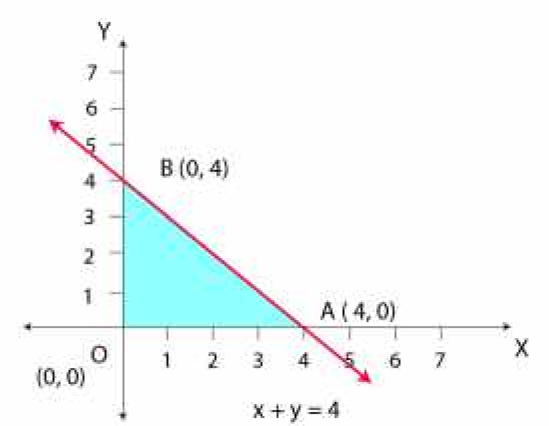
EXERCISE 12.1

1. Maximise Z = 3*x* + 4*y*

Subject to the constraints: $x + y \leq 4, x \geq 0, y \geq 0.$

Solution:

The feasible region determined by the constraints, $x + y \leq 4$, $x \geq 0$, $y \geq 0$, is given below.



O (0, 0), A (4, 0), and B (0, 4) are the corner points of the feasible region. The values of Z at these points are given below.

Corner point	Z = 3x + 4y	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	Maximum

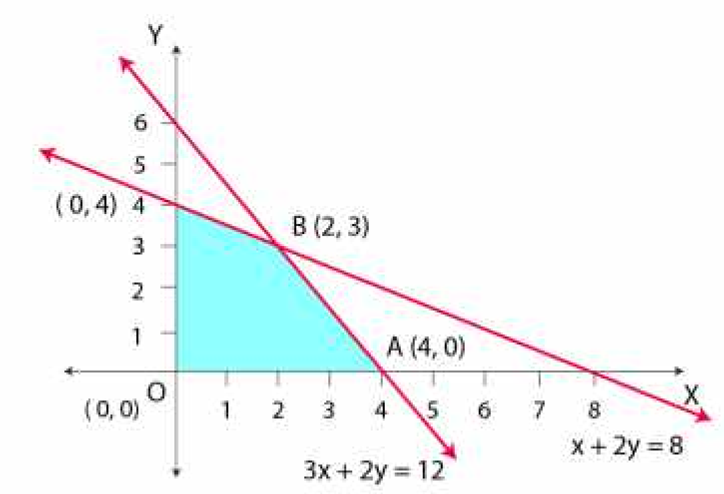
Hence, the maximum value of Z is 16 at the point B (0, 4).

2. Minimise Z = −3*x* + 4*y*

subject to $x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$.

Solution:

The feasible region determined by the system of constraints, $x+2y \leq 8,\ 3x+2y \leq 12,\ x \geq 0,\ y \geq 0$ is given below.



O (0, 0), A (4, 0), B (2, 3) and C (0, 4) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = – 3x + 4y	
O (0, 0)	0	
A (4, 0)	-12	Minimum
B (2, 3)	6	
C (0, 4)	16	

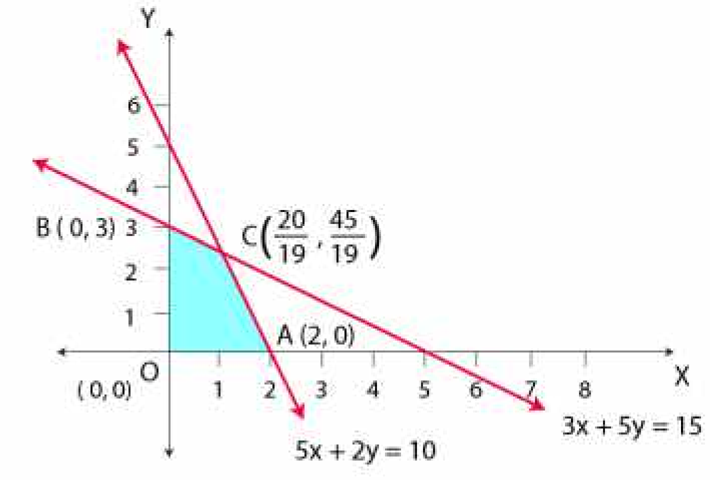
Hence, the minimum value of Z is – 12 at the point (4, 0).

3. Maximise Z = 5*x* + 3*y*

subject to $3x+5y \leq 15,\ 5x+2y \leq 10,\ x \geq 0,\ y \geq 0$.

Solution:

The feasible region determined by the system of constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$, is given below.



O (0, 0), A (2, 0), B (0, 3) and C (20 / 19, 45 / 19) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 3y	
O (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
C (20 / 19, 45 / 19)	235 / 19	Maximum

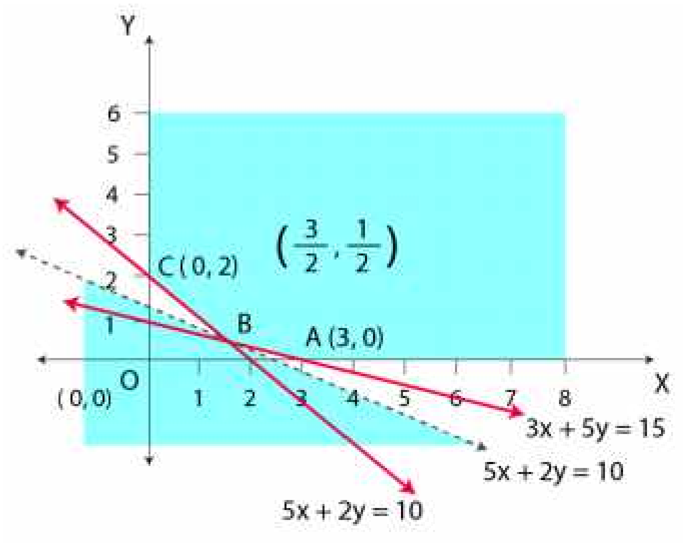
Hence, the maximum value of Z is 235 / 19 at the point (20 / 19, 45 / 19).

4. Minimise Z = 3*x* + 5*y*

such that $x + 3y \geq 3,\ x + y \geq 2,\ x, y \geq 0$.

Solution:

The feasible region determined by the system of constraints, x + 3y ≥ 3, x + y ≥ 2, and x, y ≥ 0, is given below.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), B (3 / 2, 1 / 2) and C (0, 2).

The values of Z at these corner points are given below.

Corner point	Z = 3x + 5y	
A (3, 0)	9	
B (3 / 2, 1 / 2)	7	Smallest
C (0, 2)	10	

7 may or may not be the minimum value of Z because the feasible region is unbounded.

For this purpose, we draw the graph of the inequality, 3x + 5y < 7 and check whether the resulting half-plane has common points with the feasible region or not.

Hence, it can be seen that the feasible region has no common point with 3x + 5y < 7.

