EXERCISE 9.2

1. Find the sum of odd integers from 1 to 2001.

Solution:

The odd integers from 1 to 2001 are 1, 3, 5, …1999, 2001.

It clearly forms a sequence in A.P.

Where the first term, $a = 1$

The common difference, $d = 2$

Now,

a + (n -1)d = 2001

1 + (n-1)(2) = 2001

2n – 2 = 2000

2n = 2000 + 2 = 2002

n = 1001

We know,

$S_n = n/2\ [2a + (n-1)d]$

$$S_n = \frac{1001}{2}\left[2\times1+\left(1001-1\right)\times2\right]$$
$$= \frac{1001}{2}\left[2+1000\times2\right]$$
$$= \frac{1001}{2}\times2002$$
$$= 1001\times1001$$
$$= 1002001$$

Therefore, the sum of odd numbers from 1 to 2001 is 1002001.

2. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Solution:

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, … 995.

It clearly forms a sequence in A.P.

Where the first term, $a = 105$

The common difference, $d = 5$

Now,

$a + (n-1)d = 995$

$105 + (n – 1)(5) = 995$

$105 + 5n – 5 = 995$

$5n = 995 – 105 + 5 = 895$

$n = 895/5$

$n = 179$

We know,

$S_n = n/2\ [2a + (n-1)d]$

$$\begin{aligned} S_n &= \frac{179}{2}\left[2(105)+(179-1)(5)\right] \\ &= \frac{179}{2}\left[2(105)+(178)(5)\right] \\ &= 179\left[105+(89)5\right] \\ &= (179)(105+445) \\ &= (179)(550) \\ &= 98450 \end{aligned}$$

Therefore, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 98450.

3. In an A.P, the first term is 2, and the sum of the first five terms is one-fourth of the next five terms. Show that the 20th term is –112.

Solution:

Given,

The first term (a) of an A.P = 2

Let's assume d is the common difference of the A.P.

So, the A.P. will be $2, 2 + d, 2 + 2d, 2 + 3d, \ldots$

Then,

Sum of first five terms $= 10 + 10d$

Sum of next five terms = 10 + 35*d*

From the question, we have

10 + 10d = ¼ (10 + 35d)

40 + 40d = 10 + 35d

30 = -5d

d = -6

a_{20} = a + (20 – 1)d = 2 + (19) (-6) = 2 – 114 = -112

Therefore, the 20th term of the A.P. is –112.

4. How many terms of the A.P. -6, -11/2, -5, …. are needed to give the sum –25?

Solution:

Let's consider the sum of *n* terms of the given A.P. as –25.

We known that,

S_n = n/2 [2a + (n-1)d]

where *n* = number of terms, *a* = first term, and *d* = common difference

So here, *a* = –6

d = -11/2 + 6 = (-11 + 12)/ 2 = 1/2

Thus, we have

$$-25=\frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$
$$-50=n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$
$$-50=n\left[-\frac{25}{2}+\frac{n}{2}\right]$$
$$-100=n(-25+n)$$
$$n^2-25n+100=0$$
$$n^2-5n-20n+100=0$$
$$n(n-5)-20(n-5)=0$$
$$n=20 \text{ or } 5$$

5. In an A.P., if *p*th term is 1/q and *q*th term is 1/p, prove that the sum of first *pq* terms is ½ (pq + 1) where p ≠ q.

Solution:

We know that the general term of an A.P is given by: $a_n = a + (n - 1)d$

From the question, we have

$$p^{\text{th}} \text{ term} = a_p = a + (p-1)d = \frac{1}{q} \qquad \ldots(1)$$

$$q^{\text{th}} \text{ term} = a_q = a + (q-1)d = \frac{1}{p} \qquad \ldots(2)$$

Subtracting (2) from (1), we have

$$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$(p-1-q+1)d = \frac{p-q}{pq}$$

$$(p-q)d = \frac{p-q}{pq}$$

$$d = \frac{1}{pq}$$

Using the value of d in (1), we get

$$a + (p-1)\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$$

$$S_{pq} = \frac{pq}{2}\left[2a + (pq-1)d\right]$$

$$= \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$$

$$= 1 + \frac{1}{2}(pq-1)$$

$$= \frac{1}{2}pq + 1 - \frac{1}{2} = \frac{1}{2}pq + \frac{1}{2}$$

$$= \frac{1}{2}(pq+1)$$

Therefore, the sum of first pq terms of the A.P is $\frac{1}{2}(pq+1)$

6. If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term.

Solution:

Given A.P.,

25, 22, 19, …

Here,

First term, a = 25 and

Common difference, d = 22 – 25 = -3

Also given, the sum of a certain number of terms of the A.P. is 116.

