C})=\mathrm{A} \times \Phi=\Phi$

Next,
$\mathrm{A} \times \mathrm{B}=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$
$A \times C=\{(1,5),(1,6),(2,5),(2,6)\}$
Thus,
R.H.S. $=(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})=\Phi$

Therefore, L.H.S. = R.H.S.
Hence verified
(ii) To verify: $\mathrm{A} \times \mathrm{C}$ is a subset of $\mathrm{B} \times \mathrm{D}$

First,
$A \times C=\{(1,5),(1,6),(2,5),(2,6)\}$
And,
$\mathrm{B} \times \mathrm{D}=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$
Now, it's clearly seen that all the elements of set $\mathrm{A} \times \mathrm{C}$ are the elements of set $\mathrm{B} \times \mathrm{D}$.
Thus, $A \times C$ is a subset of $B \times D$.

Hence verified
8. Let $A=\{1,2\}$ and $B=\{3,4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

## Solution:

Given,
$A=\{1,2\}$ and $B=\{3,4\}$
So,
$\mathrm{A} \times \mathrm{B}=\{(1,3),(1,4),(2,3),(2,4)\}$
Number of elements in $\mathrm{A} \times \mathrm{B}$ is $n(\mathrm{~A} \times \mathrm{B})=4$
We know that,
If C is a set with $n(\mathrm{C})=m$, then $n[\mathrm{P}(\mathrm{C})]=2^{m}$.
Thus, the set $\mathrm{A} \times \mathrm{B}$ has $2^{4}=16$ subsets.
And these subsets are as given below:
$\Phi,\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},\{(1,3),(1,4)\},\{(1,3),(2,3)\},\{(1,3),(2,4)\},\{(1,4),(2,3)\},\{(1,4),(2,4)\}$, $\{(2,3),(2,4)\},\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),(2,4)\},\{(1,3),(2,3),(2,4)\},\{(1,4),(2,3),(2,4)\},\{(1,3),(1,4)$, $(2,3),(2,4)\}$
9. Let $A$ and $B$ be two sets such that $n(A)=3$ and $n(B)=2$. If $(x, 1),(y, 2),(z, 1)$ are in $A \times B$, find $A$ and $B$, where $x, y$ and $z$ are distinct elements.

## Solution:

Given,
$n(\mathrm{~A})=3$ and $n(\mathrm{~B})=2 ;$ and $(x, 1),(y, 2),(z, 1)$ are in $\mathrm{A} \times \mathrm{B}$.
We know that,
$A=$ Set of first elements of the ordered pair elements of $A \times B$
$B=$ Set of second elements of the ordered pair elements of $A \times B$
So, clearly, $x, y$, and $z$ are the elements of A; and
1 and 2 are the elements of B.
As $n(\mathrm{~A})=3$ and $n(\mathrm{~B})=2$, it is clear that set $\mathrm{A}=\{x, y, z\}$ and set $\mathrm{B}=\{1,2\}$
10. The Cartesian product $A \times A$ has 9 elements among which are found $(-1,0)$ and $(0,1)$. Find the set $A$ and the remaining elements of $\mathbf{A} \times \mathbf{A}$.

## Solution:

We know that,
If $n(\mathrm{~A})=p$ and $n(\mathrm{~B})=q$, then $n(\mathrm{~A} \times \mathrm{B})=p q$.
Also, $n(\mathrm{~A} \times \mathrm{A})=n(\mathrm{~A}) \times n(\mathrm{~A})$
Given,
$n(\mathrm{~A} \times \mathrm{A})=9$
So, $n(\mathrm{~A}) \times n(\mathrm{~A})=9$
Thus, $n(\mathrm{~A})=3$
Also, given that the ordered pairs $(-1,0)$ and $(0,1)$ are two of the nine elements of $\mathrm{A} \times \mathrm{A}$.
And, we know in $\mathrm{A} \times \mathrm{A}=\{(a, a): a \in \mathrm{~A}\}$
Thus, $-1,0$, and 1 have to be the elements of A.
As $n(\mathrm{~A})=3$, clearly $\mathrm{A}=\{-1,0,1\}$
Hence, the remaining elements of set $\mathrm{A} \times \mathrm{A}$ are as follows:
$(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0)$, and (1, 1)

## EXERCISE 2.2

1. Let $\mathrm{A}=\{1,2,3, \ldots, 14\}$. Define a relation R from A to A by $\mathrm{R}=\{(x, y): 3 x-y=0$, where $x, y \in \mathrm{~A}\}$. Write down its domain, codomain and range.

