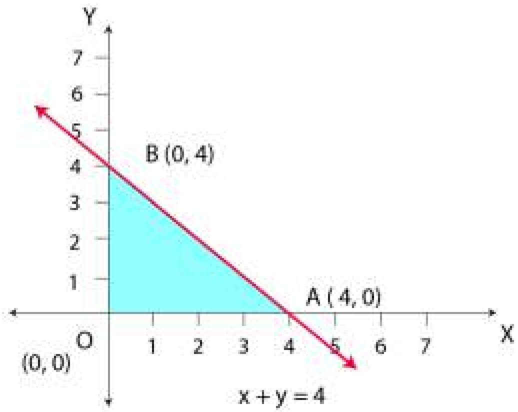
EXERCISE 12.1

1. Maximise Z = 3*x* + 4*y*

Subject to the constraints: $x + y \le 4, x \ge 0, y \ge 0.$

Solution:

The feasible region determined by the constraints, x + y ≤ 4, x ≥ 0, y ≥ 0, is given below.



O (0, 0), A (4, 0), and B (0, 4) are the corner points of the feasible region. The values of Z at these points are given below.

Corner point	Z = 3x + 4y	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	Maximum

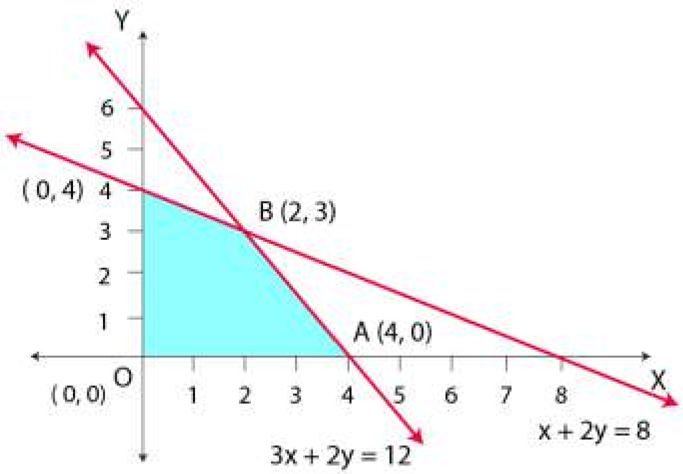
Hence, the maximum value of Z is 16 at the point B (0, 4).

2. Minimise Z = −3*x* + 4*y*

subject to $x + 2y \le 8,\ 3x + 2y \le 12,\ x \ge 0,\ y \ge 0$.

Solution:

The feasible region determined by the system of constraints, $x + 2y \le 8,\ 3x + 2y \le 12,\ x \ge 0,\ y \ge 0$ is given below.



O (0, 0), A (4, 0), B (2, 3) and C (0, 4) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = – 3x + 4y	
O (0, 0)	0	
A (4, 0)	-12	Minimum
B (2, 3)	6	
C (0, 4)	16	

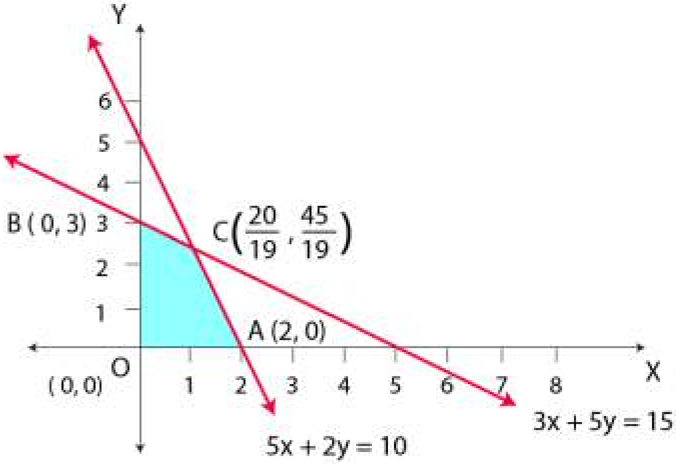
Hence, the minimum value of Z is – 12 at the point (4, 0).

