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Prove that:

1.

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

So we get

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

By further calculation

= 1/4 + 1/4 – 1

= - 1/2

= RHS

2.

$$2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$$

Solution:

Consider

$$\text{L.H.S.} = 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

By further calculation

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

It can be written as

$$= 2\times\frac{1}{4} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

So we get

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

Here

= 1/2 + 4/4

= 1/2 + 1

= 3/2

= RHS

3.

$$\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$$

Solution:

Consider

$$\text{L.H.S.} = \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$$

So we get

$$= \left(\sqrt{3}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

By further calculation

$$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3\times\frac{1}{3}$$

We get

$= 3 + 2 + 1$

$= 6$

$=$ RHS

4.

$$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$$

Solution:

Consider

$$\text{L.H.S} = 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$$

So we get

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

By further calculation

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

It can be written as

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$= 1 + 1 + 8$

$= 10$

$=$ RHS

5. Find the value of:

(i) sin 75°

(ii) tan 15°

Solution:

(i) sin 75°

It can be written as

$= \sin (45° + 30°)$

Using the formula $[\sin (x + y) = \sin x \cos y + \cos x \sin y]$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

Substituting the values

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

By further calculation

$$= \frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15°

It can be written as

$= \tan (45° – 30°)$

Using formula

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

Substituting the values

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

By further calculation

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

So we get

$$= \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

Prove the following:

6.

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$$

Solution:

Consider LHS =

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

We can write it as

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

By further simplification

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

Using the formula

2 cos A cos B = cos (A + B) + cos (A – B)

-2 sin A sin B = cos (A + B) – cos (A – B)

$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

We get

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]$$

= sin (x + y)

= RHS

7.

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

Consider

$$\text{L.H.S.}=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

By using the formula

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

So we get

$$=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$$

It can be written as

$$= \frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

= RHS

8.

$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

It can be written as

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

So we get

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$= \cot^2 x$

= RHS

9.

$$\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$$

It can be written as

= sin x cos x (tan x + cot x)

So we get

$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

= 1

= RHS

10. sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x* = cos *x*

Solution:

LHS = sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x*

By multiplying and dividing by 2

$$= \frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x + 2\cos(n+1)x\cos(n+2)x\right]$$

Using the formula

- 2 sin A sin B = cos (A + B) – cos (A – B)

2 cos A cos B = cos (A + B) + cos (A – B)

$$= \frac{1}{2}\begin{bmatrix} \cos\{(n+1)x-(n+2)x\} - \cos\{(n+1)x+(n+2)x\} \\ +\cos\{(n+1)x+(n+2)x\} + \cos\{(n+1)x-(n+2)x\} \end{bmatrix}$$

By further calculation

$$= \frac{1}{2} \times 2\cos\{(n+1)x-(n+2)x\}$$

= cos (- x)

= cos x

= RHS

11.

$$\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

Using the formula

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

So we get

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$$

It can be written as

$$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

By further calculation

$$= -2\sin\frac{\pi}{4}\sin x$$

Substituting the values

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$= -\sqrt{2}\sin x$

= RHS

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

Consider

L.H.S. = $\sin^2 6x - \sin^2 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

= ($\sin 6x + \sin 4x$) ($\sin 6x - \sin 4x$)

By further calculation

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

We get

= ($2 \sin 5x \cos x$) ($2 \cos 5x \sin x$)

It can be written as

= ($2 \sin 5x \cos 5x$) ($2 \sin x \cos x$)

= $\sin 10x \sin 2x$

= RHS

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

Consider

L.H.S. = $\cos^2 2x - \cos^2 6x$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

$= (\cos 2x + \cos 6x)\ (\cos 2x - 6x)$

By further calculation

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

We get

$= [2 \cos 4x \cos (-2x)]\ [-2 \sin 4x \sin (-2x)]$

It can be written as

$= [2 \cos 4x \cos 2x]\ [-2 \sin 4x\ (-\sin 2x)]$

So we get

$= (2 \sin 4x \cos 4x)\ (2 \sin 2x \cos 2x)$

$= \sin 8x \sin 4x$

= RHS

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

Consider

L.H.S. = $\sin 2x + 2 \sin 4x + \sin 6x$

$= [\sin 2x + \sin 6x] + 2 \sin 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

By further simplification

= 2 sin 4x cos (– 2x) + 2 sin 4x

It can be written as

= 2 sin 4x cos 2x + 2 sin 4x

Taking common terms

= 2 sin 4x (cos 2x + 1)

Using the formula

= 2 sin 4x (2 $\cos^2$ x – 1 + 1)

We get

= 2 sin 4x (2 $\cos^2$ x)

= 4$\cos^2$ x sin 4x

= R.H.S.

15. cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* – sin 3*x*)

Solution:

Consider

LHS = cot 4x (sin 5x + sin 3x)

It can be written as

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

So we get

= 2 cos 4x cos x

Similarly

R.H.S. = cot x (sin 5x – sin 3x)

It can be written as

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

So we get

= 2 cos 4x cos x

Hence, LHS = RHS.

16.

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Solution:

Consider

$$L.H.S = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

Using the formula

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

By further calculation

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

So we get

$$=-\frac{\sin 2x}{\cos 10x}$$

= RHS

17.

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$$

Solution:

Consider

$$\text{L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

Using the formula

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

By further calculation

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

So we get

$$= \frac{\sin 4x}{\cos 4x}$$

= tan 4x

= RHS

18.

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

By further calculation

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

So we get

$$= \tan\left(\frac{x-y}{2}\right)$$

= RHS

19.

$$\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

Using the formula

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

By further calculation

$$=\frac{\sin 2x}{\cos 2x}$$

So we get

= tan 2x

= RHS

20.

$$\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

Using the formula

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\cos^2 A - \sin^2 A = \cos 2A$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

By further calculation

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

So we get

= - 2 (-sin x)

= 2 sin x

= RHS

21.

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

It can be written as

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

By further calculation

$$= \frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x}$$

So we get

$$= \frac{\cos 3x\left(2\cos x + 1\right)}{\sin 3x\left(2\cos x + 1\right)}$$

= cot 3x

= RHS

22. cot x cot 2x – cot 2x cot 3x – cot 3x cot x = 1

Solution:

Consider

LHS = cot x cot 2x – cot 2x cot 3x – cot 3x cot x

It can be written as

= cot x cot 2x – cot 3x (cot 2x + cot x)

= cot x cot 2x – cot (2x + x) (cot 2x + cot x)

Using the formula

$$\cot\left(A + B\right) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$$

$$= \cot x\cot 2x - \left[\frac{\cot 2x\cot x - 1}{\cot x + \cot 2x}\right]\left(\cot 2x + \cot x\right)$$

So we get

= cot x cot 2x – (cot 2x cot x – 1)

= 1

= RHS

23.

$$\tan 4x = \frac{4\tan x\left(1 - \tan^2 x\right)}{1 - 6\tan^2 x + \tan^4 x}$$

Solution:

Consider

LHS = tan 4x = tan 2(2x)

By using the formula

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

It can be written as

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

By further simplification

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

Taking LCM

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

On further simplification

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

We get

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

It can be written as

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

$= \text{RHS}$

24. $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Solution:

Consider

LHS = cos 4x

We can write it as

= $\cos 2(2x)$

Using the formula $\cos 2A = 1 - 2 \sin^2 A$

= $1 - 2 \sin^2 2x$

Again by using the formula $\sin 2A = 2\sin A \cos A$

= $1 - 2(2 \sin x \cos x)^2$

So we get

= $1 - 8 \sin^2 x \cos^2 x$

= R.H.S.

25. $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Solution:

Consider

L.H.S. = $\cos 6x$

It can be written as

= $\cos 3(2x)$

Using the formula $\cos 3A = 4 \cos^3 A - 3 \cos A$

= $4 \cos^3 2x - 3 \cos 2x$

Again by using formula $\cos 2x = 2 \cos^2 x - 1$

= $4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)$

By further simplification

= $4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6\cos^2 x + 3$

We get

= $4 [8\cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$

By multiplication

= $32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$

On further calculation

= $32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

= R.H.S.