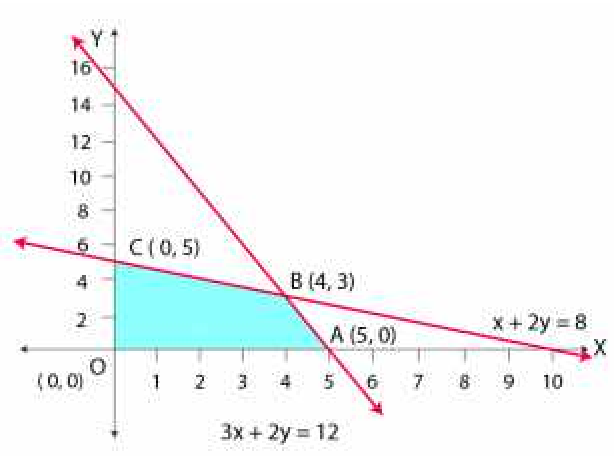
Thus, the minimum value of Z is 7 at point B (3 / 2, 1 / 2).

5. Maximise Z = 3*x* + 2*y*

subject to $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, and $y \geq 0$, is given below.



A (5, 0), B (4, 3), C (0, 5) and D (0, 0) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = 3x + 2y	
A (5, 0)	15	
B (4, 3)	18	Maximum
C (0, 5)	10	

Hence, the maximum value of Z is 18 at points (4, 3).

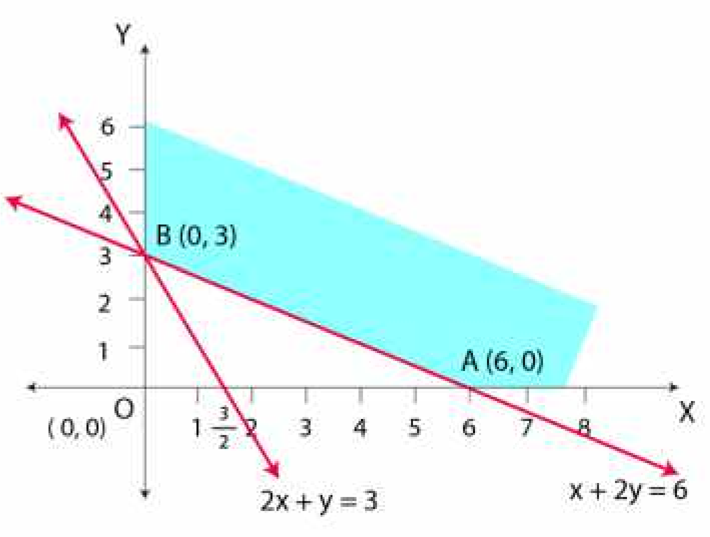
6. Minimise Z = x + 2y

subject to

$2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is given below.



A (6, 0) and B (0, 3) are the corner points of the feasible region.

The values of Z at the corner points are given below.

Corner point	Z = x + 2y
A (6, 0)	6
B (0, 3)	6

Here, the values of Z at points A and B are same. If we take any other point, such as (2, 2) on line x + 2y = 6, then Z = 6.

Hence, the minimum value of Z occurs for more than 2 points.

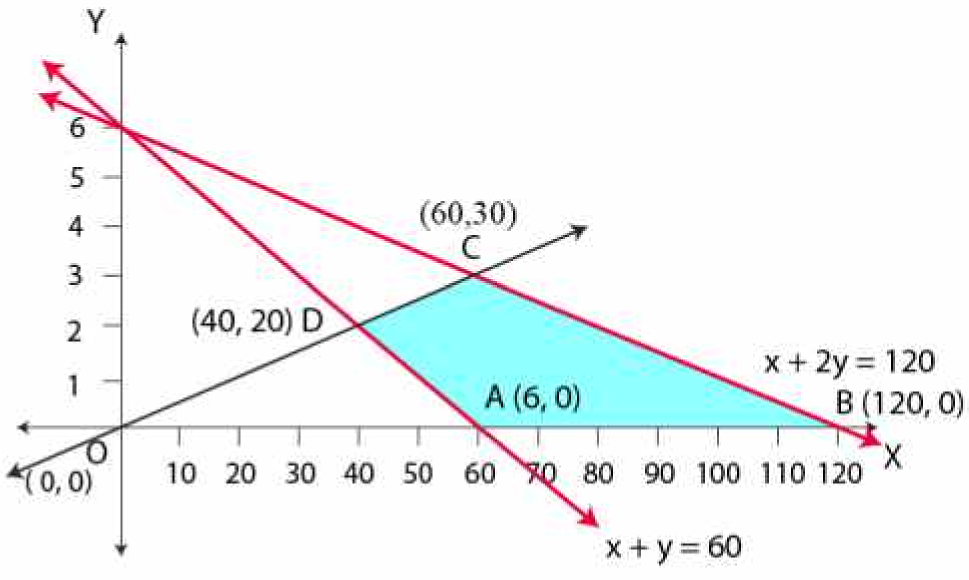
Therefore, the value of Z is minimum at every point on the line x + 2y = 6.

7. Minimise and Maximise Z = 5*x* + 10*y*

subject to $x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x, y \ge 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, is given below.



A (60, 0), B (120, 0), C (60, 30), and D (40, 20) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 10y	
A (60, 0)	300	Minimum
B (120, 0)	600	Maximum
C (60, 30)	600	Maximum
D (40, 20)	400	

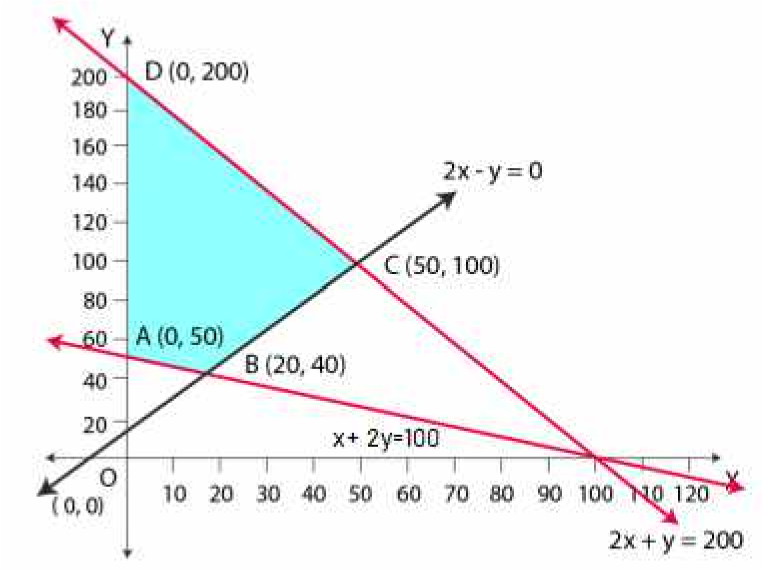
The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

8. Minimise and Maximise Z = x + 2y

subject to $x+2y \geq 100,\ 2x-y \leq 0,\ 2x+y \leq 200;\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is given below.



