EXERCISE 3.1 PAGE: 54

1. Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) – 47° 30′ (iii) 240° (iv) 520°

Solution:

(i) 25°

Here 180° = π radian

It can be written as

$$25° = \frac{\pi}{180} \times 25 \text{ radian}$$

So we get

$$= \frac{5\pi}{36} \text{ radian}$$

(ii) – 47° 30′

Here 1° = 60′

It can be written as

$$-47°\ 30' = -47\frac{1}{2} \text{ degree}$$

So we get

$$= \frac{-95}{2} \text{ degree}$$

Here 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian}$$

It can be written as

$$= \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

We get

$$-47°\ 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

Here 180° = π radian

It can be written as

$$240° = \frac{\pi}{180} \times 240 \text{ radian}$$

So we get

$$= \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

Here 180° = π radian

It can be written as

$$520^\circ = \frac{\pi}{180} \times 520 \text{ radian}$$

So we get

$$= \frac{26\pi}{9} \text{ radian}$$

2. Find the degree measures corresponding to the following radian measures (Use π = 22/7)

(i) 11/16

(ii) -4

(iii) 5π/3

(iv) 7π/6

Solution:

(i) 11/16

Here π radian = 180°

$$\frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree}$$

We can write it as

$$= \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

So we get

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree}$$

$$= \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

Take 1° = 60'

$$= 39^\circ + \frac{3 \times 60}{8} \text{ min utes}$$

We get

$$= 39^\circ + 22' + \frac{1}{2} \text{ min utes}$$

Consider 1' = 60''

= 39° 22' 30''

(ii) -4

Here π radian = 180°

$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree}$

We can write it as

$= \frac{180 \times 7(-4)}{22} \text{ degree}$

By further calculation

$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$

Take 1° = 60'

$= -229° + \frac{1 \times 60}{11} \text{ minutes}$

So we get

$= -229° + 5' + \frac{5}{11} \text{ minutes}$

Again 1' = 60''

= - 229° 5' 27''

(iii) 5π/3

Here π radian = 180°

$\frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree}$

We get

= 300°

(iv) 7π/6

Here π radian = 180°

$\frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6}$

We get

= 210°

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

It is given that

No. of revolutions made by the wheel in

1 minute = 360

1 second = 360/60 = 6

We know that

The wheel turns an angle of 2π radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of 6 × 2π radian = 12 π radian

Therefore, in one second, the wheel turns an angle of 12π radian.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use π = 22/7).

Solution:

Consider a circle of radius r unit with l unit as the arc length which subtends an angle θ radian at the centre

$\theta = l/r$

Here r = 100 cm, l = 22 cm

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree}$$

It can be written as

$$= \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$

$$= \frac{126}{10} \text{ degree}$$

So we get

$$= 12\frac{3}{5} \text{ degree}$$

Here $1^\circ = 60'$

$= 12^\circ\ 36'$

Therefore, the required angle is $12^\circ\ 36'$.

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

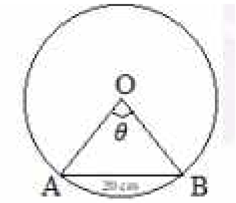
Solution:

The dimensions of the circle are

Diameter = 40 cm

Radius = 40/2 = 20 cm

Consider AB be as the chord of the circle i.e. length = 20 cm



In ΔOAB,

Radius of circle = OA = OB = 20 cm

Similarly AB = 20 cm

Hence, ΔOAB is an equilateral triangle.

θ = 60° = π/3 radian

In a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre

We get θ = 1/r

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Therefore, the length of the minor arc of the chord is 20π/3 cm.

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Consider r_1 and r_2 as the radii of the two circles.
Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 and an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.
Here 60° = π/3 radian and 75° = 5π/ 12 radian
In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre
We get
$\theta = l/r$ or $l = r\theta$
We know that

$$l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

By equating both we get

$$\frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

On further calculation

$$r_1 = \frac{r_2 5}{4}$$

So we get

$$\frac{r_1}{r_2} = \frac{5}{4}$$

Therefore, the ratio of the radii is 5: 4.

7. Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then θ = 1/r

We know that r = 75 cm

(i) l = 10 cm

So we get

θ = 10/75 radian

By further simplification

θ = 2/15 radian

(ii) l = 15 cm

So we get

θ = 15/75 radian

By further simplification

θ = 1/5 radian

(iii) l = 21 cm

So we get

θ = 21/75 radian

By further simplification

θ = 7/25 radian

EXERCISE 3.2 PAGE: 63

Find the values of other five trigonometric functions in Exercises 1 to 5.

1. cos x = -1/2, x lies in third quadrant.

Solution:

It is given that

cos x = - 1/2

sec x = 1/cos x

Substituting the values

$$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

Consider

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 - \cos^2 x$

Substituting the values

$\sin^2 x = 1 - (-1/2)^2$

$\sin^2 x = 1 - 1/4 = 3/4$

$\sin^2 x = \pm \sqrt{3}/2$

Here x lies in the third quadrant so the value of sin x will be negative

sin x = - √3/2

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2. sin x = 3/5, x lies in second quadrant.

Solution:

It is given that

sin x = 3/5

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\cos^2 x = 1 - \sin^2 x$

Substituting the values

$\cos^2 x = 1 - (3/5)^2$

$\cos^2 x = 1 - 9/25$

$\cos^2 x = 16/25$

$\cos x = \pm 4/5$

Here x lies in the second quadrant so the value of cos x will be negative

$\cos x = - 4/5$

We can write it as

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

Here

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

3. cot x = 3/4, x lies in third quadrant.

Solution:

It is given that

$\cot x = 3/4$

We can write it as

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (4/3)^2 = \sec^2 x$

Substituting the values

$1 + 16/9 = \sec^2 x$

$\cos^2 x = 25/9$

$\sec x = \pm 5/3$

Here x lies in the third quadrant so the value of sec x will be negative

$\sec x = -5/3$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

By further calculation

$$\sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4. sec x = 13/5, x lies in fourth quadrant.

Solution:

It is given that

$\sec x = 13/5$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 - \cos^2 x$

Substituting the values

$\sin^2 x = 1 - (5/13)^2$

$\sin^2 x = 1 – 25/169 = 144/169$

$\sin^2 x = \pm 12/13$

Here x lies in the fourth quadrant so the value of sin x will be negative

$\sin x = – 12/13$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

5. tan x = -5/12, x lies in second quadrant.

Solution:

It is given that

$\tan x = – 5/12$

We can write it as

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (-5/12)^2 = \sec^2 x$

Substituting the values

$1 + 25/144 = \sec^2 x$

$\sec^2 x = 169/144$

$\sec x = \pm 13/12$

Here x lies in the second quadrant so the value of sec x will be negative

$\sec x = – 13/12$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

By further calculation

$$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

Here

$$\text{cosec}\, x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the values of the trigonometric functions in Exercises 6 to 10.

6. sin 765°

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin 765° = \sin(2 \times 360° + 45°)$$

By further calculation

= sin 45°

= 1/ √ 2

7. cosec (–1410°)

Solution:

We know that values of cosec x repeat after an interval of 2π or 360°

So we get

$$\text{cosec}\ (-1410°) = \text{cosec}(-1410° + 4 \times 360°)$$

By further calculation

$$= \text{cosec}\ (-1410° + 1440°)$$

= cosec 30° = 2

8. $\tan\frac{19\pi}{3}$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

By further calculation

$$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3}$$

We get

= tan 60°

= √3

9. $\sin\left(-\frac{11\pi}{3}\right)$

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right)$$

By further calculation

$$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

10. $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$$

By further calculation

$$= \cot\frac{\pi}{4} = 1$$

EXERCISE 3.3 PAGE: 73

Prove that:

1.

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

So we get

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

By further calculation

= 1/4 + 1/4 – 1

= - 1/2

= RHS

2.

$$2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$$

Solution:

Consider

$$\text{L.H.S.} = 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

By further calculation

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

It can be written as

$$= 2 \times \frac{1}{4} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

So we get

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

Here

= 1/2 + 4/4

= 1/2 + 1

= 3/2

= RHS

3.

$$\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$$

Solution:

Consider

$$\text{L.H.S.} = \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$$

So we get

$$= \left(\sqrt{3}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

By further calculation

$$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3\times\frac{1}{3}$$

We get

$= 3 + 2 + 1$

$= 6$

$= \text{RHS}$

4.

$$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$$

Solution:

Consider

$$\text{L.H.S} = 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$$

So we get

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

By further calculation

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

It can be written as

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$= 1 + 1 + 8$

$= 10$

$= \text{RHS}$

5. Find the value of:

(i) sin 75°

(ii) tan 15°

Solution:

(i) sin 75°

It can be written as

$= \sin (45° + 30°)$

Using the formula $[\sin (x + y) = \sin x \cos y + \cos x \sin y]$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

Substituting the values

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

By further calculation

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15°

It can be written as

$= \tan (45° – 30°)$

Using formula

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

Substituting the values

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

By further calculation

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

So we get

$$= \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

Prove the following:

6.