## Solution:

The relation R from A to A is given as:
$\mathrm{R}=\{(x, y): 3 x-y=0$, where $x, y \in \mathrm{~A}\}$
$=\{(x, y): 3 x=y$, where $x, y \in \mathrm{~A}\}$
So,
$R=\{(1,3),(2,6),(3,9),(4,12)\}$
Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of $\mathrm{R}=\{1,2,3,4\}$
The whole set A is the codomain of the relation R .
Hence, Codomain of $\mathrm{R}=\mathrm{A}=\{1,2,3, \ldots, 14\}$
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of $\mathrm{R}=\{3,6,9,12\}$
2. Define a relation $R$ on the set $N$ of natural numbers by $R=\{(x, y): y=x+5, x$ is a natural number less than $4 ; x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

## Solution:

The relation $R$ is given by:
$\mathrm{R}=\{(x, y): y=x+5, x$ is a natural number less than $4, x, y \in \mathbf{N}\}$
The natural numbers less than 4 are 1,2, and 3 .
So,
$R=\{(1,6),(2,7),(3,8)\}$
Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of $R=\{1,2,3\}$

The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of $R=\{6,7,8\}$
3. $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. Define a relation $R$ from $A$ to $B$ by $R=\{(x, y)$ : the difference between $x$ and $y$ is odd; $x \in A, y \in B\}$. Write $R$ in roster form.

## Solution:

Given,
$A=\{1,2,3,5\}$ and $B=\{4,6,9\}$
The relation from $A$ to $B$ is given as
$\mathrm{R}=\{(x, y)$ : the difference between $x$ and $y$ is odd; $x \in \mathrm{~A}, y \in \mathrm{~B}\}$
Thus,
$R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$
4. The figure shows a relationship between the sets $P$ and $Q$. Write this relation
(i) in set-builder form (ii) in roster form

What is its domain and range?


Solution:
From the given figure, it's seen that
$P=\{5,6,7\}, Q=\{3,4,5\}$
The relation between P and Q :
Set-builder form
(i) $\mathrm{R}=\{(x, y): y=x-2 ; x \in \mathrm{P}\}$ or $\mathrm{R}=\{(x, y): y=x-2$ for $x=5,6,7\}$

Roster form
(ii) $\mathrm{R}=\{(5,3),(6,4),(7,5)\}$

Domain of $R=\{5,6,7\}$

Range of $\mathrm{R}=\{3,4,5\}$
5. Let $\mathrm{A}=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a, b): a, b \in \mathbf{A}, b$ is exactly divisible by $a\}$.
(i) Write $\mathbf{R}$ in roster form
(ii) Find the domain of $\mathbf{R}$
(iii) Find the range of $\mathbf{R}$

## Solution:

Given,
$\mathrm{A}=\{1,2,3,4,6\}$ and relation $\mathrm{R}=\{(a, b): a, b \in \mathrm{~A}, b$ is exactly divisible by $a\}$
Hence,
(i) $\mathrm{R}=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$
(ii) Domain of $\mathrm{R}=\{1,2,3,4,6\}$
(iii) Range of $\mathrm{R}=\{1,2,3,4,6\}$
6. Determine the domain and range of the relation $R$ defined by $R=\{(x, x+5): x \in\{0,1,2,3,4,5\}\}$.

## Solution:

Given,
Relation $\mathrm{R}=\{(x, x+5): x \in\{0,1,2,3,4,5\}\}$
Thus,
$R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$
So,

Domain of $\mathrm{R}=\{0,1,2,3,4,5\}$ and,
Range of $R=\{5,6,7,8,9,10\}$
7. Write the relation $\mathrm{R}=\left\{\left(x, x^{3}\right)\right.$ : $x$ is a prime number less than 10$\}$ in roster form.

## Solution:

Given,
Relation $\mathrm{R}=\left\{\left(x, x^{3}\right): x\right.$ is a prime number less than 10$\}$
The prime numbers less than 10 are $2,3,5$, and 7 .

Therefore,
$R=\{(2,8),(3,27),(5,125),(7,343)\}$
8. Let $A=\{x, y, z\}$ and $B=\{1,2\}$. Find the number of relations from $A$ to $B$.

