EXERCISE 3.4

Find the principal and general solutions of the following equations:

1. tan x = √3

Solution:

It is given that

tan x = √3

We know that

$$\tan\frac{\pi}{3} = \sqrt{3}$$

It can be written as

$$\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right)$$

So we get

$$= \tan\frac{\pi}{3} = \sqrt{3}$$

Hence, the principal solutions are x = π/3 and 4π/3

$$\tan x = \tan\frac{\pi}{3}$$

We get

$$x = n\pi + \frac{\pi}{3},\ \text{where } n \in Z$$

Hence, the general solution is

$$x = n\pi + \frac{\pi}{3},\ \text{where } n \in Z$$

2. sec x = 2

Solution:

It is given that

sec x = 2

We know that

$$\sec\frac{\pi}{3} = 2$$

It can be written as

$$\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right)$$

So we get

$$= \sec\frac{\pi}{3} = 2$$

Hence, the principal solutions are x = π/3 and 5π/3.

$$\sec x = \sec\frac{\pi}{3}$$

We know that sec x = 1/ cos x

$$\cos x = \cos\frac{\pi}{3}$$

So we get

$$x = 2n\pi \pm \frac{\pi}{3},\ \text{where } n \in Z$$

Hence, the general solution is

$$x = 2n\pi \pm \frac{\pi}{3},\ \text{where } n \in Z$$

3. cot x = – √3

Solution:

It is given that

cot x = - √3

We know that

$$\cot\frac{\pi}{6} = \sqrt{3}$$

It can be written as

$$\cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$$

And

$$\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$$

So we get

$$\cot\frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot\frac{11\pi}{6} = -\sqrt{3}$$

Hence, the principal solutions are x = 5π/6 and 11π/6.

$$\cot x = \cot\frac{5\pi}{6}$$

We know that cot x = 1/ tan x

$$\tan x = \tan\frac{5\pi}{6}$$

So we get

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$$

4. cosec x = – 2

Solution:

It is given that

cosec x = - 2

We know that

$$\operatorname{cosec}\frac{\pi}{6} = 2$$

It can be written as

$$\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

And

$$\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

So we get

$$\operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Hence, the principal solutions are x = 7π/6 and 11π/6.

$$\operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

We know that cosec x = 1/ sin x

$$\sin x = \sin \frac{7\pi}{6}$$

So we get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Find the general solution for each of the following equations:

5. cos 4x = cos 2x

Solution:

It is given that

cos 4x = cos 2x

We can write it as

cos 4x – cos 2x = 0

Using the formula

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right).$$

We get

$$-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

By further simplification

sin 3x sin x = 0

We can write it as

sin 3x = 0 or sin x = 0

By equating the values

3x = nπ or x = nπ, where n ∈ Z

We get

x = nπ/3 or x = nπ, where n ∈ Z

6. cos 3x + cos x – cos 2x = 0

Solution:

It is given that

cos 3x + cos x – cos 2x = 0

We can write it as

$$2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-\cos 2x=0$$

Using the formula

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

We get

2 cos 2x cos x – cos 2x = 0

By further simplification

cos 2x (2 cos x – 1) = 0

We can write it as

cos 2x = 0 or 2 cos x – 1 = 0

cos 2x = 0 or cos x = 1/2

By equating the values

$$2x=(2n+1)\frac{\pi}{2} \qquad \text{or} \qquad \cos x=\cos\frac{\pi}{3},\ \text{where } n\in Z$$

We get

$$x=(2n+1)\frac{\pi}{4} \qquad \text{or} \qquad x=2n\pi\pm\frac{\pi}{3},\ \text{where } n\in Z$$

7. sin 2x + cos x = 0

Solution:

It is given that

sin 2x + cos x = 0

We can write it as

2 sin x cos x + cos x = 0

cos x (2 sin x + 1) = 0

cos x = 0 or 2 sin x + 1 = 0

Let cos x = 0

$\cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2 \sin x + 1 = 0$

So we get

$\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6}$

We can write it as

$= \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$

$= \sin\frac{7\pi}{6}$

We get

$x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Hence, the general solution is

$(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in Z$

8. $\sec^2 2x = 1 - \tan 2x$

Solution:

It is given that

$\sec^2 2x = 1 - \tan 2x$

We can write it as

$1 + \tan^2 2x = 1 - \tan 2x$

$\tan^2 2x + \tan 2x = 0$

Taking common terms

$\tan 2x (\tan 2x + 1) = 0$

Here

$\tan 2x = 0$ or $\tan 2x + 1 = 0$

If $\tan 2x = 0$

$\tan 2x = \tan 0$

We get

$2x = n\pi + 0$, where $n \in Z$

$x = n\pi/2$, where $n \in Z$

tan 2x + 1 = 0

We can write it as

tan 2x = – 1

So we get

$$= -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{3\pi}{4}$$

Here

2x = nπ + 3π/4, where n ∈ Z

x = nπ/2 + 3π/8, where n ∈ Z

Hence, the general solution is nπ/2 or nπ/2 + 3π/8, n ∈ Z.

9. sin x + sin 3x + sin 5x = 0

Solution:

It is given that

sin x + sin 3x + sin 5x = 0

We can write it as

(sin x + sin 5x) + sin 3x = 0

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0$$

By further calculation

2 sin 3x cos (-2x) + sin 3x = 0

It can be written as

2 sin 3x cos 2x + sin 3x = 0

By taking out the common terms

sin 3x (2 cos 2x + 1) = 0

Here

sin 3x = 0 or 2 cos 2x + 1 = 0

If sin 3x = 0

$3x = n\pi$, where $n \in Z$

We get

$x = n\pi/3$, where $n \in Z$

If $2 \cos 2x + 1 = 0$

$\cos 2x = -1/2$

By further simplification

$= -\cos \pi/3$

$= \cos (\pi - \pi/3)$

So we get

$\cos 2x = \cos 2\pi/3$

Here

$$2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in Z$$

Dividing by 2

$$x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Hence, the general solution is

$$\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3}, n \in Z$$