3. Maximise Z = 5*x* + 3*y*

subject to $3x + 5y \le 15,\ 5x + 2y \le 10,\ x \ge 0,\ y \ge 0$.

Solution:

The feasible region determined by the system of constraints, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, is given below.



O (0, 0), A (2, 0), B (0, 3) and C (20 / 19, 45 / 19) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 3y	
O (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
C (20 / 19, 45 / 19)	235 / 19	Maximum

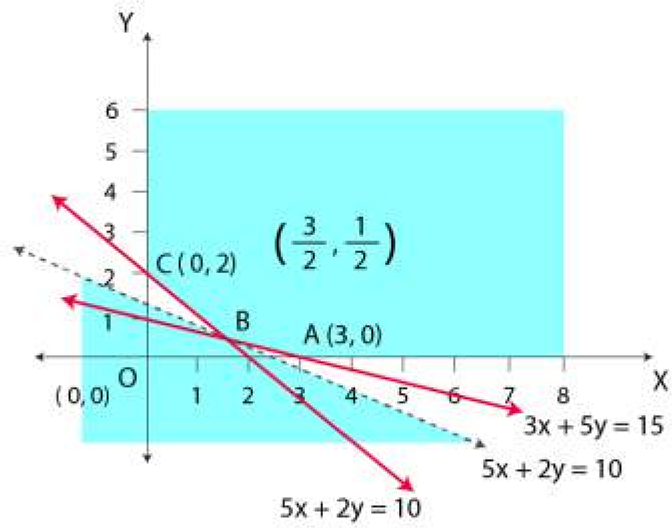
Hence, the maximum value of Z is 235 / 19 at the point (20 / 19, 45 / 19).

4. Minimise Z = 3*x* + 5*y*

such that $x+3y \geq 3,\ x+y \geq 2,\ x, y \geq 0$.

Solution:

The feasible region determined by the system of constraints, x + 3y ≥ 3, x + y ≥ 2, and x, y ≥ 0, is given below.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), B (3 / 2, 1 / 2) and C (0, 2).

The values of Z at these corner points are given below.

Corner point	Z = 3x + 5y	
A (3, 0)	9	
B (3 / 2, 1 / 2)	7	Smallest
C (0, 2)	10	

7 may or may not be the minimum value of Z because the feasible region is unbounded.

For this purpose, we draw the graph of the inequality, 3x + 5y < 7 and check whether the resulting half-plane has common points with the feasible region or not.

Hence, it can be seen that the feasible region has no common point with 3x + 5y < 7.

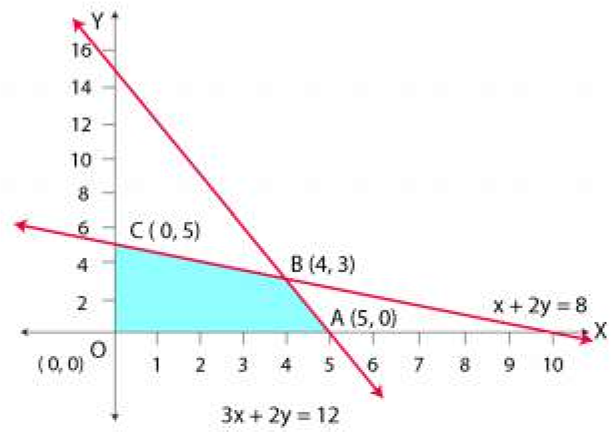
Thus, the minimum value of Z is 7 at point B (3 / 2, 1 / 2).

5. Maximise Z = 3*x* + 2*y*

subject to $x + 2y \le 10, 3x + y \le 15, x, y \ge 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, and $y \geq 0$, is given below.



A (5, 0), B (4, 3), C (0, 5) and D (0, 0) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = 3x + 2y	
A (5, 0)	15	
B (4, 3)	18	Maximum
C (0, 5)	10	

Hence, the maximum value of Z is 18 at points (4, 3).