Solution:
Given, $\mathrm{A}=\{x, y, \mathrm{z}\}$ and $\mathrm{B}=\{1,2\}$
Now,
$\mathrm{A} \times \mathrm{B}=\{(x, 1),(x, 2),(y, 1),(y, 2),(z, 1),(z, 2)\}$
As $n(A \times B)=6$, the number of subsets of $A \times B$ will be $2^{6}$.
Thus, the number of relations from A to B is $2^{6}$.
9. Let $R$ be the relation on $Z$ defined by $R=\{(a, b): a, b \in \mathbb{Z}, a-b$ is an integer $\}$. Find the domain and range of R.

## Solution:

Given,
Relation $\mathrm{R}=\{(a, b): a, b \in \mathrm{Z}, a-b$ is an integer $\}$
We know that the difference between any two integers is always an integer.
Therefore,
Domain of $\mathrm{R}=\mathrm{Z}$ and Range of $\mathrm{R}=\mathrm{Z}$

## EXERCISE 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.
(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$
(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$
(iii) $\{(1,3),(1,5),(2,5)\}$

## Solution:

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

As 2,5,8,11,14, and 17 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain $=\{2,5,8,11,14,17\}$ and range $=\{1\}$
(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

As $2,4,6,8,10,12$, and 14 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain $=\{2,4,6,8,10,12,14\}$ and range $=\{1,2,3,4,5,6,7\}$
(iii) $\{(1,3),(1,5),(2,5)\}$

It's seen that the same first element, i.e., 1 , corresponds to two different images, i.e., 3 and 5; this relation cannot be called a function.
2. Find the domain and range of the following real function:
(i) $f(x)=-|x|$ (ii) $f(x)=\sqrt{ }\left(9-x^{2}\right)$

## Solution:

(i) Given,
$f(x)=-|x|, x \in \mathrm{R}$
We know that,
$|x|=\left\{\begin{array}{l}x, x \geq 0 \\ -x, x<0\end{array}\right.$
$\therefore f(x)=-|x|=\left\{\begin{array}{l}-x, x \geq 0 \\ x, x<0\end{array}\right.$

As $f(x)$ is defined for $x \in \mathrm{R}$, the domain of $f$ is R .
It is also seen that the range of $f(x)=-|x|$ is all real numbers except positive real numbers.
Therefore, the range of $f$ is given by $(-\infty, 0]$.
(ii) $f(x)=\sqrt{ }\left(9-x^{2}\right)$

As $\sqrt{ }\left(9-x^{2}\right)$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3 , for $9-x^{2} \geq 0$.
So, the domain of $f(x)$ is $\{x:-3 \leq x \leq 3\}$ or $[-3,3]$.
Now,
For any value of $x$ in the range $[-3,3]$, the value of $f(x)$ will lie between 0 and 3 .
Therefore, the range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0,3]$.
3. A function $f$ is defined by $f(x)=2 x-5$. Write down the values of
(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

## Solution:

Given,
Function, $f(x)=2 x-5$
Therefore,
(i) $f(0)=2 \times 0-5=0-5=-5$
(ii) $f(7)=2 \times 7-5=14-5=9$
(iii) $f(-3)=2 \times(-3)-5=-6-5=-11$
4. The function ' $t$ ', which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(\mathrm{C})=\frac{9 \mathrm{C}}{5}+32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of $C$, when $t(C)=212$

## Solution:

Given function, $t(\mathrm{C})=\frac{9 \mathrm{C}}{5}+32$
So,
So,
(j) $t(0)=\frac{9 \times 0}{5}+32=0+32=32$
(ii) $t(28)=\frac{9 \times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$
(iii) $t(-10)=\frac{9 \times(-10)}{5}+32=9 \times(-2)+32=-18+32=14$
(iv) Gixen that, $t(\mathrm{C})=212$

$$
\begin{aligned}
& \therefore 212=\frac{9 C}{5}+32 \\
& \Rightarrow \frac{9 C}{5}=212-32 \\
& \Rightarrow \frac{9 C}{5}=180 \\
& \Rightarrow 9 C=180 \times 5 \\
& \Rightarrow C=\frac{180 \times 5}{9}=100
\end{aligned}
$$

Therefore, the value of $t$ when $t(\mathrm{C})=212$, is 100 .
5. Find the range of each of the following functions:
(i) $f(x)=2-3 x, x \in \mathrm{R}, x>0$
(ii) $f(x)=x^{2}+2, x$ is a real number
(iii) $f(x)=x, x$ is a real number