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$$

Solution:

Consider LHS =

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

We can write it as

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

By further simplification

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

Using the formula

2 cos A cos B = cos (A + B) + cos (A – B)

-2 sin A sin B = cos (A + B) – cos (A – B)

$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

We get

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]$$

= sin (x + y)

= RHS

7.

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

Consider

$$L.H.S.=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

By using the formula

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

So we get

$$=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$$

It can be written as

$$= \frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

= RHS

8.

$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2}+x\right)}$$

It can be written as

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

So we get

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$= \cot^2 x$

= RHS

9.

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi + x)\right] = 1$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi + x)\right]$$

It can be written as

= sin x cos x (tan x + cot x)

So we get

$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

= 1

= RHS

10. sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x* = cos *x*

Solution:

LHS = sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x*

By multiplying and dividing by 2

$$= \frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x + 2\cos(n+1)x\cos(n+2)x\right]$$

Using the formula

- 2 sin A sin B = cos (A + B) – cos (A – B)

2 cos A cos B = cos (A + B) + cos (A – B)

$$= \frac{1}{2}\begin{bmatrix} \cos\{(n+1)x-(n+2)x\} - \cos\{(n+1)x+(n+2)x\} \\ +\cos\{(n+1)x+(n+2)x\} + \cos\{(n+1)x-(n+2)x\} \end{bmatrix}$$

By further calculation

$$= \frac{1}{2} \times 2\cos\{(n+1)x-(n+2)x\}$$

= cos (- x)

= cos x

= RHS

11.

$$\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$$

Solution:

Consider

L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

Using the formula

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

So we get

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

It can be written as

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

By further calculation

$$=-2\sin\frac{\pi}{4}\sin x$$

Substituting the values

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$= -\sqrt{2}\sin x$

= RHS

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

Consider

L.H.S. = $\sin^2 6x - \sin^2 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

= $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

By further calculation

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

We get

= $(2\sin 5x \cos x)(2\cos 5x \sin x)$

It can be written as

= $(2\sin 5x \cos 5x)(2\sin x \cos x)$

= $\sin 10x \sin 2x$

= RHS

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

Consider

L.H.S. = $\cos^2 2x - \cos^2 6x$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

= $(\cos 2x + \cos 6x)(\cos 2x - 6x)$

By further calculation

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

We get

= [2 cos 4x cos (-2x)] [-2 sin 4x sin (-2x)]

It can be written as

= [2 cos 4x cos 2x] [–2 sin 4x (–sin 2x)]

So we get

= (2 sin 4x cos 4x) (2 sin 2x cos 2x)

= sin 8x sin 4x

= RHS

14. sin 2x + 2sin 4x + sin 6x = 4cos² x sin 4x

Solution:

Consider

L.H.S. = $\sin 2x + 2\sin 4x + \sin 6x$

= $[\sin 2x + \sin 6x] + 2\sin 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

By further simplification

= 2 sin 4x cos (– 2x) + 2 sin 4x

It can be written as

= 2 sin 4x cos 2x + 2 sin 4x

Taking common terms

= 2 sin 4x (cos 2x + 1)

Using the formula

= 2 sin 4x (2 $\cos^2$ x – 1 + 1)

We get

= 2 sin 4x (2 $\cos^2$ x)

= 4$\cos^2$ x sin 4x

= R.H.S.

15. cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* – sin 3*x*)

Solution:

Consider

LHS = cot 4x (sin 5x + sin 3x)

It can be written as

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x \cos x\right]$$

So we get

= 2 cos 4x cos x

Similarly

R.H.S. = cot x (sin $5x$ – sin $3x$)

It can be written as

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x \sin x\right]$$

So we get

= 2 cos 4x cos x

Hence, LHS = RHS.

