EXERCISE 7.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution:

Total number of digits possible for choosing = 9

Number of places for which a digit has to be taken = 3

As there is no repetition allowed,

⇒ No. of permutations = $^{9}_{3}P = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\times8\times7\times6!}{6!} = 504.$

2. How many 4-digit numbers are there with no digit repeated?

Solution:

To find the four-digit number (digits do not repeat),

We will have 4 places where 4-digits are to be put.

So, at the thousand's place = There are 9 ways as 0 cannot be at the thousand's place = 9 ways

At the hundredth's place = There are 9 digits to be filled as 1 digit is already taken = 9 ways

At the ten's place = There are now 8 digits to be filled as 2 digits are already taken = 8 ways

At unit's place = There are 7 digits that can be filled = 7 ways

The total number of ways to fill the four places = 9 × 9 × 8 × 7 = 4536 ways

So, a total of 4536 four-digit numbers can be there with no digits repeated.

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Solution:

An even number means that the last digit should be even.

The number of possible digits at one's place = 3 (2, 4 and 6)

⇒ Number of permutations=

$^{3}_{1}P = \frac{3!}{(3-1)!} = 3$

One of the digits is taken at one's place; the number of possible digits available = 5

⇒ Number of permutations=

$$^{5}_{2}P = \frac{5!}{(5-2)!} = \frac{5\times4\times3!}{3!} = 20.$$

Therefore, the total number of permutations =3 × 20=60

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, if no digit is repeated. How many of these will be even?

Solution:

Total number of digits possible for choosing = 5

Number of places for which a digit has to be taken = 4

As there is no repetition allowed,

⇒ Number of permutations =

$$^{5}_{4}P = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$$

The number will be even when 2 and 4 are in one's place.

The possibility of (2, 4) at one's place = 2/5 = 0.4

The total number of even numbers = 120 × 0.4 = 48

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman, assuming one person cannot hold more than one position?

Solution:

Total number of people in committee = 8

Number of positions to be filled = 2

⇒ Number of permutations =

$$^{8}_{2}P = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 56.$$

6. Find n if $^{n-1}P_3 : {}^{n}P_3 = 1 : 9$.

Solution:

Given equation can be written as

$$\frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{9}$$

By substituting the values we get

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

On simplification

$$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

⇒ n=9.

$$\Rightarrow \frac{(n-1)!}{n \text{ x } (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

⇒ n=9.

7. Find r if

(i) $^{5}P_{r} = 2\,^{6}P_{r-1}$

(ii) $^{5}P_{r} = {}^{6}P_{r-1}$

Solution:

(i) ${}^{5}_{r}P = 2\ {}^{6}_{r-1}P$

Substituting the values we get

$\Rightarrow \frac{5!}{(5-r)!} = 2\ \frac{6!}{(7-r)!}$

The above equation can be written as

$\Rightarrow \frac{(7-r)!}{(5-r)!} = 2\ \frac{6!}{5!}$

On simplifying we get

$\Rightarrow (7 – r)\ (6 – r) = 2\ (6)$

$\Rightarrow 42 - 13r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r\ (r - 10) - 3(r - 10) = 0$

$\Rightarrow (r-3)\ (r-10) = 0$

r= 3 or r=10

But r = 10 is rejected, as in ${}^{5}_{r}P$, r cannot be greater than 5.

Therefore, r = 3.

(ii) ${}^{5}_{r}P = {}^{6}_{r-1}P$

The above equation can be written as

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$

$\Rightarrow (7 - r)(6 - r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r(r - 9) - 4(r - 9) = 0$

$\Rightarrow (r-4)(r-9) = 0$

r= 4 or r=9

But r=9 is rejected, as in ${}^{5}_{r}P$, r cannot be greater than 5.

Therefore, r=4.

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:

Total number of different letters in EQUATION = 8

Number of letters to be used to form a word = 8

⇒ Number of permutations =

${}^{8}_{8}P = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$

9. How many words, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.
(i) 4 letters are used at a time,

(ii) All letters are used at a time,
(iii) All letters are used, but the first letter is a vowel.

Solution:

(i) Number of letters to be used =4

⇒ Number of permutations =

$${}^{6}_{4}P = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$$

(ii) Number of letters to be used = 6

⇒ Number of permutations =

$${}^{6}_{6}P = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$$

(iii) Number of vowels in MONDAY = 2 (O and A)

⇒ Number of permutations in vowel =

$${}^{2}_{1}P = 2$$

Now, the remaining places = 5

Remaining letters to be used =5

⇒ Number of permutations =

$${}^{5}_{5}P = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$$

Therefore, the total number of permutations = 2 × 120 =240

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution:

Total number of letters in MISSISSIPPI =11

Letter Number of occurrence

M	1
I	4
S	4
P	2

⇒ Number of permutations =

$$\frac{11!}{1!4!4!2!} = 34650$$

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations =
$$\frac{8!}{1!4!2!} = 840$$

Therefore, total number of permutations where four I's don't come together = 34650-840=33810

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) Words start with P and end with S,

(ii) Vowels are all together,
(iii) There are always 4 letters between P and S?

Solution:

(i) Total number of letters in PERMUTATIONS =12

The only repeated letter is T; 2times

The first and last letters of the word are fixed as P and S, respectively.

Number of letters remaining =12 – 2 = 10

⇒ Number of permutations =
$$\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$$

(ii) Number of vowels in PERMUTATIONS = 5 (E, U, A, I, O)

Now, we consider all the vowels together as one.

Number of permutations of vowels = 120

Now, the total number of letters = 12 – 5 + 1= 8

⇒ Number of permutations =
$$\frac{{}^{8}_{8}P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$$

Therefore, the total number of permutations = 120 × 20160 = 2419200

(iii) The number of places is as 1 2 3 4 5 6 7 8 9 10 11 12

There should always be 4 letters between P and S.

Possible places of P and S are 1 and 6, 2 and 7, 3 and 8, 4 and 9, 5 and 10, 6 and 11, 7 and 12

Possible ways =7,

Also, P and S can be interchanged,

No. of permutations =2 × 7 =14

The remaining 10 places can be filled with 10 remaining letters,

∴ No. of permutations =

$$\frac{^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$$

Therefore, the total number of permutations = 14 × 1814400 =25401600