A (0, 50), B (20, 40), C (50, 100) and D (0, 200) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = x + 2y	
A (0, 50)	100	Minimum
B (20, 40)	100	Minimum
C (50, 100)	250	
D (0, 200)	400	Maximum

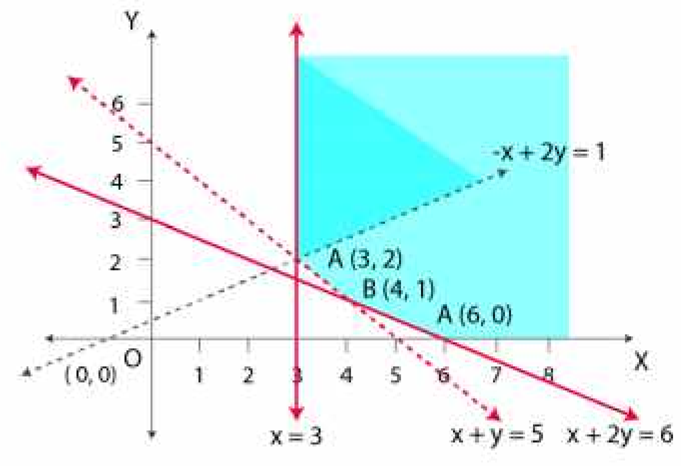
The maximum value of Z is 400 at points (0, 200), and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

9. Maximise Z = − *x* + 2*y*, subject to the constraints.

$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Solution:

The feasible region determined by the constraints, $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$ is given below.



Here, it can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1) and C (3, 2) are given below.

Corner point	Z = – x + 2y
A (6, 0)	Z = – 6
B (4, 1)	Z = – 2
C (3, 2)	Z = 1

Since the feasible region is unbounded, z = 1 may or may not be the maximum value.

For this purpose, we graph the inequality, – x + 2y > 1, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, the resulting feasible region has points in common with the feasible region.

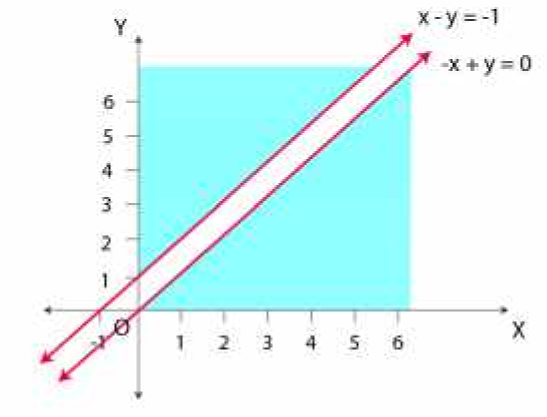
Hence, z = 1 is not the maximum value.

Z has no maximum value.

10. Maximise Z = *x* + *y*, subject to $x-y\le -1,\ -x+y\le 0,\ x,y\ge 0$.

Solution:

The region determined by the constraints $x-y\le -1,\ -x+y\le 0,\ x,y\ge 0$ is given below.



There is no feasible region, and therefore, z has no maximum value.

EXERCISE 12.2 PAGE NO: 519

1. **Reshma wishes to mix two types of food, P and Q, in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A, and 11 units of vitamin B. Food P costs Rs 60/kg, and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?**

Solution:

Let the mixture contain x kg of food P and y kg of food Q.

Hence, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Vitamin A (units/kg)	Vitamin B (units/kg	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Hence, the constraints are

$3x + 4y \geq 8$

$5x + 2y \geq 11$

The total cost of purchasing food is Z = 60x + 80y.

So, the mathematical formulation of the given problem can be written as

Minimise Z = 60x + 80y (i)

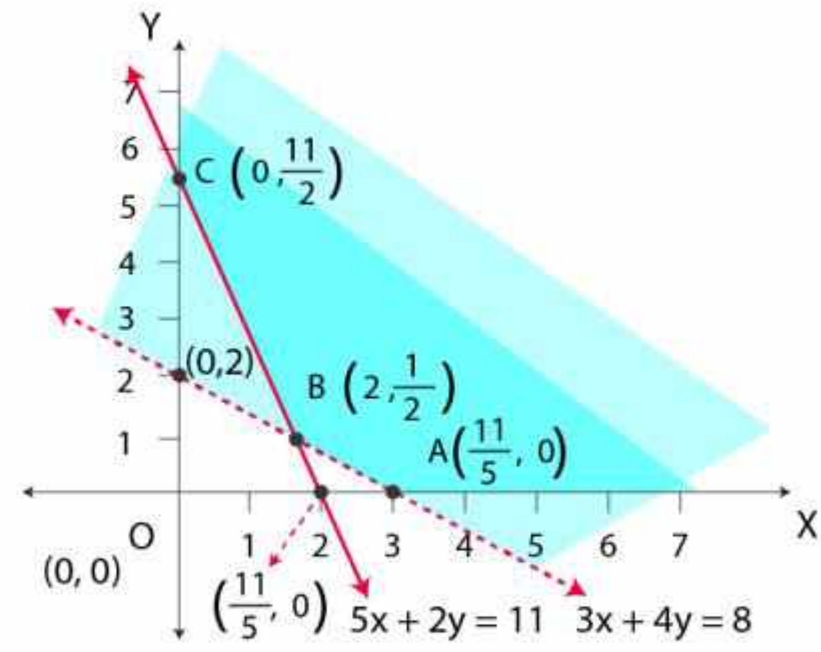
Now, subject to the constraints,

$3x + 4y \geq 8$ … (2)

$5x + 2y \geq 11$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is given below.



Clearly, we can see that the feasible region is unbounded.

A (8 / 3, 0), B (2, 1 / 2) and C (0, 11 / 2)

The values of Z at these corner points are given below.

Corner point	Z = 60x + 80 y	
A (8 / 3, 0)	160	Minimum
B (2, 1 / 2)	160	Minimum
C (0, 11 / 2)	440	

Here, the feasible region is unbounded; therefore, 160 may or may not be the minimum value of Z.

For this purpose, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $3x + 4y < 8$.

Hence, at the line segment joining the points (8 / 3, 0) and (2, 1 / 2), the minimum cost of the mixture will be Rs 160.

2. One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

Solution:

Let the first kind of cake be x and second kind of cakes be y. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as shown below.