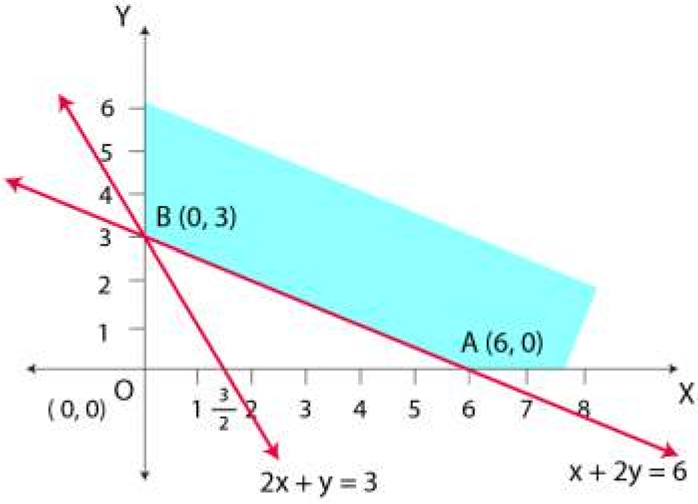
6. Minimise Z = $x + 2y$

subject to

$2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is given below.



A (6, 0) and B (0, 3) are the corner points of the feasible region.

The values of Z at the corner points are given below.

Corner point	Z = x + 2y
A (6, 0)	6
B (0, 3)	6

Here, the values of Z at points A and B are same. If we take any other point, such as (2, 2) on line x + 2y = 6, then Z = 6.

Hence, the minimum value of Z occurs for more than 2 points.

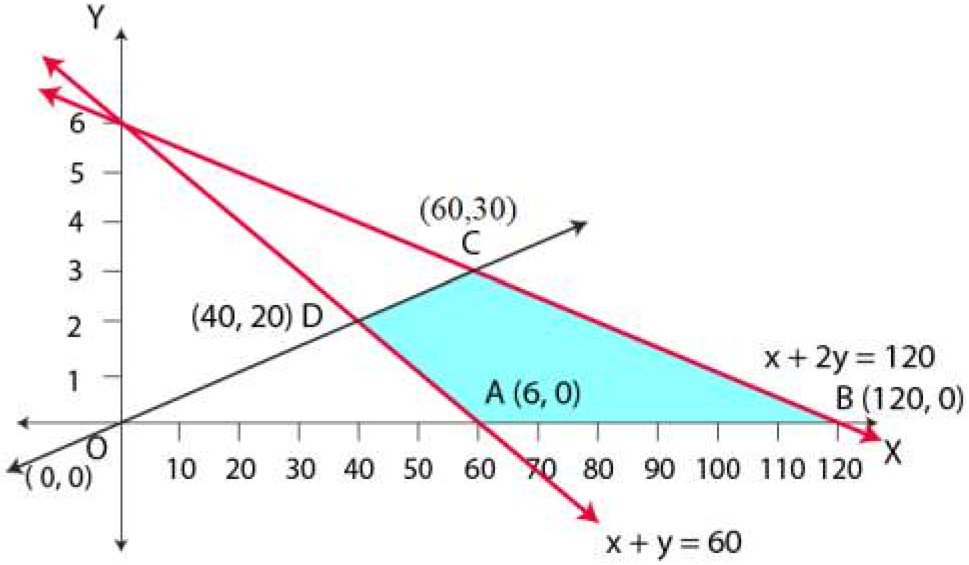
Therefore, the value of Z is minimum at every point on the line x + 2y = 6.

7. Minimise and Maximise Z = 5*x* + 10*y*

subject to $x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x, y \ge 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, is given below.