16.

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Solution:

Consider

$$\text{L.H.S} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

Using the formula

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

By further calculation

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

So we get

$$= -\frac{\sin 2x}{\cos 10x}$$

= RHS

17.

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

By further calculation

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

So we get

$$= \frac{\sin 4x}{\cos 4x}$$

= tan 4x

= RHS

18.

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

By further calculation

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

So we get

$$= \tan\left(\frac{x-y}{2}\right)$$

= RHS

19.

$$\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$$

Solution:

Consider

$$\text{L.H.S.}=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

Using the formula

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

By further calculation

$$=\frac{\sin 2x}{\cos 2x}$$

So we get

= tan 2x

= RHS

20.

$$\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$$

Solution:

Consider

$$\text{L.H.S.}=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

Using the formula

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\cos^2 A - \sin^2 A = \cos 2A$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

By further calculation

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

So we get

= - 2 (-sin x)

= 2 sin x

= RHS

21.

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

It can be written as

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

By further calculation

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

So we get

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

= cot 3x

= RHS

22. cot *x* cot 2*x* – cot 2*x* cot 3*x* – cot 3*x* cot *x* = 1

Solution:

Consider

LHS = cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x

It can be written as

= cot x cot $2x$ – cot $3x$ (cot $2x$ + cot x)

= cot x cot $2x$ – cot $(2x + x)$ (cot $2x$ + cot x)

Using the formula

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

So we get

= cot x cot 2x – (cot 2x cot x – 1)

= 1

= RHS

23.

$$\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$$

Solution:

Consider

LHS = tan 4x = tan 2(2x)

By using the formula

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

It can be written as

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

By further simplification

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

Taking LCM

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

On further simplification

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

We get

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

It can be written as

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

$= \text{RHS}$

24. $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

Solution:

Consider

LHS = cos 4x

We can write it as

$= \cos 2(2x)$

Using the formula $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\sin^2 2x$

Again by using the formula $\sin 2A = 2\sin A \cos A$

$= 1 - 2(2\sin x \cos x)^2$

So we get

$= 1 - 8\sin^2 x \cos^2 x$

= R.H.S.

25. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution:

Consider

L.H.S. = $\cos 6x$

It can be written as

$= \cos 3(2x)$

Using the formula $\cos 3A = 4\cos^3 A - 3\cos A$

$= 4\cos^3 2x - 3\cos 2x$

Again by using formula $\cos 2x = 2\cos^2 x - 1$

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$

By further simplification

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

We get

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

By multiplication

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

On further calculation

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

= R.H.S.

EXERCISE 3.4 PAGE: 78

Find the principal and general solutions of the following equations:

1. tan x = √3

Solution:

It is given that

tan x = √3

We know that

$$\tan\frac{\pi}{3} = \sqrt{3}$$

It can be written as

$$\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right)$$

So we get

$$= \tan\frac{\pi}{3} = \sqrt{3}$$

Hence, the principal solutions are x = π/3 and 4π/3

$$\tan x = \tan\frac{\pi}{3}$$

We get

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in Z$$

2. sec x = 2

Solution:

It is given that

sec x = 2

We know that

$$\sec\frac{\pi}{3} = 2$$

It can be written as

$$\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right)$$

So we get

$$= \sec\frac{\pi}{3} = 2$$

Hence, the principal solutions are x = π/3 and 5π/3.

$$\sec x = \sec\frac{\pi}{3}$$

We know that sec x = 1/ cos x

$$\cos x = \cos\frac{\pi}{3}$$

So we get

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

3. cot x = – √3

Solution:

It is given that

cot x = - √3

We know that

$$\cot\frac{\pi}{6} = \sqrt{3}$$

It can be written as

$$\cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$$

And

$$\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$$

So we get

$$\cot\frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot\frac{11\pi}{6} = -\sqrt{3}$$

Hence, the principal solutions are x = 5π/6 and 11π/6.

$$\cot x = \cot \frac{5\pi}{6}$$

We know that cot x = 1/ tan x

$$\tan x = \tan \frac{5\pi}{6}$$

So we get

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$$

4. cosec x = – 2

Solution:

It is given that

cosec x = - 2

We know that

$$\operatorname{cosec} \frac{\pi}{6} = 2$$

It can be written as

$$\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

And

$$\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

So we get

$$\operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Hence, the principal solutions are x = 7π/6 and 11π/6.

$$\operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

We know that cosec x = 1/ sin x

$$\sin x = \sin \frac{7\pi}{6}$$

So we get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Hence, the general solution is

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Find the general solution for each of the following equations:

5. cos 4x = cos 2x

Solution:

It is given that

cos 4x = cos 2x

We can write it as

cos 4x – cos 2x = 0

Using the formula

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

We get

$$-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

By further simplification

sin 3x sin x = 0

We can write it as

sin 3x = 0 or sin x = 0

By equating the values

3x = nπ or x = nπ, where n ∈ Z

We get

x = nπ/3 or x = nπ, where n ∈ Z

6. cos 3x + cos x – cos 2x = 0

Solution:

It is given that

cos 3x + cos x – cos 2x = 0

We can write it as

$$2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-\cos 2x=0$$

Using the formula

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

We get

2 cos 2x cos x – cos 2x = 0

By further simplification

cos 2x (2 cos x – 1) = 0

We can write it as

cos 2x = 0 or 2 cos x – 1 = 0

cos 2x = 0 or cos x = 1/2

By equating the values

$$2x=(2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x=\cos\frac{\pi}{3},\ \text{where } n\in Z$$

We get

$$x=(2n+1)\frac{\pi}{4} \quad \text{or} \quad x=2n\pi\pm\frac{\pi}{3},\ \text{where } n\in Z$$

7. sin 2x + cos x = 0

Solution:

It is given that

sin 2x + cos x = 0

We can write it as

2 sin x cos x + cos x = 0

cos x (2 sin x + 1) = 0

cos x = 0 or 2 sin x + 1 = 0

Let cos x = 0

$\cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2 \sin x + 1 = 0$

So we get

$\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6}$

We can write it as

$= \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$

$= \sin\frac{7\pi}{6}$

We get

$x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Hence, the general solution is

$(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in Z$

8. $\sec^2 2x = 1 - \tan 2x$

Solution:

It is given that

$\sec^2 2x = 1 - \tan 2x$

We can write it as

$1 + \tan^2 2x = 1 - \tan 2x$

$\tan^2 2x + \tan 2x = 0$

Taking common terms

$\tan 2x (\tan 2x + 1) = 0$

Here

$\tan 2x = 0$ or $\tan 2x + 1 = 0$

If $\tan 2x = 0$

$\tan 2x = \tan 0$

We get

$2x = n\pi + 0$, where $n \in Z$

$x = n\pi/2$, where $n \in Z$

tan 2x + 1 = 0

We can write it as

tan 2x = – 1

So we get

$$= -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{3\pi}{4}$$

Here

2x = nπ + 3π/4, where n ∈ Z

x = nπ/2 + 3π/8, where n ∈ Z

Hence, the general solution is nπ/2 or nπ/2 + 3π/8, n ∈ Z.

9. sin x + sin 3x + sin 5x = 0

Solution:

It is given that

sin x + sin 3x + sin 5x = 0

We can write it as

(sin x + sin 5x) + sin 3x = 0

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0$$

By further calculation

2 sin 3x cos (-2x) + sin 3x = 0

It can be written as

2 sin 3x cos 2x + sin 3x = 0

By taking out the common terms

sin 3x (2 cos 2x + 1) = 0

Here

sin 3x = 0 or 2 cos 2x + 1 = 0

If sin 3x = 0

3x = nπ, where n ∈ Z

We get

x = nπ/3, where n ∈ Z

If 2 cos 2x + 1 = 0

cos 2x = – 1/2

By further simplification

= – cos π/3

= cos (π – π/3)

So we get

cos 2x = cos 2π/3

Here

$$2x = 2n\pi \pm \frac{2\pi}{3},\ \text{where } n \in Z$$

Dividing by 2

$$x = n\pi \pm \frac{\pi}{3},\ \text{where } n \in Z$$

Hence, the general solution is

$$\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3},\ n \in Z$$

MISCELLANEOUS EXERCISE

Prove that:

1.