	Flour (g)	Fat (g)
Cakes of the first kind, x	200	25
Cakes of the second kind, y	100	50
Availability	5000	1000

So, $200x + 100y \leq 5000$

$2x + y \leq 50$

$25x + 50y \leq 1000$

$x + 2y \leq 40$

The total number of cakes Z that can be made are

$Z = x + y$

The mathematical formulation of the given problem can be written as

Maximise $Z = x + y$ (i)

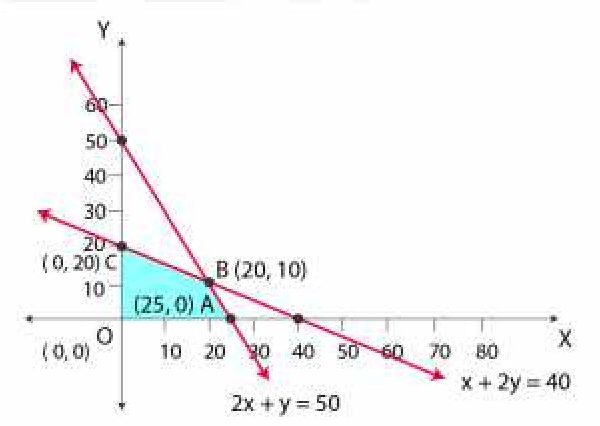
Here, subject to the constraints,

$2x + y \leq 50$ (ii)

$x + 2y \leq 40$ (iii)

$x, y \geq 0$ (iv)

The feasible region determined by the system of constraints is given below.



A (25, 0), B (20, 10), O (0, 0) and C (0, 20) are the corner points.

The values of Z at these corner points are as given below.

Corner point	Z = x + y	
A (25, 0)	25	
B (20, 10)	30	Maximum
C (0, 20)	20	
O (0,0)	0	

Hence, the maximum numbers of cakes that can be made are 30 (20 cakes of one kind and 10 cakes of other kind).

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(ii) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10, respectively, find the maximum profit of the factory when it works at full capacity.

Solution:

Let x and y be the number of rackets and the number of bats to be made.

Given that the machine time is not available for more than 42 hours.

Hence, $1.5x + 3y \leq 42$ ……………. (i)

Also, given that the craftsman's time is not available for more than 24 hours.

Hence, $3x + y \leq 24$ ………… (ii)

The factory is to work at full capacity. Hence,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we get

$x = 4$ and $y = 12$

Therefore, 4 rackets and 12 bats must be made.

(i) The given information can be compiled in a table as give below.

	Tennis Racket	Cricket Bat	Availability
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

Since the profit on a racket is Rs 20 and Rs 10.

Hence, $Z = 20x + 10y$

The mathematical formulation of the given problem can be written as

Maximise $Z = 20x + 10y$ ………….. (i)

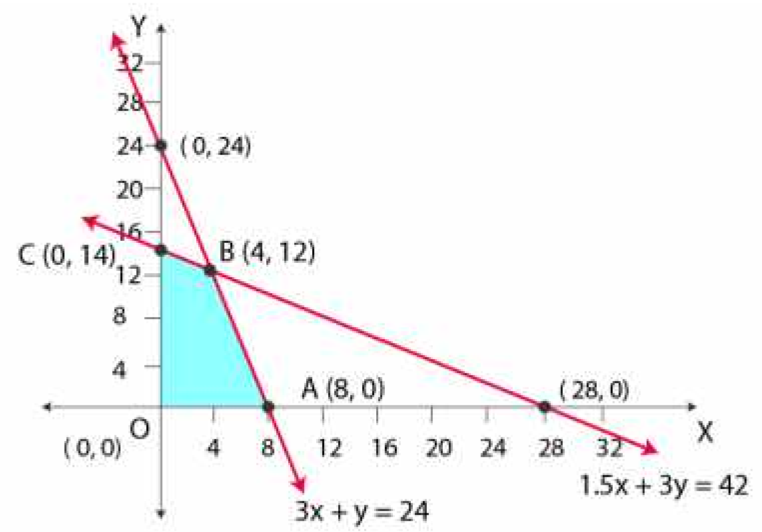
Subject to the constraints,

$1.5x + 3y \leq 42$ …………. (ii)

$3x + y \leq 24$ …………….. (iii)

$x, y \geq 0$ ………………… (iv)

The feasible region determined by the system of constraints is given below.



A (8, 0), B (4, 12), C (0, 14) and O (0, 0) are the corner points respectively.

The values of Z at these corner points are given below.

Corner point	Z = 20x + 10y	
A (8, 0)	160	
B (4, 12)	200	Maximum
C (0, 14)	140	

Therefore, the maximum profit of the factory when it works to its full capacity is Rs 200.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

Solution:

Let the manufacturer produce x package of nuts and y package of bolts. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Nuts	Bolts	Availability
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs 7.

Hence, the constraints are

$x + 3y \leq 12$

$3x + y \leq 12$

Then, the total profit is $Z = 17.5x + 7y$

The mathematical formulation of the given problem can be written as follows:

Maximise $Z = 17.5x + 7y$ …………. (1)

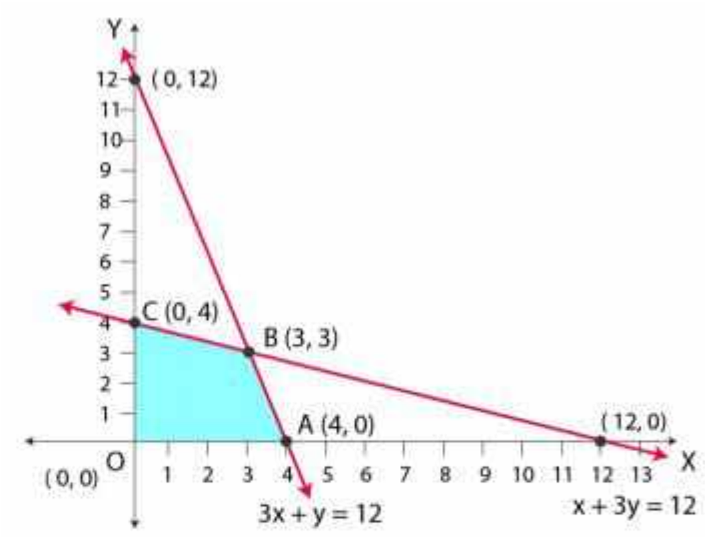
Subject to the constraints,

$x + 3y \leq 12$ …………. (2)

$3x + y \leq 12$ ………… (3)

$x, y \geq 0$ …………….. (4)

The feasible region determined by the system of constraints is given below.



A (4, 0), B (3, 3) and C (0, 4) are the corner points.

The values of Z at these corner points are given below.

Corner point	Z = 17.5x + 7y	
O (0, 0)	0	
A (4, 0)	70	
B (3, 3)	73.5	Maximum
C (0, 4)	28	

Therefore, Rs 73.50 at (3, 3) is the maximum value of Z.

Hence, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

5. A factory manufacturers two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

Solution:

On each day, let the factory manufacture x screws of type A and y screws of type B.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	$4 \times 60 = 240$

Hand Operated Machine (min)	6	3	$4 \times 60 = 240$

The profit on a package of screws A is Rs 7 and on the package screws, B is Rs 10.