A (60, 0), B (120, 0), C (60, 30), and D (40, 20) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 10y	
A (60, 0)	300	Minimum
B (120, 0)	600	Maximum
C (60, 30)	600	Maximum
D (40, 20)	400	

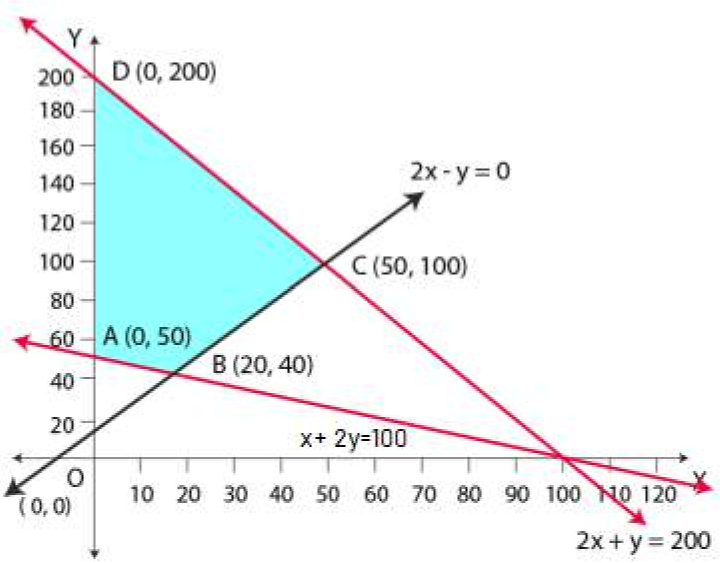
The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

8. Minimise and Maximise Z = $x + 2y$

subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200;\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is given below.



A (0, 50), B (20, 40), C (50, 100) and D (0, 200) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = x + 2y	
A (0, 50)	100	Minimum
B (20, 40)	100	Minimum
C (50, 100)	250	
D (0, 200)	400	Maximum

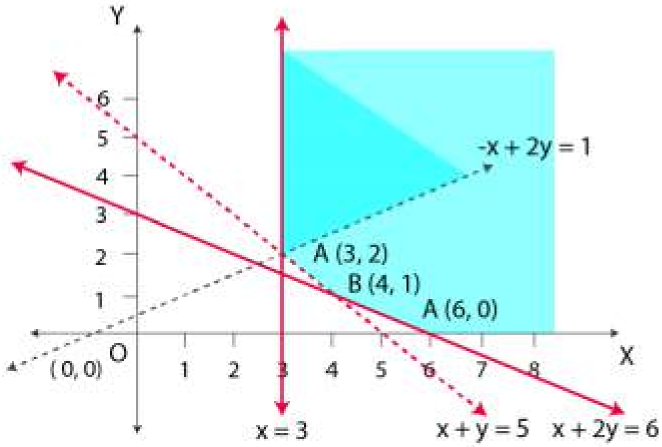
The maximum value of Z is 400 at points (0, 200), and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

9. Maximise Z = − *x* + 2*y*, subject to the constraints.

$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Solution:

The feasible region determined by the constraints, $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$ is given below.



Here, it can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1) and C (3, 2) are given below.

Corner point	Z = – x + 2y
A (6, 0)	Z = – 6
B (4, 1)	Z = – 2
C (3, 2)	Z = 1

Since the feasible region is unbounded, z = 1 may or may not be the maximum value.

For this purpose, we graph the inequality, – x + 2y > 1, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, the resulting feasible region has points in common with the feasible region.

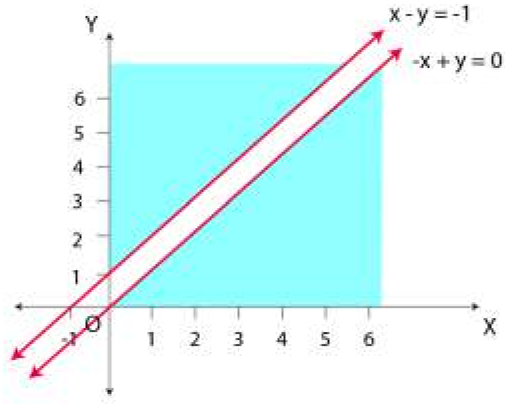
Hence, z = 1 is not the maximum value.

Z has no maximum value.

10. Maximise Z = *x* + *y*, subject to $x - y \le -1,\ -x + y \le 0,\ x, y \ge 0$.

Solution:

The region determined by the constraints $x - y \le -1,\ -x + y \le 0,\ x, y \ge 0$ is given below.



There is no feasible region, and therefore, z has no maximum value.