## Solution:

(i) Given,
$\mathrm{f}(\mathrm{x})=2-3 x, x \in \mathrm{R}, x>0$
Here the values of $f(x)$ for various values of real numbers $x>0$ can be given as

| x | 0.01 | 0.1 | 0.9 | 1 | 2 | 2.5 | 4 | 5 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x})$ | 1.97 | 1.7 | -0.7 | -1 | -4 | -5.5 | -10 | -13 | $\ldots$ |

It can be observed that the range of $f$ is the set of all real numbers less than 2 .
Range of $f=(-\infty, 2)$
We have,
$x>0$
So,
$3 x>0$
$-3 \mathrm{x}<0$ [Multiplying by -1 on both sides, the inequality sign changes]
$2-3 x<2$

Therefore, the value of $2-3 x$ is less than 2 .
Hence, Range $=(-\infty, 2)$
(ii) Given,
$f(x)=x^{2}+2, x$ is a real number
Here the values of $f(x)$ for various values of real numbers $x$ can be given as

| $x$ | 0 | $\pm 0.3$ | $\pm 0.8$ | $\pm 1$ | $\pm 2$ | $\pm 3$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 2 | 2.09 | 2.64 | 3 | 6 | 11 | $\ldots$ |

It can be oberserved that the range of $f$ is the set of all real numbers greater than 2.
Range of $f=[2, \infty)$
We know that,
$x^{2} \geq 0$
So,
$x^{2}+2 \geq 2$ [Adding 2 on both sides]
Therefore, the value of $x^{2}+2$ is always greater or equal to 2 , for x is a real number.
Hence, Range $=[2, \infty)$
(iii) Given,
$f(x)=x, x$ is a real number
Clearly, the range of $f$ is the set of all real numbers.
Thus,
Range of $f=\mathrm{R}$

## MISCELLANEOUS EXERCISE

$$
f(x)=\left\{\begin{array}{l}
x^{2}, 0 \leq x \leq 3 \\
3 x, 3 \leq x \leq 10
\end{array}\right.
$$

1. The relation $f$ is defined by

The relation $g$ is defined by $g(x)= \begin{cases}x^{2}, & 0 \leq x \leq 2 \\ 3 x, & 2 \leq x \leq 10\end{cases}$
Show that $f$ is a function and $g$ is not a function.

## Solution:

The given relation $f$ is defined as:
$f(x)=\left\{\begin{array}{l}x^{2}, 0 \leq x \leq 3 \\ 3 x, 3 \leq x \leq 10\end{array}\right.$
It is seen that for $0 \leq x<3$,
$f(x)=x^{2}$ and for $3<x \leq 10$,
$f(x)=3 x$
Also, at $x=3$
$f(x)=3^{2}=9$ or $f(x)=3 \times 3=9$
i.e., at $x=3, f(x)=9$ [Single image]

Hence, for $0 \leq x \leq 10$, the images of $f(x)$ are unique.
Therefore, the given relation is a function.
Now,
In the given relation, $g$ is defined as
$g(x)= \begin{cases}x^{2}, & 0 \leq x \leq 2 \\ 3 x, & 2 \leq x \leq 10\end{cases}$
It is seen that, for $x=2$
$g(x)=2^{2}=4$ and $g(x)=3 \times 2=6$
Thus, element 2 of the domain of the relation $g$ corresponds to two different images, i.e., 4 and 6 .
Therefore, this relation is not a function.
2. If $f(x)=x^{2}$, find
$\frac{f(1.1)-f(1)}{(1.1-1)}$

## Solution:

Given,
$f(x)=x^{2}$
Hence,

$$
\frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^{2}-(1)^{2}}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1
$$

3. Find the domain of the function
$f(x)=\frac{x^{2}+2 x+1}{x^{2}-8 x+12}$

## Solution:

Given function,
$f(x)=\frac{x^{2}+2 x+1}{x^{2}-8 x+12}$.
$f(x)=\frac{x^{2}+2 x+1}{x^{2}-8 x+12}=\frac{x^{2}+2 x+1}{(x-6)(x-2)}$
It's clearly seen that the function $f$ is defined for all real numbers except at $x=6$ and $x=2$, as the denominator becomes zero otherwise.