The number of terms is n.

So, we have

$S_n = n/2\ [2a + (n-1)d] = 116$

$116 = n/2\ [2(25) + (n-1)(-3)]$

$116 \times 2 = n\ [50 – 3n + 3]$

$232 = n\ [53 – 3n]$

$232 = 53n – 3n^2$

$3n^2 – 53n + 232 = 0$

$3n^2 – 24n – 29n + 232 = 0$

$3n(n – 8) – 29(n – 8) = 0$

$(3n – 29)(n – 8) = 0$

Hence,

n = 29/3 or n = 8

As n can only be an integral value, n = 8

Thus, the 8^{th} term is the last term of the A.P.

$a_8 = 25 + (8 – 1)(-3)$

$= 25 – 21$

$= 4$

7. Find the sum to *n* terms of the A.P., whose k^{th} term is 5*k* + 1.

Solution:

Given, the k^{th} term of the A.P. is $5k + 1$.

k^{th} term $= a_k = a + (k – 1)d$

And,

$a + (k - 1)d = 5k + 1$

$a + kd - d = 5k + 1$

On comparing the coefficient of k, we get $d = 5$

$a - d = 1$

$a - 5 = 1$

$\Rightarrow a = 6$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$$
$$= \frac{n}{2}\left[12 + 5n - 5\right]$$
$$= \frac{n}{2}(5n + 7)$$

8. If the sum of n terms of an A.P. is ($pn + qn^2$), where p and q are constants, find the common difference.

Solution:

We know that,

$S_n = n/2\ [2a + (n-1)d]$

From the question, we have

$$\frac{n}{2}\left[2a + (n-1)d\right] = pn + qn^2$$
$$\frac{n}{2}\left[2a + nd - d\right] = pn + qn^2$$
$$na + n^2\frac{d}{2} - n \cdot \frac{d}{2} = pn + qn^2$$

On comparing the coefficients of n^2 on both sides, we get

$d/2 = q$

Hence, $d = 2q$

Therefore, the common difference of the A.P. is $2q$.

9. The sums of *n* terms of two arithmetic progressions are in the ratio 5*n* + 4: 9*n* + 6. Find the ratio of their 18th terms.

Solution:

Let a_1, a_2, and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression, respectively.

Then, from the question, we have

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{5n+4}{9n+6}$$

$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6} \qquad ...(1)$$

Substituting $n = 35$ in (1), we get

$$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321} \qquad ...(2)$$

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P}} = \frac{a_1+17d_1}{a_2+17d_2} \qquad ...(3)$$

From (2) and (3), we have

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}} = \frac{179}{321}$$

Therefore, the ratio of 18^{th} term of both the A.P.s is 179: 321.

10. If the sum of the first *p* terms of an A.P. is equal to the sum of the first *q* terms, then find the sum of the first (*p* + *q*) terms.

Solution:

Let's take *a* and *d* to be the first term and the common difference of the A.P., respectively.

Then, it is given that

$$S_p = \frac{p}{2}\left[2a+(p-1)d\right]$$

$$S_q = \frac{q}{2}\left[2a+(q-1)d\right]$$

From the question, we have

$$\frac{p}{2}\left[2a+(p-1)d\right]=\frac{q}{2}\left[2a+(q-1)d\right]$$

$$p\left[2a+(p-1)d\right]=q\left[2a+(q-1)d\right]$$

$$2ap+pd(p-1)=2aq+qd(q-1)$$

$$2a(p-q)+d\left[p(p-1)-q(q-1)\right]=0$$

$$2a(p-q)+d\left[p^2-p-q^2+q\right]=0$$

$$2a(p-q)+d\left[(p-q)(p+q)-(p-q)\right]=0$$

$$2a(p-q)+d\left[(p-q)(p+q-1)\right]=0$$

$$2a+d(p+q-1)=0$$

$$\Rightarrow d=\frac{-2a}{p+q-1} \qquad \text{.......... (i)}$$

So the sum of (p + q) terms will be,

$$S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)\cdot d\right]$$

$$S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)\left(\frac{-2a}{p+q-1}\right)\right] \quad \text{[From (i)]}$$

$$=\frac{p+q}{2}\left[2a-2a\right]$$

$$=0$$

Therefore, the sum of (p + q) terms of the A.P. is 0.

