MISCELLANEOUS EXERCISE

1. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Solution:

The word DAUGHTER has 3 vowels A, E, and U and 5 consonants D, G, H, T and R.

The three vowels can be chosen in ${}^3C_2$ as only two vowels are to be chosen.

Similarly, the five consonants can be chosen in ${}^5C_3$ ways.

∴ The number of choosing 2 vowels and 5 consonants would be ${}^3C_2 \times {}^5C_3$

$$= \frac{3!}{2!\,(3-2)!} \times \frac{5!}{3!\,(5-3)!} = \frac{3!}{2!1!} \times \frac{5!}{3!2!}$$

= 30

∴ The total number of ways of is 30.

Each of these 5 letters can be arranged in 5 ways to form different words = ${}^5P_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Total number of words formed would be = 30 × 120 = 3600

2. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Solution:

In the word EQUATION, there are 5 vowels (A, E, I, O, U) and 3 consonants (Q, T, N).

The numbers of ways in which 5 vowels can be arranged are ${}^5C_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5\times4\times3\times2\times1}{0!} = \frac{120}{1} = 120$$ …………… (i)

Similarly, the numbers of ways in which 3 consonants can be arranged are ${}^3P_3$

$$\Rightarrow \frac{3!}{(3-3)!} = \frac{3\times2\times1}{0!} = \frac{6}{1} = 6$$ …………….. (ii)

There are two ways in which vowels and consonants can appear together.

(AEIOU) (QTN) or (QTN) (AEIOU)

∴ The total number of ways in which vowel and consonant can appear together are $2 \times {}^5C_5 \times {}^3C_3$

∴ $2 \times 120 \times 6 = 1440$

3. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:
(i) Exactly 3 girls?

(ii) At least 3 girls?

(iii) At most 3 girls?

Solution:

(i) Given exactly 3 girls.

The total numbers of girls are 4.

Out of which, 3 are to be chosen.

∴ The number of ways in which choice would be made = ${}^4C_3$

Numbers of boys are 9 out of which 4 are to be chosen which is given by ${}^9C_4$

Total ways of forming the committee with exactly three girls.

$= {}^4C_3 \times {}^9C_4$

$$= \frac{4!}{3!(4-3)!} \times \frac{9!}{4!(9-4)!} = \frac{4!}{3!1!} \times \frac{9!}{4!5!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times5\times4\times3\times2\times1} = 504$$

(ii) Given at least 3 girls.

There are two possibilities for making a committee choosing at least 3 girls.

There are 3 girls and 4 boys, or there are 4 girls and 3 boys.

Choosing three girls we have done in (i)

Choosing four girls and 3 boys would be done in ${}^4C_4$ ways.

And choosing 3 boys would be done in ${}^9C_3$

Total ways = ${}^4C_4 \times {}^9C_3$

$$= \frac{4!}{4!\,(4-4)!} \times \frac{9!}{3!\,(9-3)!} = \frac{4!}{4!\,0!} \times \frac{9!}{3!\,6!} = \frac{9 \times 8 \times 7 \times 6!}{3 \times 2 \times 1 \times 6!} = 84$$

The total number of ways of making the committee are

$504 + 84 = 588$

(iii) Given at most 3 girls

In this case, the numbers of possibilities are

0 girl and 7 boys

1 girl and 6 boys

2 girls and 5 boys

3 girls and 4 boys

Number of ways to choose 0 girl and 7 boys = ${}^4C_0 \times {}^9C_7$

$$= \frac{4!}{0!\,(4-0)!} \times \frac{9!}{7!\,2!} = \frac{4!}{4!} \times \frac{9 \times 8 \times 7!}{7! \times 2 \times 1} = \frac{72}{2} = 36$$

Number of ways of choosing 1 girl and 6 boys = ${}^4C_1 \times {}^9C_6$

$$\frac{4!}{1!\,3!} \times \frac{9!}{6!\,3!} = \frac{4 \times 3!}{3!} \times \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1} = 336$$

Number of ways of choosing 2 girls and 5 boys = ${}^4C_2 \times {}^9C_5$

$$\frac{4!}{2!\,2!} \times \frac{9!}{5!\,4!} = \frac{4!}{2 \times 1 \times 2 \times 1} \times \frac{9 \times 7 \times 8 \times 6 \times 5!}{5!\,4!} = 756$$

The number of choosing 3 girls and 4 boys has been done in (1)

= 504

The total number of ways in which a committee can have at most 3 girls are = 36 + 336 + 756 + 504 = 1632

4. If the different permutations of all the letters of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starts with E?

Solution:

In a dictionary, words are listed alphabetically, so to find the words

Listed before E should start with the letter either A, B, C or D.

But the word EXAMINATION doesn't have B, C or D.

Hence, the words should start with the letter A

The remaining 10 places are to be filled in by the remaining letters of the word EXAMINATION which are E, X, A, M, 2N, T, 2I, 0

Since the letters are repeating, the formula used would be

$$= \frac{n!}{p_1!\, p_2!\, p_{3!}}$$

Where n is the remaining number of letters, p_1 and p_2 are the number of times the repeated terms occurs.

$$= \frac{10!}{2!\,2!} = 907200$$

The number of words in the list before the word starting with E

= words starting with letter A = 907200

5. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9, which are divisible by 10 and no digit is repeated?