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

Solution:

$$\text{L.H.S.}=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

Using the formula

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

So we get

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$$

By further calculation

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

We get

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

Taking out the common terms

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

It can be written as

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

On further calculation

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

We get

$$= 2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$$

= 0

= RHS

2. (sin 3*x* + sin *x*) sin *x* + (cos 3*x* – cos *x*) cos *x* = 0

Solution:

Consider

LHS = (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

By further calculation

= sin 3x sin x + $\sin^2$ x + cos 3x cos x – $\cos^2$ x

Taking out the common terms

= cos 3x cos x + sin 3x sin x – ($\cos^2$ x – $\sin^2$ x)

Using the formula

cos (A – B) = cos A cos B + sin A sin B

= cos (3x – x) – cos 2x

So we get

= cos 2x – cos 2x

= 0

= RHS

3.

$$(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$$

Solution:

Consider

LHS = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

By expanding using formula we get

= $\cos^2$ x + $\cos^2$ y + 2 cos x cos y + $\sin^2$ x + $\sin^2$ y – 2 sin x sin y

Grouping the terms

= ($\cos^2$ x + $\sin^2$ x) + ($\cos^2$ y + $\sin^2$ y) + 2 (cos x cos y – sin x sin y)

Using the formula cos (A + B) = (cos A cos B – sin A sin B)

= 1 + 1 + 2 cos (x + y)

By further calculation

= 2 + 2 cos (x + y)

Taking 2 as common

= 2 [1 + cos (x + y)]

From the formula cos 2A = 2 $\cos^2$ A – 1

$$= 2\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right]$$

We get

$$= 4\cos^2\left(\frac{x+y}{2}\right)$$

= RHS

4.

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x-y}{2}$$

Solution:

LHS = (cos x – cos y) ^{2} + (sin x – sin y) ^{2}

By expanding using formula

= $\cos^2$ x + $\cos^2$ y – 2 cos x cos y + $\sin^2$ x + $\sin^2$ y – 2 sin x sin y

Grouping the terms

= ($\cos^2$ x + $\sin^2$ x) + ($\cos^2$ y + $\sin^2$ y) – 2 (cos x cos y + sin x sin y)

Using the formula cos (A – B) = cos A cos B + sin A sin B

= 1 + 1 – 2 [cos (x – y)]

By further calculation

= 2 [1 – cos (x – y)]

From formula cos 2A = 1 – 2 $\sin^2$ A

$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right]$$

We get

$$= 4\sin^2\left(\frac{x-y}{2}\right)$$

= RHS

5. sin x + sin 3x + sin 5x + sin 7x = 4 cos x cos 2x sin 4x

Solution:

Consider

LHS = sin x + sin 3x + sin 5x + sin 7x

Grouping the terms

= (sin x + sin 5x) + (sin 3x + sin 7x)

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

So we get

$$= 2\sin\left(\frac{x+5x}{2}\right)\cdot\cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$$

By further calculation

= 2 sin 3x cos (-2x) + 2 sin 5x cos (-2x)

We get

= 2 sin 3x cos 2x + 2 sin 5x cos 2x

Taking out the common terms

= 2 cos 2x [sin 3x + sin 5x]

Using the formula we can write it as

$$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cdot\cos\left(\frac{3x-5x}{2}\right)\right]$$

We get

= 2 cos 2x [2 sin 4x. cos (-x)]

= 4 cos 2x sin 4x cos x

= RHS

6.

$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Solution:

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

By further calculation

$$= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 3x]}$$

Taking out the common terms

$$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$$

We get

= tan 6x

= RHS

7.

$$\sin 3x + \sin 2x - \sin x = 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$$

Solution:

LHS = sin 3x + sin 2x – sin x

It can be written as

= sin 3x + (sin 2x – sin x)

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right]$$

By further simplification

$$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

Using formula sin 2A = 2 sin A cos B

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

Taking out the common terms

$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$$

From the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right]$$

By further simplification

$$= 2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$$

We get

$$= 4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)$$

= RHS

8. Find sin x/2, cos x/2 and tan x/2 in each of the following:

$\tan x = -\frac{4}{3}$, x in quadrant II

Solution:

It is given that

x is in quadrant II

$$\frac{\pi}{2} < x < \pi$$

Dividing by 2

$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Hence, sin x/2, cos x/2 and tan x/2 are all positive.