Hence, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem can be written as

Maximise $Z = 7x + 10y$ …………. (i)

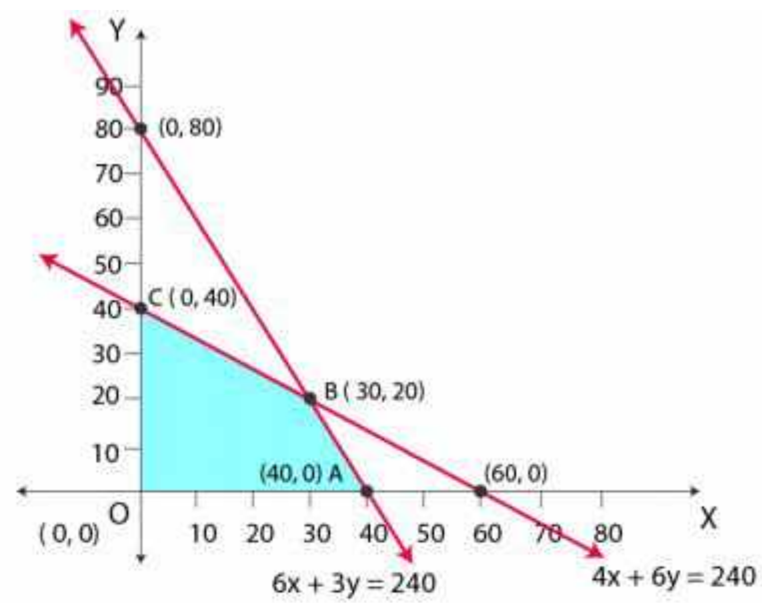
Subject to the constraints,

$4x + 6y \leq 240$ …………. (ii)

$6x + 3y \leq 240$ …………. (iii)

$x, y \geq 0$ ……………… (iv)

The feasible region determined by the system of constraints is given below.



A (40, 0), B (30, 20) and C (0, 40) are the corner points.

The value of Z at these corner points is given below.

Corner point	Z = 7x + 10y	
A (40, 0)	280	
B (30, 20)	410	Maximum
C (0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Hence, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shade that he produces, how should he schedule his daily production in order to maximise his profit?

Solution:

Let the cottage industry manufacture x pedestal lamps and y wooden shades, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Hence, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, Z = 5x + 3y …………….. (i)

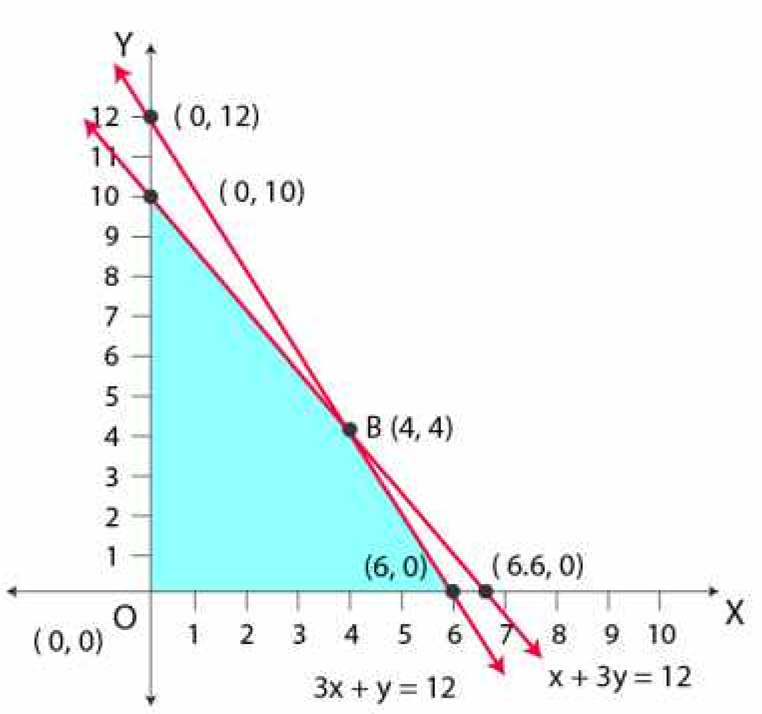
Subject to the constraints,

$2x + y \leq 12$ ………… (ii)

$3x + 2y \leq 20$ ………… (iii)

$x, y \geq 0$ ………… (iv)

The feasible region determined by the system of constraints is given below.



A (6, 0), B (4, 4) and C (0, 10) are the corner points.

The value of Z at these corner points is given below.

Corner point	Z = 5x + 3y	
A (6, 0)	30	
B (4, 4)	32	Maximum

C (0, 10)	30	

The maximum value of Z is 32 at points (4, 4).

Therefore, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximise his profits.

7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

Solution:

Let the company manufacture x souvenirs of type A and y souvenirs of type B, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Hence, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ i.e.,

$5x + 4y \leq 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem can be written as

Maximise $Z = 5x + 6y$ …………… (i)

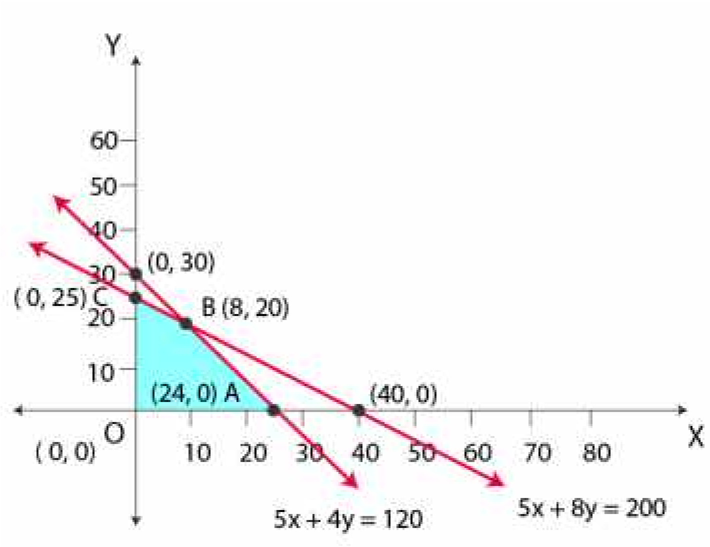
Subject to the constraints,

$5x + 8y \leq 200$ ……………. (ii)

$5x + 4y \leq 120$ ………….. (iii)

$x, y \geq 0$ ………….. (iv)

The feasible region determined by the system of constraints is given below.



A (24, 0), B (8, 20) and C (0, 25) are the corner points.

The values of Z at these corner points are given below.

Corner point	Z = 5x + 6y	
A (24, 0)	120	
B (8, 20)	160	Maximum
C (0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Hence, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000, respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4,500 and on the portable model is Rs 5,000.

Solution:

Let the merchant stock x desktop models and y portable models, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

Given that the cost of desktop model is Rs 25,000 and of a portable model is Rs 40,000.