Therefore, the domain of $f$ is $\mathrm{R}-\{2,6\}$.
4. Find the domain and the range of the real function $\boldsymbol{f}$ defined by $\boldsymbol{f}(\mathbf{x})=\sqrt{ }(\mathbf{x}-1)$.

## Solution:

Given real function,
$f(x)=\sqrt{ }(x-1)$
Clearly, $\sqrt{ }(x-1)$ is defined for $(x-1) \geq 0$
So, the function $f(x)=\sqrt{ }(x-1)$ is defined for $x \geq 1$

Thus, the domain of $f$ is the set of all real numbers greater than or equal to 1 .
Domain of $f=[1, \infty)$
Now,
As $x \geq 1 \Rightarrow(x-1) \geq 0 \Rightarrow \sqrt{ }(x-1) \geq 0$
Thus, the range of $f$ is the set of all real numbers greater than or equal to 0 .
Range of $f=[0, \infty)$
5. Find the domain and the range of the real function $f$ defined by $f(x)=|x-1|$.

## Solution:

Given a real function,
$f(x)=|x-1|$
Clearly, the function $|x-1|$ is defined for all real numbers.
Hence,
Domain of $f=\mathrm{R}$
Also, for $x \in R,|x-1|$ assumes all real numbers.
Therefore, the range of $f$ is the set of all non-negative real numbers.
6. Let $f=\left\{\left(x, \frac{x^{2}}{1+x^{2}}\right): x \in \mathbf{R}\right\}$ be a function from $\mathbf{R}$ into $\mathbf{R}$. Determine the range of $f$.

## Solution:

Given function,

$$
f=\left\{\left(x, \frac{x^{2}}{1+x^{2}}\right): x \in \mathbf{R}\right\}
$$

Substituting values and determining the images, we have

$$
=\left\{(0,0),\left( \pm 0.5, \frac{1}{5}\right),\left( \pm 1, \frac{1}{2}\right),\left( \pm 1.5, \frac{9}{13}\right),\left( \pm 2, \frac{4}{5}\right),\left(3, \frac{9}{10}\right),\left(4, \frac{16}{17}\right), \ldots\right\}
$$

The range of $f$ is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[As the denominator is greater than the numerator.]

Or,
We know that, for $x \in R$,
$x^{2} \geq 0$
Then,
$x^{2}+1 \geq x^{2}$
$1 \geq x^{2} /\left(x^{2}+1\right)$
Therefore, the range of $f=[0,1)$
7. Let $f, g: \mathbf{R} \rightarrow \mathbf{R}$ be defined, respectively by $f(x)=x+1, g(x)=2 x-3$. Find $f+g, f-g$ and $f / g$.

Solution:
Given the functions $f, g: \mathrm{R} \rightarrow \mathrm{R}$ is defined as
$f(x)=x+1, g(x)=2 x-3$
Now,
$(f+g)(x)=f(x)+g(x)=(x+1)+(2 x-3)=3 x-2$
Thus, $(f+g)(x)=3 x-2$
$(f-g)(x)=f(x)-g(x)=(x+1)-(2 x-3)=x+1-2 x+3=-x+4$
Thus, $(f-g)(x)=-x+4$
$f / g(\mathrm{x})=f(\mathrm{x}) / g(\mathrm{x}), \mathrm{g}(\mathrm{x}) \neq 0, \mathrm{x} \in \mathrm{R}$
$f / g(\mathrm{x})=x+1 / 2 x-3,2 x-3 \neq 0$
Thus, $f / g(\mathrm{x})=x+1 / 2 x-3, x \neq 3 / 2$
8. Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function from $Z$ to $Z$ defined by $f(x)=a x+b$, for some integers $a, b$. Determine $a, b$.

## Solution:

Given, $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$
And the function defined as, $f(x)=a x+b$
For $(1,1) \in f$
We have, $f(1)=1$
So, $a \times 1+b=1$
$a+b=1$
And for $(0,-1) \in f$
We have $f(0)=-1$
$a \times 0+b=-1$
$b=-1$
On substituting $b=-1$ in (i), we get
$a+(-1)=1 \Rightarrow a=1+1=2$.
Therefore, the values of $a$ and $b$ are 2 and -1 , respectively.
9. Let R be a relation from N to N defined by $\mathrm{R}=\left\{(a, b): a, b \in \mathrm{~N}\right.$ and $\left.a=b^{2}\right\}$. Are the following true?
(i) $(a, a) \in \mathbf{R}$, for all $a \in \mathbf{N}$
(ii) $(a, b) \in \mathbf{R}$, implies $(b, a) \in \mathbf{R}$
(iii) $(a, b) \in \mathbf{R},(b, c) \in \mathbf{R}$ implies $(a, c) \in \mathbf{R}$

Justify your answer in each case.