Solution:

The number is divisible by 10 if the unit place has 0 in it.

The 6-digit number is to be formed out of which unit place is fixed as 0.

The remaining 5 places can be filled by 1, 3, 5, 7 and 9.

Here, n = 5

And the numbers of choice available are 5.

So, the total ways in which the rest of the places can be filled are ${}^5P_5$

$$= \frac{5!}{(5-5)!} \times 1 = \frac{5!}{1} \times 1 = 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$$

6. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Solution:

We know that there are 5 vowels and 21 consonants in the English alphabet.

Choosing two vowels out of 5 would be done in ${}^5C_2$ ways.

Choosing 2 consonants out of 21 can be done in ${}^{21}C_2$ ways.

The total number of ways to select 2 vowels and 2 consonants

= ${}^5C_2 \times {}^{21}C_2$

$$\Rightarrow \frac{5!}{2!\,3!} \times \frac{21!}{2!\,19!} = \frac{5 \times 4 \times 3!}{2!\,3!} \times \frac{21 \times 20 \times 19!}{2 \times 1 \times 19!} = 2100$$

Each of these four letters can be arranged in four ways $^4P_4$

$$\Rightarrow \frac{4!}{0!} = 4 \times 3 \times 2 \times 1 = 24 \text{ ways}$$

Total numbers of words that can be formed are

24 × 2100 = 50400

7. In an examination, a question paper consists of 12 questions divided into two parts, i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Solution:

The student can choose 3 questions from part I and 5 from part II

Or

4 questions from part I and 4 from part II

5 questions from part 1 and 3 from part II

3 questions from part I and 5 from part II can be chosen in

$= {}^5C_3 \times {}^7C_5$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!} = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 210$$

4 questions from part I and 4 from part II can be chosen in

$= {}^5C_4 \times {}^7C_4$

$$= \frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!} = \frac{5 \times 4!}{4!} \times \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = 175$$

5 questions from part 1 and 3 from part II can be chosen in

$= {}^5C_5 \times {}^7C_3$

$$= \frac{5!}{5!\,0!} \times \frac{7!}{3!\,4!} = 1 \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

Now the total number of ways in which a student can choose the questions are

= 210 + 175 + 35 = 420

8. Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Solution:

We have a deck of cards that has 4 kings.

The numbers of remaining cards are 52.

Ways of selecting a king from the deck = ${}^4C_1$

Ways of selecting the remaining 4 cards from 48 cards= ${}^{48}C_4$

The total number of selecting the 5 cards having one king always

= ${}^4C_1 \times {}^{48}C_4$

$$= \frac{4!}{1!\,3!} \times \frac{48!}{4!\,44!} = \frac{4 \times 3!}{3!} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4 \times 3 \times 2 \times 1 \times 44!} = 778320$$

9. It is required to seat 5 men and 4 women in a row so that the women occupy even places. How many such arrangements are possible?

Solution:

Given there is a total of 9 people.

Women occupy even places, which means they will be sitting in 2^{nd}, 4^{th}, 6^{th} and 8^{th} place where as men will be sitting in 1^{st}, 3^{rd}, 5^{th},7^{th} and 9^{th} place.

4 women can sit in four places and ways they can be seated= ${}^4P_4$

$$= \frac{4!}{(4-4)!} = \frac{4 \times 3 \times 2 \times 1}{0!} = 24$$

5 men can occupy 5 seats in 5 ways.

The number of ways in which these can be seated = ${}^5P_5$

$$= \frac{5!}{(5-5)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$$

The total numbers of sitting arrangements possible are

24 × 120 = 2880

10. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

Solution:

In this question, we get 2 options, which are

(i) Either all 3 will go

Then, the remaining students in the class are: 25 – 3 = 22

The number of students remained to be chosen for party = 7

Number of ways to choose the remaining 22 students = $^{22}C_7$

$$= \frac{22!}{7!15!} = 170544$$

(ii) None of them will go

The students going will be 10.

Remaining students eligible for going = 22

The number of ways in which these 10 students can be selected are $^{22}C_{10}$

$$= \frac{22!}{10!\,12!} = 646646$$

The total number of ways in which students can be chosen is

= 170544 + 646646 = 817190

11. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Solution:

In the given word ASSASSINATION, there are 4 'S'. Since all the 4 'S' have to be arranged together, let us take them as one unit.

The remaining letters are= 3 'A', 2 'I', 2 'N', T

The number of letters to be arranged is 9 (including 4 'S').

Using the formula

$\frac{n!}{p_1!p_2!p_3!}$ where n is the number of terms and p_1, p_2 p_3 are the number of times the repeating letters repeat themselves.

Here, p_1= 3, p_2= 2, p_3 = 2

Putting the values in formula we get

$$\frac{10!}{3!\,2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 2\ \times 1 \times 1} = 151200$$