However, the merchant can invest a maximum of Rs 70 lakhs.

Hence, $25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

Hence, $x + y \leq 250$

The profit on a desktop model is 4500, and the profit on a portable model is Rs 5000.

Total profit, $Z = 4500x + 5000y$

Therefore, the mathematical formulation of the given problem is

Maximum $Z = 4500x + 5000y$ ………… (i)

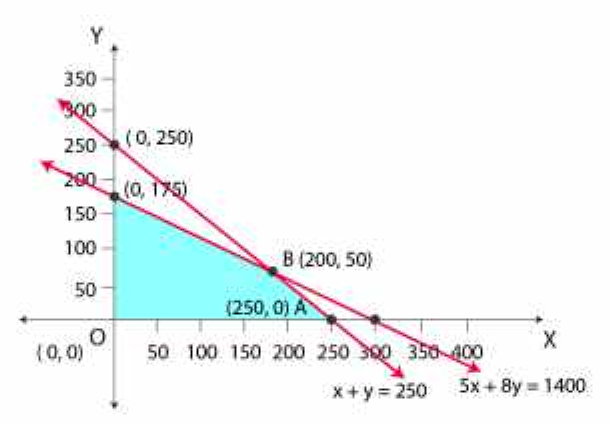
Subject to the constraints,

$5x + 8y \leq 1400$ ………… (ii)

$x + y \leq 250$ ………….. (iii)

$x, y \geq 0$ …………. (iv)

The feasible region determined by the system of constraints is given below.



A (250, 0), B (200, 50) and C (0, 175) are the corner points.

The values of Z at these corner points are given below.

Corner point	Z = 4500x + 5000y	
A (250, 0)	1125000	
B (200, 50)	1150000	Maximum
C (0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Therefore, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 11,50,000.

9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for the diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Solution:

Let the diet contain x units of food F_1 and y units of food F_2. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Vitamin A (units)	Mineral (units)	Cost per unit (Rs)
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit, and of food, F_2 is Rs 6 per unit.

Hence, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

The total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem can be written as

Minimise $Z = 4x + 6y$ …………… (i)

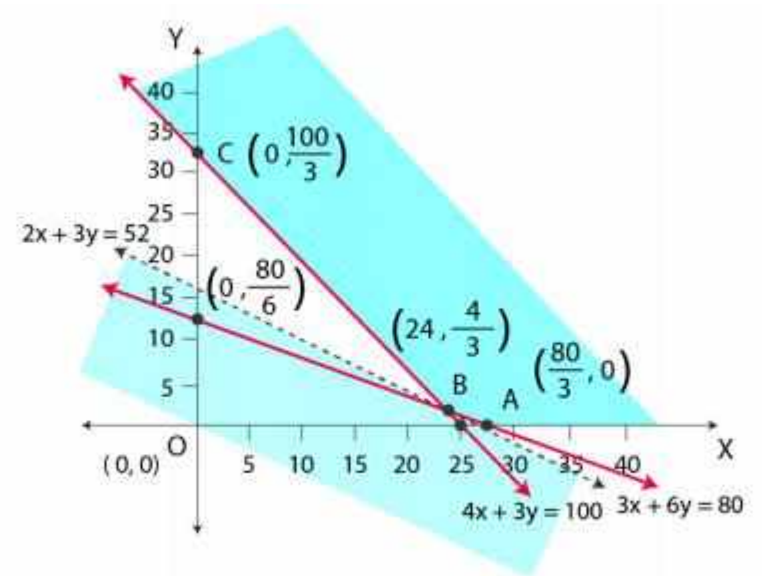
Subject to the constraints,

$3x + 6y \geq 80$ ………… (ii)

$4x + 3y \geq 100$ ………. (iii)

$x, y \geq 0$ …………. (iv)

The feasible region determined by the constraints is given below.



We can see that the feasible region is unbounded.

A (80 / 3, 0), B (24, 4 / 3), and C (0, 100 / 3) are the corner points.

The values of Z at these corner points are given below.

Corner point	Z = 4x + 6y	

A (80 / 3, 0)	320 / 3 = 106.67	
B (24, 4 / 3)	104	Minimum
C (0, 100 / 3)	200	

Here, the feasible region is unbounded, so 104 may or not be the minimum value of Z.

For this purpose, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $2x + 3y < 52$.

Hence, the minimum cost of the mixture will be Rs 104.

10. There are two types of fertilisers, F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid, and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution:

Let the farmer buy x kg of fertiliser F_1 and y kg of fertiliser F_2. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs / kg)
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen, and F_2 consists of 5% nitrogen.

However, the farmer requires at least 14 kg of nitrogen.

So, 10% of x + 5% of y $\geq$ 14

$x / 10 + y / 20 \geq 14$

By L.C.M we get

$2x + y \geq 280$

F_1 consists of 6% phosphoric acid, and F_2 consists of 10% phosphoric acid.

However, the farmer requires at least 14 kg of phosphoric acid.

So, 6% of x + 10 % of y ≥ 14

$6x / 100 + 10y / 100 \geq 14$

$3x + 5y \geq 700$

The total cost of fertilisers, $Z = 6x + 5y$

The mathematical formulation of the given problem can be written as

Minimise $Z = 6x + 5y$ ………….. (i)

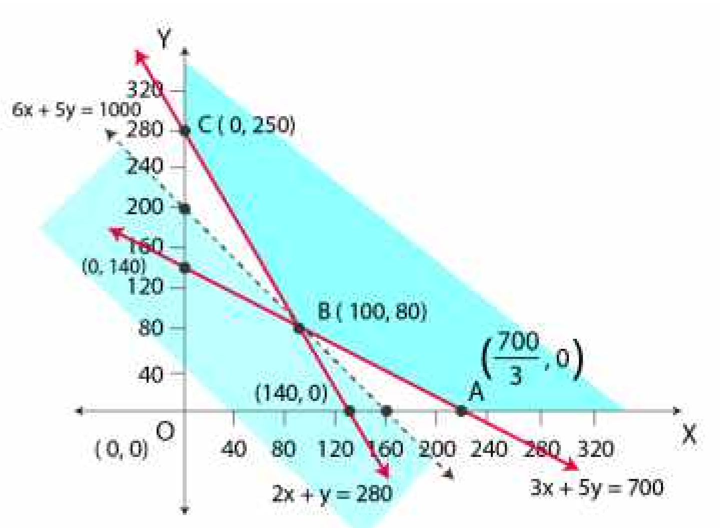
Subject to the constraints,

$2x + y \geq 280$ ……… (ii)

$3x + 5y \geq 700$ ………. (iii)

$x, y \geq 0$ ………… (iv)

The feasible region determined by the system of constraints is given below.



Here, we can see that the feasible region is unbounded.

A (700 / 3, 0), B (100, 80) and C (0, 280) are the corner points.

The values of Z at these points are given below.

