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1. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.
(a) z = 2

(b) x + y + z = 1

(c) 2x + 3y – z = 5

(d) 5y + 8 = 0

Solution:

(a) z = 2

Given:

The equation of the plane, z = 2 or 0x + 0y + z = 2 …. (1)

Direction ratio of the normal (0, 0, 1)

By using the formula,

$\sqrt{[(0)^2 + (0)^2 + (1)^2]} = \sqrt{1}$

= 1

Now,

Divide both the sides of equation (1) by 1, we get

0x/(1) + 0y/(1) + z/1 = 2

So this is of the form lx + my + nz = d

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are 0, 0, 1

Distance (d) from the origin is 2 units

(b) x + y + z = 1

Given:

The equation of the plane, x + y + z = 1…. (1)

Direction ratio of the normal (1, 1, 1)

By using the formula,

$\sqrt{[(1)^2 + (1)^2 + (1)^2]} = \sqrt{3}$

Now,

Divide both the sides of equation (1) by $\sqrt{3}$, we get

$x/(\sqrt{3}) + y/(\sqrt{3}) + z/(\sqrt{3}) = 1/\sqrt{3}$

So this is of the form $lx + my + nz = d$

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are $1/\sqrt{3}$, $1/\sqrt{3}$, $1/\sqrt{3}$

Distance (d) from the origin is $1/\sqrt{3}$ units

(c) $2x + 3y – z = 5$

Given:

The equation of the plane, $2x + 3y – z = 5$…. (1)

Direction ratio of the normal (2, 3, -1)

By using the formula,

$\sqrt{[(2)^2 + (3)^2 + (-1)^2]} = \sqrt{14}$

Now,

Divide both the sides of equation (1) by $\sqrt{14}$, we get

$2x/(\sqrt{14}) + 3y/(\sqrt{14}) – z/(\sqrt{14}) = 5/\sqrt{14}$

So this is of the form $lx + my + nz = d$

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are $2/\sqrt{14}$, $3/\sqrt{14}$, $-1/\sqrt{14}$

Distance (d) from the origin is $5/\sqrt{14}$ units

(d) $5y + 8 = 0$

Given:

The equation of the plane, $5y + 8 = 0$

$-5y = 8$ or

$0x – 5y + 0z = 8$…. (1)

Direction ratio of the normal (0, -5, 0)

By using the formula,

$\sqrt{[(0)^2 + (-5)^2 + (0)^2]} = \sqrt{25}$

= 5

Now,

Divide both the sides of equation (1) by 5, we get

0x/(5) – 5y/(5) – 0z/(5) = 8/5

So this is of the form lx + my + nz = d

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are 0, -1, 0

Distance (d) from the origin is 8/5 units

2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector

$3\hat{i} + 5\hat{j} - 6\hat{k}.$

Solution:

Given:

The vector $3\hat{i} + 5\hat{j} - 6\hat{k}.$

Vector eq. of the plane with position vector $\vec{r}$ is

$\vec{r}.\hat{n} = d$ … (1)

So,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{9+25+36}}$$

$$= \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}$$

So,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{9+25+36}}$$

$$= \frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$$

Substituting in equation (1), we get

$$\vec{r}.\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}} = 7$$

$$\vec{r}.3\hat{i}+5\hat{j}-6\hat{k} = 7\sqrt{70}$$

∴ The required vector equation is $\vec{r}.3\hat{i}+5\hat{j}-6\hat{k} = 7\sqrt{70}$

3. Find the Cartesian equation of the following planes:

(a) $\vec{r}.(\hat{i}+\hat{j}-\hat{k}) = 2$

Solution:

Given:

The equation of the plane.