11. Sum of the first *p, q* and *r* terms of an A.P. are *a, b* and *c,* respectively.

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$ **Prove that**

Solution:

Let a_1 and d be the first term and the common difference of the A.P., respectively.

Then, according to the question, we have

$$S_p = \frac{p}{2}\left[2a_1 + (p-1)d\right] = a$$

$$\Rightarrow 2a_1 + (p-1)d = \frac{2a}{p} \qquad ...(1)$$

$$S_q = \frac{q}{2}\left[2a_1 + (q-1)d\right] = b$$

$$\Rightarrow 2a_1 + (q-1)d = \frac{2b}{q} \qquad ...(2)$$

$$S_r = \frac{r}{2}\left[2a_1 + (r-1)d\right] = c$$

$$\Rightarrow 2a_1 + (r-1)d = \frac{2c}{r} \qquad ...(3)$$

Now, subtracting (2) from (1), we get

$$(p-1)d - (q-1)d = \frac{2a}{p} - \frac{2b}{q}$$

$$d(p-1-q+1) = \frac{2aq-2bp}{pq}$$

$$d(p-q) = \frac{2aq-2bp}{pq}$$

$$d = \frac{2(aq-bp)}{pq(p-q)} \qquad ....(4)$$

Then, subracting (3) from (2), we get

$$(q-1)d-(r-1)d=\frac{2b}{q}-\frac{2c}{r}$$

$$d(q-1-r+1)=\frac{2b}{q}-\frac{2c}{r}$$

$$d(q-r)=\frac{2br-2qc}{qr}$$

$$d=\frac{2(br-qc)}{qr(q-r)} \qquad ...(5)$$

On equating both the values of d obtained in (4) and (5), we get

$$\frac{aq-bp}{pq(p-q)}=\frac{br-qc}{qr(q-r)}$$

$$\frac{aq-bp}{p(p-q)}=\frac{br-qc}{r(q-r)}$$

$$r(q-r)(aq-bp)=p(p-q)(br-qc)$$

$$r(aq-bp)(q-r)=p(br-qc)(p-q)$$

$$(aqr-bpr)(q-r)=(bpr-cpq)(p-q)$$

Dividing both sides by pqr, we have

$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$

$$\frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Hence, the given result is proved.

12. The ratio of the sums of *m* and *n* terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and the n^{th} term is $(2m-1) : (2n-1)$.

Solution:

Let’s consider that *a* and *b* are the first term and the common difference of the A.P., respectively.

Then, from the question, we have

$$\frac{\text{Sum of m terms}}{\text{Sum of n terms}}=\frac{m^2}{n^2}$$

$$\frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}$$

$$\frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}\qquad \text{.......}\ (1)$$

Putting $m = 2m - 1$ and $n = 2n - 1$ in (1), we get

$$\frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$

$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1}\qquad \text{......}\ (2)$$

Now,

$$\frac{m^{th}\ \text{term of A.P.}}{n^{th}\ \text{term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d}\qquad \text{......}\ (3)$$

From (2) and (3), we have

$$\frac{m^{th}\ \text{term of A.P}}{n^{th}\text{term of A.P}}=\frac{2m-1}{2n-1}$$

Hence, the given result is proved.

13. If the sum of *n* terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of *m*.

Solution:

Let's consider *a* and *b* to be the first term and the common difference of the A.P., respectively.

$a_m = a + (m - 1)d = 164$ … (1)

The sum of the terms is given by,

$S_n = n/2\ [2a + (n-1)d]$

$\frac{n}{2}\left[2a + nd - d\right] = 3n^2 + 5n$

$na + \frac{d}{2}n^2 - \frac{d}{2}n = 3n^2 + 5n$

$\frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n = 3n^2 + 5n$

On comparing the coefficient of n^2 on both sides, we get

$\frac{d}{2} = 3$

$\Rightarrow d = 6$

On comparing the coefficient of n on both sides, we get

$a - \frac{d}{2} = 5$

$a - 3 = 5$

$a = 8$

Hence, from (1), we get

$8 + (m - 1)\,6 = 164$

$(m - 1)\,6 = 164 - 8 = 156$

$m - 1 = 26$

$m = 27$

Therefore, the value of m is 27.

