## EXERCISE 11.1

1. If a line makes angles $90^{\circ}, 135^{\circ}, 45^{\circ}$ with the $x, y$ and $z$-axes, respectively, find its direction cosines.

## Solution:

Let the direction cosines of the line be $1, m$ and $n$.
Here let $\alpha=90^{\circ}, \beta=135^{\circ}$ and $\gamma=45^{\circ}$
So,
$1=\cos \alpha, m=\cos \beta$ and $n=\cos \gamma$
So, the direction cosines are
$1=\cos 90^{\circ}=0$
$m=\cos 135^{\circ}=\cos \left(180^{\circ}-45^{\circ}\right)=-\cos 45^{\circ}=-1 / \sqrt{ } 2$
$\mathrm{n}=\cos 45^{\circ}=1 / \sqrt{ } 2$
$\therefore$ The direction cosines of the line are $0,-1 / \sqrt{ } 2,1 / \sqrt{ } 2$
2. Find the direction cosines of a line which makes equal angles with the coordinate axes.

## Solution:

Given:

Angles are equal.
So, let the angles be $\alpha, \beta, \gamma$
Let the direction cosines of the line be $1, m$ and $n$.
$1=\cos \alpha, m=\cos \beta$ and $n=\cos \gamma$
Here, given $\alpha=\beta=\gamma$ (Since, line makes equal angles with the coordinate axes) ... (1)
The direction cosines are
$1=\cos \alpha, m=\cos \beta$ and $n=\cos \gamma$
We have,
$\mathrm{l}^{2}+\mathrm{m}^{2}+\mathrm{n}^{2}=1$
$\cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma=1$
From (1) we have,
$\cos ^{2} \alpha+\cos ^{2} \alpha+\cos ^{2} \alpha=1$
$3 \cos ^{2} \alpha=1$
$\operatorname{Cos} \alpha= \pm \sqrt{ }(1 / 3)$
$\therefore$ The direction cosines are
$l= \pm \sqrt{ }(1 / 3), m= \pm \sqrt{ }(1 / 3), n= \pm \sqrt{ }(1 / 3)$
3. If a line has the direction ratios $\mathbf{- 1 8}, \mathbf{1 2}, \mathbf{- 4}$, then what are its direction cosines?

## Solution:

Given:
Direction ratios as $-18,12,-4$
Where, $a=-18, b=12, c=-4$
Let us consider the direction ratios of the line as $\mathrm{a}, \mathrm{b}$ and c

Then the direction cosines are
$\frac{a}{\sqrt{a^{2}+b^{2}+c^{2}}}, \frac{b}{\sqrt{a^{2}+b^{2}+c^{2}}}, \frac{c}{\sqrt{a^{2}+b^{2}+c^{2}}}$
Where,
$\sqrt{a^{2}+b^{2}+c^{2}}=\sqrt{(-18)^{2}+12^{2}+(-4)^{2}}$

$$
\begin{aligned}
& =\sqrt{324+144+16} \\
& =\sqrt{484} \\
& =22
\end{aligned}
$$

$\therefore$ The direction cosines are
$-18 / 22,12 / 22,-4 / 22=>-9 / 11,6 / 11,-2 / 11$
4. Show that the points $(2,3,4),(-1,-2,1),(5,8,7)$ are collinear.

## Solution:

If the direction ratios of two lines segments are proportional, then the lines are collinear.

Given:
$\mathrm{A}(2,3,4), \mathrm{B}(-1,-2,1), \mathrm{C}(5,8,7)$
Direction ratio of line joining $\mathrm{A}(2,3,4)$ and $\mathrm{B}(-1,-2,1)$, are
$(-1-2),(-2-3),(1-4)=(-3,-5,-3)$

Where, $a_{1}=-3, b_{1}=-5, c_{1}=-3$
Direction ratio of line joining B $(-1,-2,1)$ and $C(5,8,7)$ are
$(5-(-1)),(8-(-2)),(7-1)=(6,10,6)$
Where, $a_{2}=6, b_{2}=10$ and $c_{2}=6$
Now,

$$
\begin{aligned}
& \frac{a_{2}}{a_{1}}=\frac{6}{-3}=-2 \\
& \frac{b_{2}}{b_{1}}=\frac{10}{-5}=-2
\end{aligned}
$$

And
$\frac{c_{2}}{c_{1}}=\frac{6}{-3}=-2$
$\therefore \mathrm{A}, \mathrm{B}, \mathrm{C}$ are collinear.
5. Find the direction cosines of the sides of the triangle whose vertices are (3,5,-4), (-1, 1, 2) and (-5, -5, -2).

