EXERCISE 5.1 PAGE NO: 103

Express each of the complex numbers given in Exercises 1 to 10 in the form a + ib.

1. (5i) (-3/5i)

Solution:

(5i) (-3/5i) = 5 x (-3/5) x i^2

= -3 x -1 [i^2 = -1]

= 3

Hence,

(5i) (-3/5i) = 3 + i0

2. $i^9 + i^{19}$

Solution:

$i^9 + i^{19} = (i^2)^4. i + (i^2)^9. i$

= $(-1)^4$. i + $(-1)^9$ .i

= 1 x i + -1 x i

= i – i

= 0

Hence,

$i^9 + i^{19}$ = 0 + i0

3. i^{-39}

Solution:

i^{-39} = 1/ i^{39} = 1/ $i^{4 x 9 + 3}$ = 1/ (1^9 x i^3) = 1/ i^3 = 1/ (-i) [i^4 = 1, i^3 = -I and i^2 = -1]

Now, multiplying the numerator and denominator by i we get

i^{-39} = 1 x i / (-i x i)

= i/ 1 = i

Hence,

i^{-39} = 0 + i

4. $3(7 + i7) + i(7 + i7)$

Solution:

$3(7 + i7) + i(7 + i7) = 21 + i21 + i7 + i^2 7$

$= 21 + i28 - 7$ [$i^2 = -1$]

$= 14 + i28$

Hence,

$3(7 + i7) + i(7 + i7) = 14 + i28$

5. $(1 - i) - (-1 + i6)$

Solution:

$(1 - i) - (-1 + i6) = 1 - i + 1 - i6$

$= 2 - i7$

Hence,

$(1 - i) - (-1 + i6) = 2 - i7$

6.

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$

Solution:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Hence,

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)=\frac{-19}{5}-\frac{21}{10}i$$

7. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Solution:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Hence,

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)=\frac{17}{3}+i\frac{5}{3}$$

8. $(1 – i)^4$

Solution:

$(1 – i)^4 = [(1 – i)^2]^2$

$= [1 + i^2 – 2i]^2$

$= [1 – 1 – 2i]^2$ $[i^2 = -1]$

$= (-2i)^2$

$= 4(-1)$

$= -4$

Hence, $(1 – i)^4 = -4 + 0i$

9. $(1/3 + 3i)^3$

Solution:

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]$$
$$=\frac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]$$
$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$
$$=\frac{-242}{27}-26i$$

Hence, $(1/3 + 3i)^3 = -242/27 - 26i$

10. $(-2 - 1/3i)^3$

Solution:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Hence,

$(-2 - 1/3i)^3 = -22/3 - 107/27i$

Find the multiplicative inverse of each of the complex numbers given in Exercises 11 to 13.

11. 4 – 3i

Solution:

Let's consider $z = 4 - 3i$

Then,

$= 4 + 3i$ and

$|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Thus, the multiplicative inverse of $4 - 3i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

12. √5 + 3*i*

Solution:

Let's consider $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$

$|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Thus, the multiplicative inverse of $\sqrt{5} + 3i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

13. – i

Solution:

Let's consider $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Thus, the multiplicative inverse of $-i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

14. Express the following expression in the form of a + ib:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Solution:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Hence,

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)} = 0+\frac{-7\sqrt{2}i}{2}$$