Let $\vec{r}$ be the position vector of P(x, y, z) is given by

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$

So,

$$\vec{r}.\left(\hat{i}+\hat{j}-\hat{k}\right) = 2$$

Substituting the value of $\vec{r}$, we get

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+\hat{j}-\hat{k}\right) = 2$$

∴ The Cartesian equation is

$$x + y - z = 2$$

(b) $\vec{r}.(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

Solution:

Let $\vec{r}$ be the position vector of P(x, y, z) is given by

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So,

$\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{4}k\right) = 1$

Substituting the value of $\vec{r}$, we get

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$

∴ The Cartesian equation is

2x + 3y - 4z = 1

(c) $\vec{r}.[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$

Solution:

Let $\vec{r}$ be the position vector of P(x, y, z) is given by

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So,

$\vec{r}.\left[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}\right] = 15$

Substituting the value of $\vec{r}$, we get

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}\right] = 15$

∴ The Cartesian equation is

(s – 2t) x + (3 – t) y + (2s + t) z = 15

4. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

(a) 2x + 3y + 4z – 12 = 0

(b) 3y + 4z – 6 = 0

(c) x + y + z = 1

(d) 5y + 8 = 0

Solution:

(a) 2x + 3y + 4z – 12 = 0

Let the coordinate of the foot of ⊥ P from the origin to the given plane be P(x, y, z).

$2x + 3y + 4z = 12$ …. (1)

Direction ratio are (2, 3, 4)

$\sqrt{[(2)^2 + (3)^2 + (4)^2]} = \sqrt{(4 + 9 + 16)}$

$= \sqrt{29}$

Now,

Divide both the sides of equation (1) by $\sqrt{29}$, we get

$2x/(\sqrt{29}) + 3y/(\sqrt{29}) + 4z/(\sqrt{29}) = 12/\sqrt{29}$

So this is of the form $lx + my + nz = d$

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are $2/\sqrt{29}$, $3/\sqrt{29}$, $4/\sqrt{29}$

Coordinate of the foot (ld, md, nd) =

$= [(2/\sqrt{29})\ (12/\sqrt{29}), (3/\sqrt{29})\ (12/\sqrt{29}), (4/\sqrt{29})\ (12/\sqrt{29})]$

$= 24/29, 36/29, 48/29$

(b) $3y + 4z – 6 = 0$

Let the coordinate of the foot of ⊥ P from the origin to the given plane be P(x, y, z).

$0x + 3y + 4z = 6$ …. (1)

Direction ratio are (0, 3, 4)

$\sqrt{[(0)^2 + (3)^2 + (4)^2]} = \sqrt{(0 + 9 + 16)}$

$= \sqrt{25}$

$= 5$

Now,

Divide both the sides of equation (1) by 5, we get

$0x/(5) + 3y/(5) + 4z/(5) = 6/5$

So this is of the form $lx + my + nz = d$

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are 0/5, 3/5, 4/5

Coordinate of the foot (ld, md, nd) =

= [(0/5) (6/5), (3/5) (6/5), (4/5) (6/5)]

= 0, 18/25, 24/25

(c) x + y + z = 1

Let the coordinate of the foot of ⊥ P from the origin to the given plane be P(x, y, z).

x + y + z = 1 …. (1)

Direction ratio are (1, 1, 1)

$\sqrt{[(1)^2 + (1)^2 + (1)^2]} = \sqrt{(1 + 1 + 1)}$

$= \sqrt{3}$

Now,

Divide both the sides of equation (1) by √3, we get

1x/(√3) + 1y/(√3) + 1z/(√3) = 1/√3

So this is of the form lx + my + nz = d

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are 1/√3, 1/√3, 1/√3

Coordinate of the foot (ld, md, nd) =

= [(1/√3) (1/√3), (1/√3) (1/√3), (1/√3) (1/√3)]

= 1/3, 1/3, 1/3

(d) 5y + 8 = 0

Let the coordinate of the foot of ⊥ P from the origin to the given plane be P(x, y, z).

