MISCELLANEOUS EXERCISE

1.

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Solution:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=\left[-1-i\right]^3$$
$$=(-1)^3\left[1+i\right]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i\left(1+i\right)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-\left[1-i+3i-3\right]$$
$$=-\left[-2+2i\right]$$
$$=2-2i$$

2. For any two complex numbers z_1 and z_2, prove that

Re (z_1z_2) = Re z_1 Re z_2 – Im z_1 Im z_2

Solution:

Lets's assume $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ as two complex numbers

Product of these complex numbers, z_1z_2

$$\begin{aligned} z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1] \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2) \end{aligned}$$

Now,

$$\mathrm{Re}(z_1z_2) = x_1x_2 - y_1y_2$$

$$\Rightarrow \mathrm{Re}(z_1z_2) = \mathrm{Re}\,z_1\,\mathrm{Re}\,z_2 - \mathrm{Im}\,z_1\,\mathrm{Im}\,z_2$$

Hence, proved.

3. Reduce to the standard form.

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

Solution:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad [\text{On multiplying numerator and denominator by } (14 + 5i)]$$

$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$

$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

Hence, this is the required standard form.

4.

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Solution:

Given,

$$x - iy = \sqrt{\frac{a-ib}{c-id}}$$

$$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}} \left[\text{On multiplying numerator and denominator by } (c+id)\right]$$

$$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

So,

$$(x-iy)^2 = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$x^2 - y^2 - 2ixy = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2}, \quad -2xy = \frac{ad-bc}{c^2+d^2} \qquad (1)$$

$$\left(x^2+y^2\right)^2 = \left(x^2-y^2\right)^2 + 4x^2y^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{ad-bc}{c^2+d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$= \frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{\left(c^2+d^2\right)^2}$$

$$= \frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{\left(c^2+d^2\right)^2}$$

$$= \frac{a^2\left(c^2+d^2\right)+b^2\left(c^2+d^2\right)}{\left(c^2+d^2\right)^2}$$

$$= \frac{\left(c^2+d^2\right)\left(a^2+b^2\right)}{\left(c^2+d^2\right)^2}$$

$$= \frac{a^2+b^2}{c^2+d^2}$$

- Hence Proved

5. Convert the following into the polar form:

(i) $\frac{1+7i}{(2-i)^2}$ **, (ii)** $\frac{1+3i}{1-2i}$

Solution:

(i) Here, $z = \frac{1+7i}{(2-i)^2}$

$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$

$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$ [Multiplying by its conjugate in the numerator and denominator]

$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$

$= -1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$r^2(\cos^2\theta + \sin^2\theta) = 2$

$r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

So,

$\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

Expressing as, $z = r\cos\theta + i\,r\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Therefore, this is the required polar form.

(ii) Let, $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i+3i-6}{1+4}$$

$$= \frac{-5+5i}{5} = -1+i$$

Now,

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$r^2(\cos^2\theta + \sin^2\theta) = 2$

$r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$$\cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

Expressing as, $z = r\cos\theta + i\,r\sin\theta$

$$z = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Therefore, this is the required polar form.

Solve each of the equations in Exercises 6 to 9.

6. $3x^2 - 4x + 20/3 = 0$

Solution:

Given the quadratic equation, **$3x^2 - 4x + 20/3 = 0$**

It can be re-written as: $9x^2 - 12x + 20 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 9$, $b = -12$, and $c = 20$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18}$$
$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$

7. $x^2 - 2x + 3/2 = 0$

Solution:

Given the quadratic equation, $x^2 - 2x + 3/2 = 0$

It can be re-written as $2x^2 - 4x + 3 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -4$, and $c = 3$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{2 \pm \sqrt{2}\, i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

8. $27x^2 - 10x + 1 = 0$

Solution:

Given the quadratic equation, $27x^2 - 10x + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 27$, $b = -10$, and $c = 1$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}\, i}{54}$$
$$= \frac{5 \pm \sqrt{2}\, i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

9. $21x^2 - 28x + 10 = 0$

Solution:

Given the quadratic equation, $21x^2 - 28x + 10 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 21$, $b = -28$, and $c = 10$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}\, i}{42}$$
$$= \frac{28 \pm 2\sqrt{14}\, i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

10. If $z_1 = 2 - i$, $z_2 = 1 + i$, find

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$$

Solution:

Given, $z_1 = 2 - i$, $z_2 = 1 + i$

So,

$$\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i}\times\frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2-i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad [i^2=-1]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2+1^2} = \sqrt{2}$$

Hence, the value of $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ is $\sqrt{2}$.

11.

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Solution:

Given,

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

Comparing the real and imaginary parts, we have

$$a = \frac{x^2-1}{2x^2+1} \text{ and } b = \frac{2x}{2x^2+1}$$

$$\therefore a^2 + b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$$

$$= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x+1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(2x^2+1)^2}$$

$$= \frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence proved,

$$a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$$

12. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find

(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, **(ii)** $\text{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

Solution:

Given,

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\overline{z}_1 = 2 + i$

$\therefore \dfrac{z_1 z_2}{\overline{z}_1} = \dfrac{-3+4i}{2+i}$

On multiplying numerator and denominator by (2 - i), we get

$$\frac{z_1 z_2}{\overline{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

Comparing the real parts, we have

$$\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$$

(ii) $\dfrac{1}{z_1 \overline{z}_1} = \dfrac{1}{(2-i)(2+i)} = \dfrac{1}{(2)^2 + (1)^2} = \dfrac{1}{5}$

