EXERCISE 5.2 PAGE NO: 108

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. z = – 1 – i √3

Solution:

Given,

$z = -1 - i\sqrt{3}$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we get

$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$$

$$r^2 = 4 \qquad [\cos^2\theta + \sin^2\theta = 1]$$

$$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

Thus, modulus = 2

So, we have

$2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$

$$\cos\theta = \frac{-1}{2} \text{ and } \sin\theta = \frac{-\sqrt{3}}{2}$$

As the values of both sin θ and cos θ are negative, θ lies in III Quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Therefore, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

2. z = -√3 + i

Solution:

Given,

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding , we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$

$r^2 = 3 + 1 = 4$ $\quad\quad [\cos^2\theta + \sin^2\theta = 1]$

$r = \sqrt{4} = 2$ $\quad\quad$ [Conventionally, $r > 0$]

Thus, modulus = 2

So,

$2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\quad\quad$ [As θ lies in the II quadrant]

Therefore, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form:

3. 1 – i

Solution:

Given complex number,
$1 - i$
Let $r\cos\theta = 1$ and $r\sin\theta = -1$
On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} = \text{Modulus} \quad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

So,

$$1 - i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

Hence, this is the required polar form.

4. – 1 + i

Solution:

Given complex number,

$-1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Hence. it can be written as

$$-1+i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

5. – 1 – i

Solution:

Given complex number,

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1+1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

Hence, it can be written as

$$-1-i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

6. – 3

Solution:

Given complex number,
-3
Let $r \cos \theta = -3$ and $r \sin \theta = 0$
On squaring and adding, we get

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2$$

$$r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 9$$

$$r^2 = 9$$

$$r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$

So,

$$3 \cos \theta = -3 \text{ and } 3 \sin \theta = 0$$

$$\Rightarrow \cos \theta = -1 \text{ and } \sin \theta = 0$$

$$\therefore \theta = \pi$$

Hence, it can be written as

$$-3 = r \cos \theta + i r \sin \theta = 3 \cos \pi + i3 \sin \pi = 3 (\cos \pi + i \sin \pi)$$

This is the required polar form.

7. 3 + *i*

Solution:

Given complex number,

$\sqrt{3}+i$

Let $r \cos \theta = \sqrt{3}$ and $r \sin \theta = 1$

On squaring and adding, we get

$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left(\sqrt{3}\right)^2 + 1^2$

$r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 3+1$

$r^2 = 4$

$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$

So,

$2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$

Hence, it can be written as

$$\sqrt{3}+i = r\cos\theta + i\,r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6}$$
$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

8. ***i***

Solution:

Given complex number, i

Let $r \cos\theta = 0$ and $r \sin \theta = 1$

On squaring and adding, we get

$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2$

$r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 1$

$r^2 = 1$

$r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$

So,

$\cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \frac{\pi}{2}$

Hence, it can be written as

$$i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.