0x – 5y + 0z = 8 …. (1)

Direction ratio are (0, -5, 0)

$\sqrt{[(0)^2 + (-5)^2 + (0)^2]} = \sqrt{(0 + 25 + 0)}$

$= \sqrt{25}$

= 5

Now,

Divide both the sides of equation (1) by 5, we get

0x/(5) – 5y/(5) + 0z/(5) = 8/5

So this is of the form lx + my + nz = d

Where, l, m, n are the direction cosines and d is the distance

∴ The direction cosines are 0, -1, 0

Coordinate of the foot (ld, md, nd) =

= [(0/5) (8/5), (-5/5) (8/5), (0/5) (8/5)]

= 0, -8/5, 0

5. Find the vector and Cartesian equations of the planes

(a) that passes through the point (1, 0, –2) and the normal to the plane is

$\hat{i}+\hat{j}-\hat{k}.$

(b) that passes through the point (1,4, 6) and the normal vector to the plane is

$\hat{i}-2\hat{j}+\hat{k}.$

Solution:

(a) That passes through the point (1, 0, –2) and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}.$

Let the position vector of the point (1, 0, -2) be

$\vec{a}=\left(1\hat{i}+0\hat{j}-2\hat{k}\right)$

We know that Normal $\vec{N}$ ⊥ to the plane is given as:

$\vec{N}=\hat{i}+\hat{j}-\hat{k}$

Vector equation of the plane is given as:

$\left(\vec{r}-\vec{a}\right).\vec{N}=0$

Now,

x – 1 – 2y + 8 + z – 6 = 0

x – 2y + z + 1 = 0

x – 2y + z = -1

∴ The required Cartesian equation of the plane is x – 2y + z = -1

$$\left(\vec{r}-\left(\hat{i}-2\hat{k}\right)\right).\hat{i}+\hat{j}-\hat{k}=0 \quad \dots (1)$$

Since,

$\vec{r} = x\hat{i}+ y\hat{j}+ z\hat{k}$

So equation (1) becomes,

$$\left(x\hat{i}+ y\hat{j}+ z\hat{k}-\hat{i}+2\hat{k}\right).\hat{i}+\hat{j}-\hat{k}=0$$

$$\left[(x-1)\hat{i}+ y\hat{j}+(z+2)\hat{k}\right].\hat{i}+\hat{j}-\hat{k}=0$$

$x-1+y-z-2=0$

$x+y-z-3=0$

∴ The required Cartesian equation of the plane is $x+y-z=3$

(b) That passes through the point (1, 4, 6) and the normal vector to the plane is $\hat{i}-2\hat{j}+\hat{k}$.

Let the position vector of the point (1, 0, -2) be

$\vec{a}=\left(1\hat{i}+4\hat{j}+6\hat{k}\right)$

We know that Normal $\vec{N}$ ⊥ to the plane is given as:

$\vec{N}= \hat{i}-2\hat{j}+ \hat{k}$

Vector equation of the plane is given as:

$(\vec{r}-\vec{a}).\vec{N}=0$

Now,

$$\left(\vec{r}-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right).\hat{i}-2\hat{j}+\hat{k}=0 \quad \dots (1)$$

Since,

$\vec{r} = x\hat{i}+ y\hat{j}+ z\hat{k}$

So equation (1) becomes,

$$\left(x\hat{i}+ y\hat{j}+ z\hat{k}-\hat{i}-4\hat{j}-6\hat{k}\right).\hat{i}-2\hat{j}+\hat{k}=0$$

$$\left[(x-1)\hat{i}+(y-4)\hat{j}+(z-6)\hat{k}\right].\hat{i}-2\hat{j}+\hat{k}=0$$

$x-1-2y+8+z-6=0$

x – 2y + z + 1 = 0

x – 2y + z = -1

∴ The required Cartesian equation of the plane is x – 2y + z = -1

6. Find the equations of the planes that passes through three points.
(a) (1, 1, –1), (6, 4, –5), (–4, –2, 3)

(b) (1, 1, 0), (1, 2, 1), (–2, 2, –1)

Solution:

Given:

The points are (1, 1, -1), (6, 4, -5), (-4, -2, 3).