Corner point	$Z = 6x + 5y$	
A (700 / 3, 0)	1400	
B (100, 80)	1000	Minimum
C (0, 280)	1400	

Here, the feasible region is unbounded; hence, 1000 may or may not be the minimum value of Z.

For this purpose, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $6x + 5y < 1000$

Hence, 100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 should be used to minimise the cost. The minimum cost is Rs 1000.

11. The corner points of the feasible region are determined by the following system of linear inequalities:

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $p = q$

(B) $p = 2q$

(C) $p = 3q$

(D) $q = 3p$

Solution:

The maximum value of Z is unique.

Here, it is given that the maximum value of Z occurs at two points, (3, 4) and (0, 5).

Value of Z at (3, 4) = Value of Z at (0, 5)

$p(3) + q(4) = p(0) + q(5)$

$3p + 4q = 5q$

$3p = 5q – 4q$

$3p = q$ or $q = 3p$

Therefore, the correct answer is option (D).

MISCELLANEOUS EXERCISE PAGE NO: 607

1. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Solution:

Let the diet contain x and y packets of foods P and Q, respectively. Hence,

$x \geq 0$ and $y \geq 0$

The mathematical formulation of the given problem is given below.

Maximise $z = 6x + 3y$ ………….. (i)

Subject to the constraints,

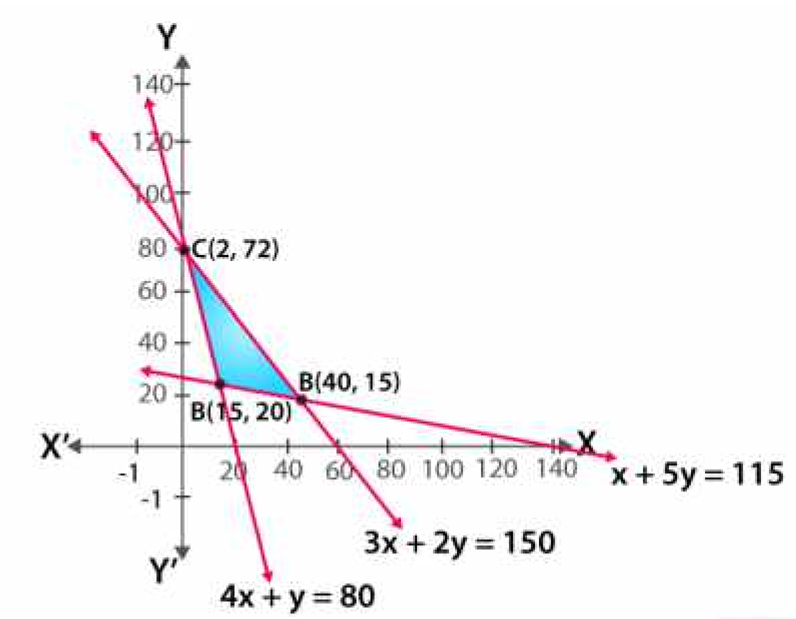
$4x + y \geq 80$ …………. (ii)

$x + 5y \geq 115$ ………. (iii)

$3x + 2y \leq 150$ ………… (iv)

$x, y \geq 0$ …………… (v)

The feasible region determined by the system of constraints is given below.



A (15, 20), B (40, 15) and C (2, 72) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 6x + 3y	
A (15, 20)	150	
B (40, 15)	285	Maximum
C (2, 72)	228	

So, the maximum value of z is 285 at (40, 15).

Hence, to maximise the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used.

The maximum amount of vitamin A in the diet is 285 units.

2. A farmer mixes two brands, P and Q, of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag, contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag. What is the minimum cost of the mixture per bag?

Solution:

Let the farmer mix x bags of brand P and y bags of brand Q, respectively.

The given information can be compiled in a table, as given below.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Vitamin C (units/kg)	Cost (Rs/kg)
Food P	3	2.5	2	250
Food Q	1.5	11.25	3	200
Requirement (units/kg)	18	45	24	

The given problem can be formulated as given below.

Minimise z = 250x + 200y ………… (i)

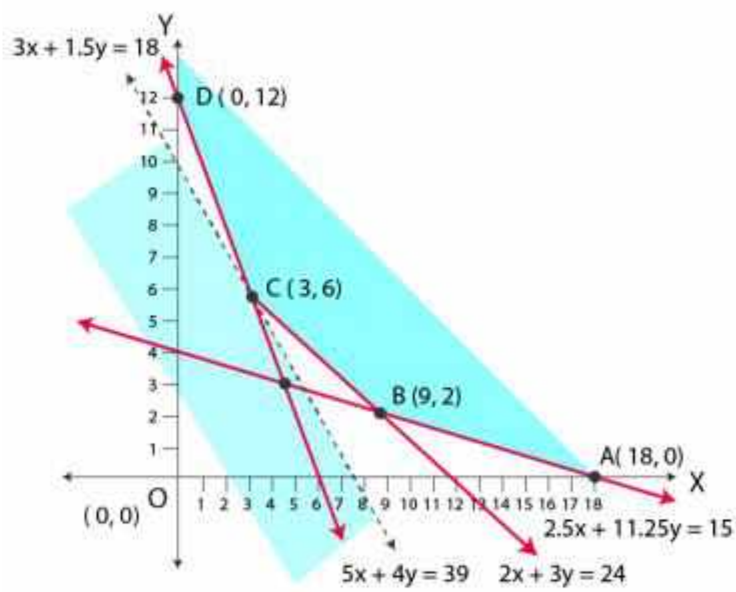
$3x + 1.5y \geq 18$ ………….. (ii)

$2.5x + 11.25y \geq 45$ ……….. (iii)

$2x + 3y \geq 24$ ………….. (iv)

$x, y \geq 0$ ………. (v)

The feasible region determined by the system of constraints is given below.



A (18, 0), B (9, 2), C (3, 6) and D (0, 12) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	$z = 250x + 200y$	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	Minimum
D (0, 12)	2400	

Here, the feasible region is unbounded; hence, 1950 may or may not be the minimum value of z.

For this purpose, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $5x + 4y < 39$

Hence, at points (3, 6), the minimum value of z is 1950.

Therefore, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimise the cost to Rs 1950.

3. A dietician wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food are given below.

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	**1**	**2**	**3**
Y	**2**	**2**	**1**

One kg of food X costs Rs 16, and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet.

Solution:

Let the mixture contain x kg of food X and y kg of food Y, respectively.

The mathematical formulation of the given problem can be written as given below.