## Solution:

Given relation $\mathrm{R}=\left\{(a, b): a, b \in \mathrm{~N}\right.$ and $\left.a=b^{2}\right\}$
(i) It can be seen that $2 \in \mathrm{~N}$; however, $2 \neq 2^{2}=4$.

Thus, the statement " $(a, a) \in \mathrm{R}$, for all $a \in \mathrm{~N}$ " is not true.
(ii) Its clearly seen that $(9,3) \in N$ because $9,3 \in N$ and $9=3^{2}$.

Now, $3 \neq 9^{2}=81$; therefore, $(3,9) \notin \mathrm{N}$
Thus, the statement " $(a, b) \in \mathrm{R}$, implies $(b, a) \in \mathrm{R}$ " is not true.
(iii) It's clearly seen that $(16,4) \in R,(4,2) \in R$ because $16,4,2 \in N$ and $16=4^{2}$ and $4=2^{2}$.

Now, $16 \neq 2^{2}=4$; therefore, $(16,2) \notin \mathrm{N}$
Thus, the statement " $(a, b) \in \mathrm{R},(b, c) \in \mathrm{R}$ implies $(a, c) \in \mathrm{R}$ " is not true.
10. Let $\mathrm{A}=\{1,2,3,4\}, \mathrm{B}=\{1,5,9,11,15,16\}$ and $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$. Are the following true?
(i) $f$ is a relation from $\mathbf{A}$ to B (ii) $f$ is a function from A to B

Justify your answer in each case.

## Solution:

Given,
$A=\{1,2,3,4\}$ and $B=\{1,5,9,11,15,16\}$
So,
$\mathrm{A} \times \mathrm{B}=\{(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),(2,1),(2,5),(2,9),(2,11),(2,15),(2,16),(3,1),(3,5),(3,9)$, $(3,11),(3,15),(3,16),(4,1),(4,5),(4,9),(4,11),(4,15),(4,16)\}$

Also, given that,
$f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$
(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $\mathrm{A} \times \mathrm{B}$.

It's clearly seen that $f$ is a subset of $\mathrm{A} \times \mathrm{B}$.
Therefore, $f$ is a relation from A to B.
(ii) As the same first element, i.e., 2 corresponds to two different images ( 9 and 11), relation $f$ is not a function.
11. Let $f$ be the subset of $Z \times Z$ defined by $f=\{(a b, a+b): a, b \in \mathbb{Z}\}$. Is $f$ a function from $Z$ to $Z$ : justify your answer.

## Solution:

Given relation, $f$ is defined as
$f=\{(a b, a+b): a, b \in \mathrm{Z}\}$
We know that a relation $f$ from a set A to a set B is said to be a function if every element of set A has unique images in set B.

As $2,6,-2,-6 \in \mathrm{Z},(2 \times 6,2+6),(-2 \times-6,-2+(-6)) \in f$
i.e., $(12,8),(12,-8) \in f$

It's clearly seen that the same first element, 12 , corresponds to two different images (8 and -8 ).
Therefore, the relation $f$ is not a function.
12. Let $\mathrm{A}=\{9,10,11,12,13\}$ and let $f: \mathrm{A} \rightarrow \mathrm{N}$ be defined by $f(n)=$ the highest prime factor of $n$. Find the range of $f$.

## Solution:

Given,
$\mathrm{A}=\{9,10,11,12,13\}$
Now, $f: \mathbf{A} \rightarrow \mathbf{N}$ is defined as
$f(n)=$ The highest prime factor of $n$

So,
Prime factor of $9=3$

Prime factors of $10=2,5$
Prime factor of $11=11$

Prime factors of $12=2,3$
Prime factor of $13=13$
Thus, it can be expressed as
$f(9)=$ The highest prime factor of $9=3$
$f(10)=$ The highest prime factor of $10=5$
$f(11)=$ The highest prime factor of $11=11$
$f(12)=$ The highest prime factor of $12=3$
$f(13)=$ The highest prime factor of $13=13$
The range of $f$ is the set of all $f(n)$, where $n \in \mathrm{~A}$.
Therefore,
Range of $f=\{3,5,11,13\}$