Let,

$$= \begin{vmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{vmatrix}$$

= 1(12 – 10) – 1(18 – 20) -1 (-12 + 16)

= 2 + 2 – 4

= 0

Since, the value of determinant is 0.

∴ The points are collinear as there will be infinite planes passing through the 3 given points.

(b) (1, 1, 0), (1, 2, 1), (–2, 2, –1)

The given points are (1, 1, 0), (1, 2, 1), (-2, 2, -1).

Let,

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix}$$

= 1(-2 - 2) – 1(-1 + 2)

= -4 – 1

= -5 ≠ 0

Since, there passes a unique plane from the given 3 points.

Equation of the plane passing through the points, (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), i.e.,

$$= \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

Let us substitute the values and simplify

$$= \begin{vmatrix} x - 1 & y - 1 & z \\ x_2 - 1 & y_2 - 1 & z_2 \\ x_3 - 1 & y_3 - 1 & z_3 \end{vmatrix}$$

$$= \begin{vmatrix} x - 1 & y - 1 & z \\ 1 - 1 & 2 - 1 & 1 \\ -2 - 1 & 2 - 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} x - 1 & y - 1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix}$$

=> (x - 1) (-2) – (y - 1) (3) + 3z = 0

=> -2x + 2 – 3y + 3 + 3z = 0

= 2x + 3y – 3z = 5

∴ The required equation of the plane is 2x + 3y – 3z = 5.

7. Find the intercepts cut off by the plane 2x + y – z = 5.

Solution:

Given:

The plane $2x + y – z = 5$

Let us express the equation of the plane in intercept form

$x/a + y/b + z/c = 1$

Where a, b, c are the intercepts cut-off by the plane at x, y and z axes, respectively.

$2x + y – z = 5$ …. (1)

Now dividing both the sides of equation (1) by 5, we get

$2x/5 + y/5 – z/5 = 5/5$

$2x/5 + y/5 – z/5 = 1$

$x/(5/2) + y/5 + z/(-5) = 1$

Here, $a = 5/2$, $b = 5$ and $c = -5$

∴ The intercepts cut-off by the plane are 5/2, 5 and -5.

8. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

Solution:

We know that the equation of the plane ZOX is $y = 0$

So, the equation of plane parallel to ZOX is of the form, $y = a$

Since the y-intercept of the plane is 3, $a = 3$

∴ The required equation of the plane is $y = 3$

9. Find the equation of the plane through the intersection of the planes $3x – y + 2z – 4 = 0$ and $x + y + z – 2 = 0$ and the point (2, 2, 1).

Solution:

Given:

Equation of the plane passes through the intersection of the plane is given by

$(3x – y + 2z – 4) + \lambda (x + y + z – 2) = 0$ and the plane passes through the points (2, 2, 1).

So, $(3 \times 2 – 2 + 2 \times 1 – 4) + \lambda (2 + 2 + 1 – 2) = 0$

$2 + 3\lambda = 0$

$3\lambda = -2$

$\lambda = -2/3$ …. (1)

Upon simplification, the required equation of the plane is given as

$(3x - y + 2z - 4) - 2/3\ (x + y + z - 2) = 0$

$(9x - 3y + 6z - 12 - 2x - 2y - 2z + 4)/3 = 0$

$7x - 5y + 4z - 8 = 0$

∴ The required equation of the plane is $7x - 5y + 4z - 8 = 0$

10. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7,\ \vec{r}.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ **and through the point (2, 1, 3).**

Solution:

Let the vector equation of the plane passing through the intersection of the planes are

$\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$

Here,

$\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) - 7 = 0$ …. (1)

$\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9 = 0$ …. (2)