14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Solution:

Let's assume A_1, A_2, A_3, A_4, and A_5 to be five numbers between 8 and 26 such that 8, A_1, A_2, A_3, A_4, A_5, 26 are in an A.P.

Here, we have,

$a = 8$, $b = 26$, $n = 7$

So,

$26 = 8 + (7 - 1)\,d$

$6d = 26 - 8 = 18$

$d = 3$

Now,

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Therefore, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

15. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between *a* and *b*, then find the value of *n*.

Solution:

The A.M between a and b is given by (a + b)/2

Then, according to the question,

$$\frac{a+b}{2} = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$$
$$(a+b)\left(a^{n-1} + b^{n-1}\right) = 2\left(a^n + b^n\right)$$
$$a^n + ab^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$$
$$ab^{n-1} + a^{n-1}b = a^n + b^n$$
$$ab^{n-1} - b^n = a^n - a^{n-1}b$$
$$b^{n-1}(a-b) = a^{n-1}(a-b)$$
$$b^{n-1} = a^{n-1}$$
$$\left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$$
$$n - 1 = 0$$
$$n = 1$$

Thus, the value of n is 1.

16. Between 1 and 31, *m* numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and (*m* – 1)th numbers is 5: 9. Find the value of *m*.

Solution:

Let's consider $a_1, a_2, \ldots a_m$ be *m* numbers such that 1, $a_1, a_2, \ldots a_m$, 31 is an A.P.

And here,

$a = 1$, $b = 31$, $n = m + 2$

So, $31 = 1 + (m + 2 – 1)(d)$

$30 = (m + 1)d$

d = 30/ (m + 1) ……. (1)

Now,

$a_1 = a + d$

$a_2 = a + 2d$

$a_3 = a + 3d$ …

Hence, $a_7 = a + 7d$

$a_{m-1} = a + (m - 1)\, d$

According to the question, we have

$$\frac{a+7d}{a+(m-1)d}=\frac{5}{9}$$

$$\frac{1+7\left(\frac{30}{(m+1)}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)}=\frac{5}{9} \qquad \text{[From (1)]}$$

$$\frac{m+1+7(30)}{m+1+30(m-1)}=\frac{5}{9}$$

$$\frac{m+1+210}{m+1+30m-30}=\frac{5}{9}$$

$$\frac{m+211}{31m-29}=\frac{5}{9}$$

$$9m+1899=155m-145$$

$$155m-9m=1899+145$$

$$146m=2044$$

$$m=14$$

Therefore, the value of m is 14.

17. A man starts repaying a loan as the first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount will he pay in the 30th instalment?

Solution:

Given,

The first instalment of the loan is Rs 100.

The second instalment of the loan is Rs 105, and so on as the instalment increases by Rs 5 every month.

Thus, the amount that the man repays every month forms an A.P.

And then, A.P. is 100, 105, 110, …

Where the first term, $a = 100$

Common difference, $d = 5$

So, the 30th term in this A.P. will be

$A_{30} = a + (30 – 1)d$

$= 100 + (29)(5)$

$= 100 + 145$

$= 245$

Therefore, the amount to be paid in the 30th instalment will be Rs 245.

18. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

Solution:

It's understood from the question that the angles of the polygon will form an A.P. with a common difference $d = 5°$ and first term $a = 120°$.

And we know that the sum of all angles of a polygon with n sides is $180° (n – 2)$.

Thus, we can say

$$S_n = 180^\circ(n-2)$$

$$\frac{n}{2}\left[2a+(n-1)d\right] = 180^\circ(n-2)$$

$$\frac{n}{2}\left[240^\circ+(n-1)5^\circ\right] = 180(n-2)$$

$$n\left[240+(n-1)5\right] = 360(n-2)$$

$$240n+5n^2-5n = 360n-720$$

$$5n^2+235n-360n+720 = 0$$

$$5n^2-125n+720 = 0$$

$$n^2-25n+144 = 0$$

$$n^2-16n-9n+144 = 0$$

$$n(n-16)-9(n-16) = 0$$

$$(n-9)(n-16) = 0$$

$$n = 9 \text{ or } 16$$

Thus, a polygon having 9 and 16 sides will satisfy the condition in the question.