Solution:
Given:
The vertices are $(3,5,-4),(-1,1,2)$ and $(-5,-5,-2)$.


The direction cosines of the two points passing through $A\left(x_{1}, y_{1}, z_{1}\right)$ and $B\left(x_{2}, y_{2}, z_{2}\right)$ is given by $\left(x_{2}-x_{1}\right),\left(y_{2}-y_{1}\right),\left(z_{2}-z_{1}\right)$

Firstly let us find the direction ratios of AB
Where, $\mathrm{A}=(3,5,-4)$ and $\mathrm{B}=(-1,1,2)$
Ratio of $A B=\left[\left(x_{2}-x_{1}\right)^{2},\left(y_{2}-y_{1}\right)^{2},\left(z_{2}-z_{1}\right)^{2}\right]$
$=(-1-3),(1-5),(2-(-4))=-4,-4,6$
Then by using the formula,
$\sqrt{ }\left[\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}\right]$
$\sqrt{ }\left[(-4)^{2}+(-4)^{2}+(6)^{2}\right]=\sqrt{ }(16+16+36)$
$=\sqrt{ } 68$
$=2 \sqrt{ } 17$
Now let us find the direction cosines of the line $A B$
By using the formula,

$$
\frac{\left(x_{2}-x_{1}\right)}{A B}, \frac{\left(y_{2}-y_{1}\right)}{A B}, \frac{\left(z_{2}-z_{1}\right)}{A B}
$$

$-4 / 2 \sqrt{ } 17,-4 / 2 \sqrt{ } 17,6 / 2 \sqrt{ } 17$
Or $-2 / \sqrt{ } 17,-2 / \sqrt{ } 17,3 / \sqrt{ } 17$
Similarly,
Let us find the direction ratios of BC
Where, $\mathrm{B}=(-1,1,2)$ and $\mathrm{C}=(-5,-5,-2)$
Ratio of $A B=\left[\left(x_{2}-x_{1}\right)^{2},\left(y_{2}-y_{1}\right)^{2},\left(z_{2}-z_{1}\right)^{2}\right]$
$=(-5+1),(-5-1),(-2-2)=-4,-6,-4$
Then by using the formula,
$\sqrt{ }\left[\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}\right]$
$\sqrt{ }\left[(-4)^{2}+(-6)^{2}+(-4)^{2}\right]=\sqrt{ }(16+36+16)$
$=\sqrt{ } 68$
$=2 \sqrt{ } 17$
Now, let us find the direction cosines of the line AB

By using the formula,

$$
\frac{\left(x_{2}-x_{1}\right)}{A B}, \frac{\left(y_{2}-y_{1}\right)}{A B}, \frac{\left(z_{2}-z_{1}\right)}{A B}
$$

$-4 / 2 \sqrt{ } 17,-6 / 2 \sqrt{ } 17,-4 / 2 \sqrt{ } 17$
Or $-2 / \sqrt{ } 17,-3 / \sqrt{ } 17,-2 / \sqrt{ } 17$
Similarly,
Let us find the direction ratios of CA
Where, $\mathrm{C}=(-5,-5,-2)$ and $\mathrm{A}=(3,5,-4)$
Ratio of $A B=\left[\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)^{2},\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2},\left(\mathrm{z}_{2}-\mathrm{z}_{1}\right)^{2}\right]$
$=(3+5),(5+5),(-4+2)=8,10,-2$
Then, by using the formula,
$\sqrt{ }\left[\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)^{2}+\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2}+\left(\mathrm{z}_{2}-\mathrm{z}_{1}\right)^{2}\right]$
$\sqrt{ }\left[(8)^{2}+(10)^{2}+(-2)^{2}\right]=\sqrt{ }(64+100+4)$
$=\sqrt{ } 168$
$=2 \sqrt{ } 42$

Now, let us find the direction cosines of the line $A B$
By using the formula,
$\frac{\left(x_{2}-x_{1}\right)}{A B}, \frac{\left(y_{2}-y_{1}\right)}{A B}, \frac{\left(z_{2}-z_{1}\right)}{A B}$
$8 / 2 \sqrt{ } 42,10 / 2 \sqrt{ } 42,-2 / 2 \sqrt{ } 42$
Or $4 / \sqrt{ } 42,5 / \sqrt{ } 42,-1 / \sqrt{ } 42$

