EXERCISE 3.1 PAGE: 54

1. Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) – 47° 30′ (iii) 240° (iv) 520°

Solution:

(i) 25°

Here 180° = π radian

It can be written as

$$25° = \frac{\pi}{180} \times 25 \text{ radian}$$

So we get

$$= \frac{5\pi}{36} \text{ radian}$$

(ii) – 47° 30'

Here 1° = 60'

It can be written as

$$-47°\ 30' = -47\frac{1}{2} \text{ degree}$$

So we get

$$= \frac{-95}{2} \text{ degree}$$

Here 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian}$$

It can be written as

$$= \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

We get

$$-47°\ 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

Here 180° = π radian

It can be written as

$$240° = \frac{\pi}{180} \times 240 \text{ radian}$$

So we get

$$= \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

Here 180° = π radian

It can be written as

$$520° = \frac{\pi}{180} \times 520 \text{ radian}$$

So we get

$$= \frac{26\pi}{9} \text{ radian}$$

2. Find the degree measures corresponding to the following radian measures (Use π = 22/7)

(i) 11/16

(ii) -4

(iii) 5π/3

(iv) 7π/6

Solution:

(i) 11/16

Here π radian = 180°

$$\frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree}$$

We can write it as

$$= \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

So we get

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree}$$

$$= \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

Take $1^{\circ} = 60'$

$$= 39° + \frac{3 \times 60}{8} \text{ min utes}$$

We get

$$= 39° + 22' + \frac{1}{2} \text{ min utes}$$

Consider $1' = 60''$

$= 39^{\circ}\ 22'\ 30''$

(ii) -4

Here π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree}$$

We can write it as

$$= \frac{180 \times 7(-4)}{22} \text{ degree}$$

By further calculation

$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$

Take $1^o = 60'$

$$= -229^\circ + \frac{1 \times 60}{11} \text{ minutes}$$

So we get

$$= -229^\circ + 5' + \frac{5}{11} \text{ minutes}$$

Again $1' = 60''$

$= -229^o\ 5'\ 27''$

(iii) $5\pi/3$

Here π radian = 180°

$$\frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree}$$

We get

$= 300^o$

(iv) $7\pi/6$

Here π radian = 180°

$$\frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6}$$

We get

$= 210^o$

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

It is given that

No. of revolutions made by the wheel in

1 minute = 360

1 second = 360/60 = 6

We know that

The wheel turns an angle of 2π radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian = $12\ \pi$ radian

Therefore, in one second, the wheel turns an angle of 12π radian.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = 22/7$).

Solution:

Consider a circle of radius r unit with l unit as the arc length which subtends an angle θ radian at the centre

$\theta = l / r$

Here $r = 100$ cm, $l = 22$ cm

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree}$$

It can be written as

$$= \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$

$$= \frac{126}{10} \text{ degree}$$

So we get

$$= 12\frac{3}{5} \text{ degree}$$

Here $1^\circ = 60'$

$= 12^\circ 36'$

Therefore, the required angle is $12^\circ 36'$.

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

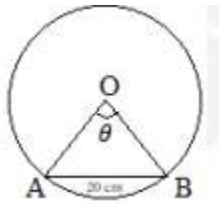
Solution:

The dimensions of the circle are

Diameter = 40 cm

Radius = 40/2 = 20 cm

Consider AB be as the chord of the circle i.e. length = 20 cm



In ΔOAB,

Radius of circle = OA = OB = 20 cm

Similarly AB = 20 cm

Hence, ΔOAB is an equilateral triangle.

$\theta = 60° = \pi/3$ radian

In a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ radian at the centre

We get θ = 1/r

$$\frac{\pi}{3}=\frac{\widehat{AB}}{20}\Rightarrow \widehat{AB}=\frac{20\pi}{3}\text{ cm}$$

Therefore, the length of the minor arc of the chord is 20π/3 cm.

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Consider r_1 and r_2 as the radii of the two circles.
Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 and an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.
Here 60° = π/3 radian and 75° = 5π/ 12 radian
In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre
We get
$\theta = l/ r$ or $l = r\theta$
We know that

$$l=\frac{r_1\pi}{3}\text{ and }l=\frac{r_2 5\pi}{12}$$

By equating both we get

$$\frac{r_1\pi}{3}=\frac{r_2 5\pi}{12}$$

On further calculation

$$r_1=\frac{r_2 5}{4}$$

So we get

$$\frac{r_1}{r_2}=\frac{5}{4}$$

Therefore, the ratio of the radii is 5: 4.

7. Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then θ = 1/r

We know that r = 75 cm

(i) l = 10 cm

So we get

θ = 10/75 radian

By further simplification

θ = 2/15 radian

(ii) l = 15 cm

So we get

θ = 15/75 radian

By further simplification

θ = 1/5 radian

(iii) l = 21 cm

So we get

θ = 21/75 radian

By further simplification

θ = 7/25 radian