The equation of any plane through the intersection of the planes given in equations (1) and (2) is given by,

The equation of any plane through the intersection of the planes given in equations (1) and (2) is given by,

$$\left[\vec{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7\right]+\lambda\ \left[\vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0$$

$$\vec{r}\left[\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\left(2\lambda\ \hat{i}+5\lambda\ \hat{j}+3\lambda\ \hat{k}\right)\right]-7-9\lambda\ \ =0$$

$$\vec{r}.\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]-7-9\lambda\ \ =0 \quad \ldots (3)$$

Since the plane passes through points (2, 1, 3)

$$\left(2\hat{i}+\hat{j}+3\hat{k}\right).\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]-7-9\lambda\ \ =0$$

$4 + 4\lambda + 2 + 5\lambda - 9 + 9\lambda - 7 - 9\lambda = 0$

$9\lambda = 10$

$\lambda = 10/9$

Sow, substituting $\lambda = 10/9$ in equation (1) we get,

$$\vec{r}.\left[\left(2+\frac{20}{9}\right)\hat{i}+\left(2+\frac{50}{9}\right)\hat{j}+\left(-3+\frac{30}{9}\right)\hat{k}\right]-7-9\frac{10}{9}\ =0$$

$$\vec{r}.\left[\left(2+\frac{20}{9}\right)\hat{i}+\left(2+\frac{50}{9}\right)\hat{j}+\left(-3+\frac{30}{9}\right)\hat{k}\right]-17=0$$

$$\vec{r}.\left[\left(2+\frac{20}{9}\right)\hat{i}+\left(2+\frac{50}{9}\right)\hat{j}+\left(-3+\frac{30}{9}\right)\hat{k}\right]=17$$

$$\vec{r}\left[\frac{38}{9}\hat{i}+\frac{68}{9}\hat{j}+\frac{3}{9}\hat{k}\right]=17$$

$$\vec{r}\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$$

$\therefore$ The required equation of the plane is $\vec{r}\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$

11. Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0.

Solution:

Let the equation of the plane that passes through the two-given planes

x + y + z = 1 and 2x + 3y + 4z = 5 is

$(x + y + z – 1) + \lambda\ (2x + 3y + 4z – 5) = 0$

$(2\lambda + 1)\ x + (3\lambda + 1)\ y + (4\lambda + 1)\ z - 1 - 5\lambda = 0$…… (1)

So the direction ratio of the plane is $(2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$

And direction ratio of another plane is (1, -1, 1)

Since, both the planes are ⊥

So by substituting in $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(2\lambda + 1 \times 1) + (3\lambda + 1 \times (-1)) + (4\lambda + 1 \times 1) = 0$

$2\lambda + 1 - 3\lambda - 1 + 4\lambda + 1 = 0$

$3\lambda + 1 = 0$

$\lambda = -1/3$

Substitute the value of λ in equation (1) we get,

$$\left(2\frac{(-1)}{3} + 1\right)x + \left(3\frac{(-1)}{3} + 1\right)y + \left(4\frac{(-1)}{3} + 1\right)z - 1 - 5\frac{(-1)}{3} = 0$$

$$\frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$$

$x - z + 2 = 0$

∴ The required equation of the plane is $x - z + 2 = 0$

12. Find the angle between the planes whose vector equations are

$$\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5,\ \vec{r}.(3\hat{i} - 3\hat{j} + 5\hat{k}) = 3.$$

Solution:

Given:

The equation of the given planes are

$\vec{r}\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=5$

If n_1 and n_2 are normal to the planes, then

$\vec{r_1}.\vec{n_1}=d_2$ and $\vec{r_2}.\vec{n_2}=d_2$

Angle between two planes is given as

$$\cos\theta=\left|\frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|}\right|$$

$$=\left|\frac{6-6-15}{\sqrt{4+4+9}\sqrt{9+9+25}}\right|$$

$$=\left|\frac{-15}{\sqrt{17}\sqrt{43}}\right|$$

$$\theta=\cos^{-1}\left(\frac{15}{\sqrt{17}\sqrt{43}}\right)$$

$$=\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$$

$$\theta=\cos^{-1}\left(\frac{15}{\sqrt{17}\sqrt{43}}\right)$$

$$=\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$$

∴ The angle is $\cos^{-1}(15/\sqrt{731})$

13. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

(a) 7x + 5y + 6z + 30 = 0 and 3x – y – 10z + 4 = 0

(b) 2x + y + 3z – 2 = 0 and x – 2y + 5 = 0

(c) 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0

(d) 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0

(e) 4x + 8y + z – 8 = 0 and y + z – 4 = 0

Solution:

(a) 7x + 5y + 6z + 30 = 0 and 3x – y – 10z + 4 = 0

Given:

The equation of the given planes are

7x + 5y + 6z + 30 = 0 and 3x – y – 10z + 4 = 0

Two planes are ⊥ if the direction ratio of the normal to the plane is

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

21 – 5 – 60

-44 ≠ 0

Both the planes are not ⊥ to each other.

Now, two planes are || to each other if the direction ratio of the normal to the plane is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{7}{3} \neq \frac{5}{-1} \neq \frac{6}{-10}$$ [both the planes are not || to each other]

Now, the angle between them is given by

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$\cos\theta = \frac{-44}{\sqrt{49 + 25 + 36}\sqrt{9 + 1 + 100}}$$

$$= \frac{-44}{\sqrt{110}\sqrt{110}}$$

$$= \frac{-44}{110}$$

$$\theta = \cos^{-1}\frac{2}{5}$$

$$= \frac{-44}{\sqrt{110}\sqrt{110}}$$

$$= \frac{-44}{110}$$

$$\theta = \cos^{-1}\frac{2}{5}$$

∴ The angle is $\cos^{-1}$ (2/5)

(b) 2x + y + 3z – 2 = 0 and x – 2y + 5 = 0

Given:

The equation of the given planes are

2x + y + 3z – 2 = 0 and x – 2y + 5 = 0

Two planes are ⊥ if the direction ratio of the normal to the plane is

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

2 × 1 + 1 × (-2) + 3 × 0

= 0

∴ The given planes are ⊥ to each other.

(c) 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0

Given:

The equation of the given planes are

2x – 2y + 4z + 5 =0 and x – 2y + 5 = 0

We know that, two planes are ⊥ if the direction ratio of the normal to the plane is

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

6 + 6 + 24

36 ≠ 0

∴ Both the planes are not ⊥ to each other.

Now let us check, both planes are || to each other if the direction ratio of the normal to the plane is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2}{3} = \frac{-2}{-3} = \frac{4}{6}$$
$$\frac{2}{3} = \frac{2}{3} = \frac{2}{3}$$

∴ The given planes are || to each other.

(d) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

Given:

The equation of the given planes are

$2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

We know that, two planes are ⊥ if the direction ratio of the normal to the plane is

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$2 \times 2 + (-1) \times (-1) + 3 \times 3$

$14 \neq 0$

∴ Both the planes are not ⊥ to each other.

Now, let us check two planes are || to each other if the direction ratio of the normal to the plane is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2}{2} = \frac{-1}{-1} = \frac{3}{3}$$
$$\frac{1}{1} = \frac{1}{1} = \frac{1}{1}$$

∴ The given planes are || to each other.

(e) $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

Given:

The equation of the given planes are

$4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

We know that, two planes are ⊥ if the direction ratio of the normal to the plane is

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$0 + 8 + 1$

$9 \neq 0$

∴ Both the planes are not ⊥ to each other.

Now let us check, two planes are || to each other if the direction ratio of the normal to the plane is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{4}{0} \neq \frac{8}{1} \neq \frac{1}{1}$$

∴ Both the planes are not || to each other.