$$\tan x = -\frac{4}{3}$$

From the formula $\sec^2 x = 1 + \tan^2 x$

Substituting the values

$\sec^2 x = 1 + (-4/3)^2$

We get

= 1 + 16/9 = 25/9

Here

$$\cos^2 x = \frac{9}{25}$$

$$\cos x = \pm\frac{3}{5}$$

Here x is in quadrant II, cos x is negative.

cos x = -3/5

From the formula

$$\cos x = 2\cos^2\frac{x}{2} - 1$$

Substituting the values

$$\frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$$

By further calculation

$$2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$$

$$2\cos^2\frac{x}{2} = \frac{2}{5}$$

$$\cos^2\frac{x}{2} = \frac{1}{5}$$

We get

$$\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$$

$$\cos\frac{x}{2} = \frac{\sqrt{5}}{5}$$

From the formula

$$\sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1$$

Substituting the value

$$\sin^2\frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

By further calculation

$$\sin^2\frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

We get

$$\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$$

$$\sin\frac{x}{2} = \frac{2\sqrt{5}}{5}$$

Here

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)}=2$$

Hence, the respective values of sin x/2, cos x/2 and tan x/2 are

$$\frac{2\sqrt{5}}{5},\ \frac{\sqrt{5}}{5},\ \text{and } 2$$

9. cos x = -1/3, x in quadrant III

Solution:

It is given that

x is in quadrant III

$$\pi < x < \frac{3\pi}{2}$$

Dividing by 2

$$\frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}$$

Hence, cos x/2 and tan x/2 are negative where sin x/2 is positive.

$$\cos x = -\frac{1}{3}$$

From the formula cos x = 1 – 2 $\sin^2$ x/2

We get

$\sin^2$ x/2 = (1 – cos x)/ 2

Substituting the values

$$\sin^2\frac{x}{2}=\frac{1-\left(-\frac{1}{3}\right)}{2}=\frac{\left(1+\frac{1}{3}\right)}{2}$$

We get

$$=\frac{\frac{4}{3}}{2}=\frac{2}{3}$$

Here

$$\sin\frac{x}{2}=\frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin\frac{x}{2}=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$$

Using the formula

$$\cos x=2\cos^2\frac{x}{2}-1$$

We get

$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}$$

Substituting the values

$$=\frac{1+\left(-\frac{1}{3}\right)}{2}=\frac{\left(\frac{3-1}{3}\right)}{2}$$

$$=\frac{\left(\frac{2}{3}\right)}{2}=\frac{1}{3}$$

We get

$$\cos\frac{x}{2}=-\frac{1}{\sqrt{3}}$$

By further calculation

$$\cos\frac{x}{2}=-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{3}$$

Here

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)}=-\sqrt{2}$$

Therefore, the respective values of sin x/2, cos x/2 and tan x/2 are

$$\frac{\sqrt{6}}{3},\ \frac{-\sqrt{3}}{3},\text{ and }-\sqrt{2}$$

10. sin x = 1/4, x in quadrant II

Solution:

It is given that

x is in quadrant II

$$\frac{\pi}{2} < x < \pi$$

Dividing by 2

$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Hence, sin x/2, cos x/2 and tan x/2 are positive.

$$\sin x = \frac{1}{4}$$

From the formula $\cos^2 x = 1 - \sin^2 x$

We get

$\cos^2 x = 1 - (1/4)^2$

Substituting the values

$\cos^2 x = 1 - 1/16 = 15/16$

We get

$$\cos x = -\frac{\sqrt{15}}{4}$$

Here

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

Substituting the values

$$= \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 + \sqrt{15}}{8}$$

We get

$$\sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

Multiplying and dividing by 2

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

By further calculation

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

Here

$$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$$

By substituting the values

$$= \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

We get

$$\cos \frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

By multiplying and dividing by 2

$$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$$

It can be written as

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

We know that

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

Substituting the values

$$= \frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

By multiplying and dividing the terms

$$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

We get

$$= \sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}} = \frac{8+2\sqrt{15}}{2}$$

$= 4 + \sqrt{15}$

Therefore, the respective values of sin x/2, cos x/2 and tan x/2 are

$$\frac{\sqrt{8+2\sqrt{15}}}{4},\ \frac{\sqrt{8-2\sqrt{15}}}{4} \text{ and } 4+\sqrt{15}$$