On comparing the imaginary part, we get

$$\mathrm{Im}\left(\frac{1}{z_1 \overline{z}_1}\right) = 0$$

13. Find the modulus and argument of the complex number.

$$\frac{1+2i}{1-3i}$$

Solution:

Let $z = \frac{1+2i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$

$$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$$

Let $z = r\cos\theta + ir\sin\theta$

So,

$$r\cos\theta = \frac{-1}{2} \text{ and } r\sin\theta = \frac{1}{2}$$

On squaring and adding, we get

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad \left[\text{Conventionally, } r > 0\right]$$

$$r = \frac{1}{\sqrt{2}}$$

Now,

$$\frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad \left[\text{As } \theta \text{ lies in the II quadrant}\right]$$

14. Find the real numbers *x* and *y* if (*x* – *iy*) (3 + 5*i*) is the conjugate of – 6 – 24*i*.

Solution:

Let's assume z = (*x* – *iy*) (3 + 5*i*)

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x+5y) + i(5x-3y)$$

$$\therefore \overline{z} = (3x+5y) - i(5x-3y)$$

Also given, $\overline{z} = -6 - 24i$

And,

(3x + 5y) – *i*(5x – 3y) = -6 -24*i*

On equating real and imaginary parts, we have

3x + 5y = -6 …… (i)

$5x – 3y = 24$ …… (ii)

Performing (i) x 3 + (ii) x 5, we get

$(9x + 15y) + (25x – 15y) = -18 + 120$

$34x = 102$

$x = 102/34 = 3$

Putting the value of *x* in equation (i), we get

$3(3) + 5y = -6$

$5y = -6 – 9 = -15$

$y = -3$

Therefore, the values of *x* and *y* are 3 and –3, respectively.

15. Find the modulus of

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}$$

Solution:

$$\begin{aligned}\frac{1+i}{1-i}-\frac{1-i}{1+i} &= \frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}\\ &= \frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}\\ &= \frac{4i}{2}=2i\end{aligned}$$

$$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right| = |2i| = \sqrt{2^2} = 2$$

16. If $(x + iy)^3 = u + iv$, then show that

$$\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$$

Solution:

Given,

$(x+iy)^3 = u+iv$

$x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$

$x^3+i^3y^3+3x^2yi+3xy^2i^2 = u+iv$

$x^3-iy^3+3x^2yi-3xy^2 = u+iv$

$(x^3-3xy^2)+i(3x^2y-y^3)=u+iv$

On equating real and imaginary parts, we get

$u=x^3-3xy^2,\ v=3x^2y-y^3$

$$\frac{u}{x}+\frac{v}{y}=\frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$

$$=\frac{x(x^2-3y^2)}{x}+\frac{y(3x^2-y^2)}{y}$$

$$=x^2-3y^2+3x^2-y^2$$

$$=4x^2-4y^2$$

$$=4(x^2-y^2)$$

$$\therefore \frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$$

Hence proved.

17. If α and β are different complex numbers with |β| = 1, then find

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$$

Solution:

Let $\alpha = a + ib$ and $\beta = x + iy$

Given, $|\beta| = 1$

So, $\sqrt{x^2 + y^2} = 1$

$\Rightarrow x^2 + y^2 = 1 \quad \text{... (i)}$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)}\right|$$

$$= \left|\frac{(x - a) + i(y - b)}{1 - (ax + aiy - ibx + by)}\right|$$

$$= \left|\frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)}\right|$$

$$= \frac{|(x - a) + i(y - b)|}{|(1 - ax - by) + i(bx - ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x - a)^2 + (y - b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}$$

$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(y^2 + x^2) - 2ax - 2by}}$$

$$= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(y^2 + x^2) - 2ax - 2by}}$$

$$= \frac{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 + b^2 - 2ax - 2by}} \qquad [\text{Using (1)}]$$

$$= 1$$

$$\therefore \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = 1$$

18. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.

Solution:

$$|1-i|^x = 2^x$$
$$\left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\left(\sqrt{2}\right)^x = 2^x$$
$$2^{\frac{x}{2}} = 2^x$$
$$\frac{x}{2} = x$$
$$x = 2x$$
$$2x - x = 0$$
$$x = 0$$

Therefore, 0 is the only integral solution of the given equation.

Hence, the number of non-zero integral solutions of the given equation is 0.

19. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Solution:

Given,

$$(a+ib)(c+id)(e+if)(g+ih) = A+iB$$
$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$
$$\Rightarrow |(a+ib)|\times|(c+id)|\times|(e+if)|\times|(g+ih)| = |A+iB| \qquad [|z_1z_2| = |z_1||z_2|]$$
$$\sqrt{a^2+b^2}\times\sqrt{c^2+d^2}\times\sqrt{e^2+f^2}\times\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we get

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Hence proved.

20. If, then find the least positive integral value of m.

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

Solution:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$
$$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$$
$$\left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$
$$\left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$
$$\left(\frac{1-1+2i}{2}\right)^m = 1$$
$$\left(\frac{2i}{2}\right)^m = 1$$
$$i^m = 1$$

Hence, $m = 4k$, where k is some integer.

Thus, the least positive integer is 1.

Therefore, the least positive integral value of m is 4 (= 4 × 1).