Now let us find the angle between them, which is given as

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\cos\theta = \frac{4 \times 0 + 8 \times 1 + 1 \times 1}{\sqrt{16 + 64 + 1}\sqrt{0 + 1 + 1}}$$

$$= \frac{9}{9\sqrt{2}}$$

$$\theta = \cos^{-1}\frac{9}{9\sqrt{2}}$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 45^0$$

∴ The angle is 45^o.

14. In the following cases, find the distance of each of the given points from the corresponding given plane.
Point Plane
(a) (0, 0, 0) 3x – 4y + 12 z = 3

(b) (3, -2, 1) 2x – y + 2z + 3 = 0

(c) (2, 3, -5) x + 2y – 2z = 9

(d) (-6, 0, 0) 2x – 3y + 6z – 2 = 0

Solution:

(a) Point Plane

(0, 0, 0) 3x – 4y + 12 z = 3

We know that, distance of point $P(x_1, y_1, z_1)$ from the plane $Ax + By + Cz – D = 0$ is given as:

$$d = \left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

Given point is (0, 0, 0) and the plane is $3x – 4y + 12z = 3$

$$d = \left|\frac{0+0+0+3}{\sqrt{9+16+144}}\right|$$

$$= \left|\frac{3}{\sqrt{169}}\right|$$

$$= \frac{3}{13}$$

Given point is (0, 0, 0) and the plane is $3x – 4y + 12z = 3$

$$d = \left|\frac{0+0+0+3}{\sqrt{9+16+144}}\right|$$

$$= \left|\frac{3}{\sqrt{169}}\right|$$

$= |3/\sqrt{169}|$

$= 3/13$

∴ The distance is 3/13.

(b) Point Plane

(3, -2, 1) 2x – y + 2z + 3 = 0

We know that, distance of point $P(x_1, y_1, z_1)$ from the plane Ax + By + Cz – D = 0 is given as:

$$d = \left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

Given point is (0, 0, 0) and the plane is 3x – 4y + 12z = 3

$$d = \left|\frac{0 + 0 + 0 + 3}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{3}{\sqrt{169}}\right|$$

$$= \frac{3}{13}$$

Given point is (3, -2, 1) and the plane is 2x – y + 2z + 3 = 0

$$d = \left|\frac{6 + 2 + 2 + 3}{\sqrt{4 + 1 + 4}}\right|$$

= |13/√9|

= 13/3

∴ The distance is 13/3.

(c) Point Plane

(2, 3, -5) x + 2y – 2z = 9

We know that, distance of point $P(x_1, y_1, z_1)$ from the plane Ax + By + Cz – D = 0 is given as:

$$d = \left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

Given point is (0, 0, 0) and the plane is 3x – 4y + 12z = 3

$$d = \left|\frac{0 + 0 + 0 + 3}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{3}{\sqrt{169}}\right|$$

$$= \frac{3}{13}$$

Given point is (2, 3, -5) and the plane is x + 2y – 2z = 9

$$d = \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

= |9/√9|

= 9/3

= 3

∴ The distance is 3.

(d) Point Plane

(-6, 0, 0) 2x – 3y + 6z – 2 = 0

We know that, distance of point P(x_1, y_1, z_1) from the plane Ax + By + Cz – D = 0 is given as:

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

Given point is (0, 0, 0) and the plane is 3x – 4y + 12z = 3

$$d = \left| \frac{0 + 0 + 0 + 3}{\sqrt{9 + 16 + 144}} \right|$$

$$= \left| \frac{3}{\sqrt{169}} \right|$$

$$= \frac{3}{13}$$

Given point is (-6, 0, 0) and the plane is 2x – 3y + 6z – 2 = 0

$$d = \left| \frac{-12 - 0 + 0 - 2}{\sqrt{4 + 9 + 36}} \right|$$

= |14/√49|

= 14/7

= 2

∴ The distance is 2.