EXERCISE 3.2 PAGE: 63

Find the values of other five trigonometric functions in Exercises 1 to 5.

1. cos x = -1/2, x lies in third quadrant.

Solution:

It is given that
cos x = - 1/2
sec x = 1/cos x
Substituting the values

$$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

Consider
$\sin^2 x + \cos^2 x = 1$
We can write it as
$\sin^2 x = 1 - \cos^2 x$
Substituting the values
$\sin^2 x = 1 - (-1/2)^2$
$\sin^2 x = 1 - 1/4 = 3/4$
$\sin^2 x = \pm \sqrt{3}/2$
Here x lies in the third quadrant so the value of sin x will be negative
$\sin x = - \sqrt{3}/2$
We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2. sin x = 3/5, x lies in second quadrant.

Solution:

It is given that

sin x = 3/5

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\cos^2 x = 1 – \sin^2 x$

Substituting the values

$\cos^2 x = 1 – (3/5)^2$

$\cos^2 x = 1 – 9/25$

$\cos^2 x = 16/25$

$\cos x = \pm 4/5$

Here x lies in the second quadrant so the value of cos x will be negative

$\cos x = - 4/5$

We can write it as

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

Here

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

3. cot x = 3/4, x lies in third quadrant.

Solution:

It is given that

$\cot x = 3/4$

We can write it as

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (4/3)^2 = \sec^2 x$

Substituting the values

$1 + 16/9 = \sec^2 x$

$\cos^2 x = 25/9$

$\sec x = \pm 5/3$

Here x lies in the third quadrant so the value of sec x will be negative

$\sec x = -5/3$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

By further calculation

$$\sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4. sec x = 13/5, x lies in fourth quadrant.

Solution:

It is given that

$\sec x = 13/5$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 - \cos^2 x$

Substituting the values

$\sin^2 x = 1 - (5/13)^2$

$\sin^2 x = 1 – 25/169 = 144/169$

$\sin^2 x = \pm 12/13$

Here x lies in the fourth quadrant so the value of sin x will be negative

sin x = – 12/13

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

5. tan x = -5/12, x lies in second quadrant.

Solution:

It is given that

tan x = – 5/12

We can write it as

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (-5/12)^2 = \sec^2 x$

Substituting the values

$1 + 25/144 = \sec^2 x$

$\sec^2 x = 169/144$

sec x = ± 13/12

Here x lies in the second quadrant so the value of sec x will be negative

sec x = – 13/12

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

By further calculation

$$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the values of the trigonometric functions in Exercises 6 to 10.

6. sin 765°

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin 765° = \sin\left(2 \times 360° + 45°\right)$$

By further calculation

= sin 45°

= 1/ √ 2

7. cosec (–1410°)

Solution:

We know that values of cosec x repeat after an interval of 2π or 360°

So we get

$$\operatorname{cosec}\ \left(-1410°\right) = \operatorname{cosec}\left(-1410° + 4 \times 360°\right)$$

By further calculation

$$= \operatorname{cosec}\ \left(-1410° + 1440°\right)$$

= cosec 30° = 2

8. $\tan\frac{19\pi}{3}$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

By further calculation

$$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3}$$

We get

$= \tan 60^\circ$

$= \sqrt{3}$

9. $\sin\left(-\frac{11\pi}{3}\right)$

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right)$$

By further calculation

$$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

10. $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$$

By further calculation

$$= \cot\frac{\pi}{4} = 1$$