Minimise $z = 16x + 20y$ …….. (i)

Subject to the constraints,

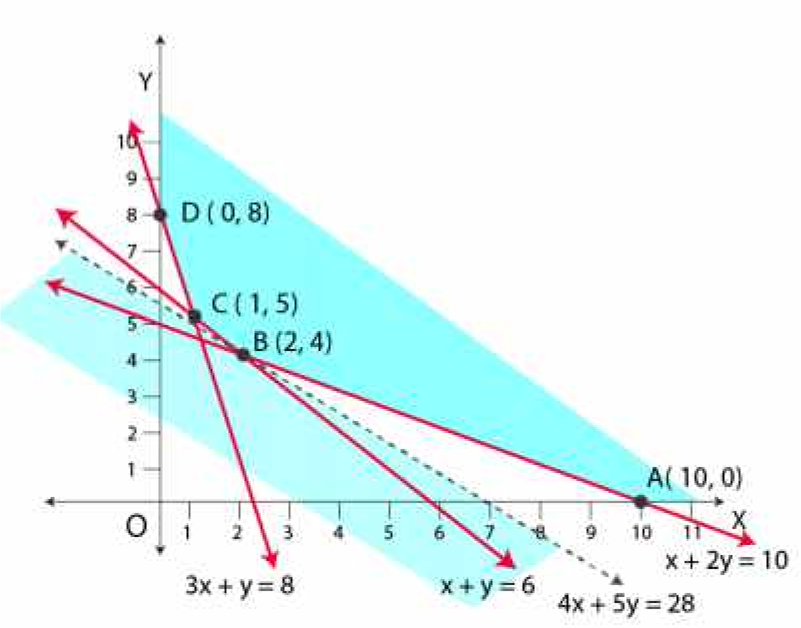
$x + 2y \geq 10$ …………. (ii)

$x + y \geq 6$ ………… (iii)

$3x + y \geq 8$ …………. (iv)

$x, y \geq 0$ ………… (v)

The feasible region determined by the system of constraints is given below.



A (10, 0), B (2, 4), C (1, 5) and D (0, 8) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 16x + 20y	
A (10, 0)	160	
B (2, 4)	112	Minimum
C (1, 5)	116	
D (0, 8)	160	

Since the feasible region is unbounded, hence, 112 may or may not be the minimum value of z.

For this purpose, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $4x + 5y < 28$

Hence, the minimum value of z is 112 at (2, 4).

Therefore, the mixture should contain 2 kg of food X and 4 kg of food Y.

The minimum cost of the mixture is Rs 112.

4. A manufacturer makes two types of toys, A and B. Three machines are needed for this purpose, and the time (in minutes) required for each toy on the machines is given below.

Types of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Solution:

Let x and y toys of type A and type B be manufactured in a day, respectively.

The given problem can be formulated as given below.

Maximise $z = 7.5x + 5y$ ………….. (i)

Subject to the constraints,

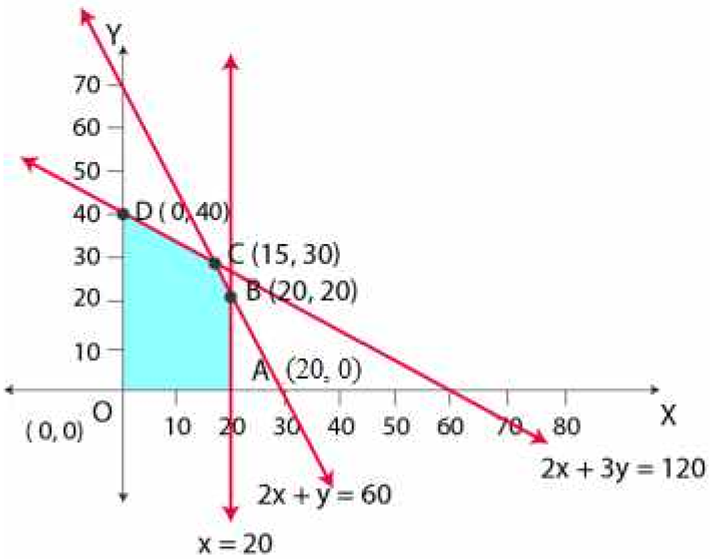
$\mathbf{2x + y} \leq 60$ …………. (ii)

$x \leq 20$ ……….. (iii)

$2x + 3y \leq 120$ ……….. (iv)

$x, y \geq 0$ ……………. (v)

The feasible region determined by the constraints is given below.



A (20, 0), B (20, 20), C (15, 30) and D (0, 40) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 7.5x + 5y	
A (20, 0)	150	
B (20, 20)	250	
C (15, 30)	262.5	Maximum
D (0, 40)	200	

262.5 at (15, 30) is the maximum value of z.

Hence, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximise the profit.

5. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

Solution:

Let the airline sell x tickets of executive class and y tickets of economy class, respectively.

The mathematical formulation of the given problem can be written as given below.

Maximise z = 1000x + 600y ………… (i)

Subject to the constraints,

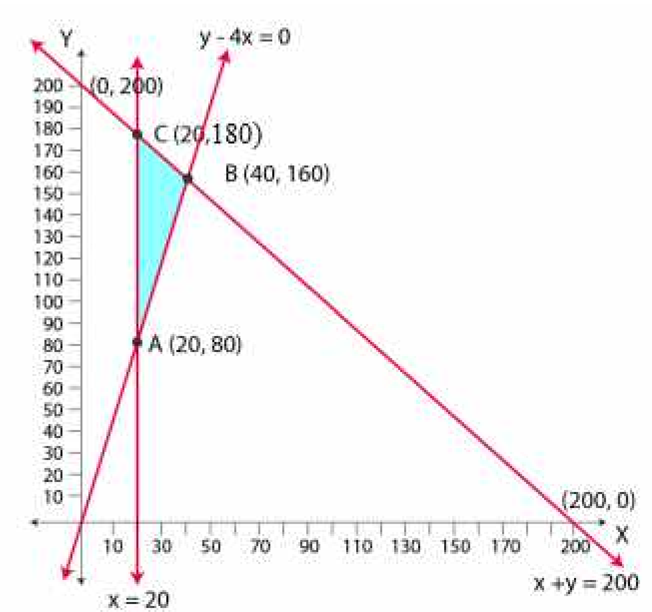
x + y ≤ 200 …………….. (ii)

x ≥ 20 ………… (iii)

y – 4x ≥ 0 …………… (iv)

x, y ≥ 0 ……………(v)

The feasible region determined by the constraints is given below.



A (20, 80), B (40, 160) and C (20, 180) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 1000x + 600y	
A (20, 80)	68000	
B (40, 160)	136000	Maximum

C (20, 180)	128000	

136000 at (40, 160) is the maximum value of z.

Therefore, 40 tickets for the executive class and 160 tickets for the economy class should be sold to maximise the profit, and the maximum profit is Rs 1,36,000.

6. Two godowns, A and B, have grain capacities of 100 quintals and 50 quintals, respectively. They supply to 3 ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals, respectively. The cost of transportation per quintal from the godowns to the shops is given in the following table:

Transportation cost per quintal (in Rs)		
From / To	**A**	**B**
D	**6**	**4**
E	**3**	**2**
F	**2.50**	**3**

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Solution:

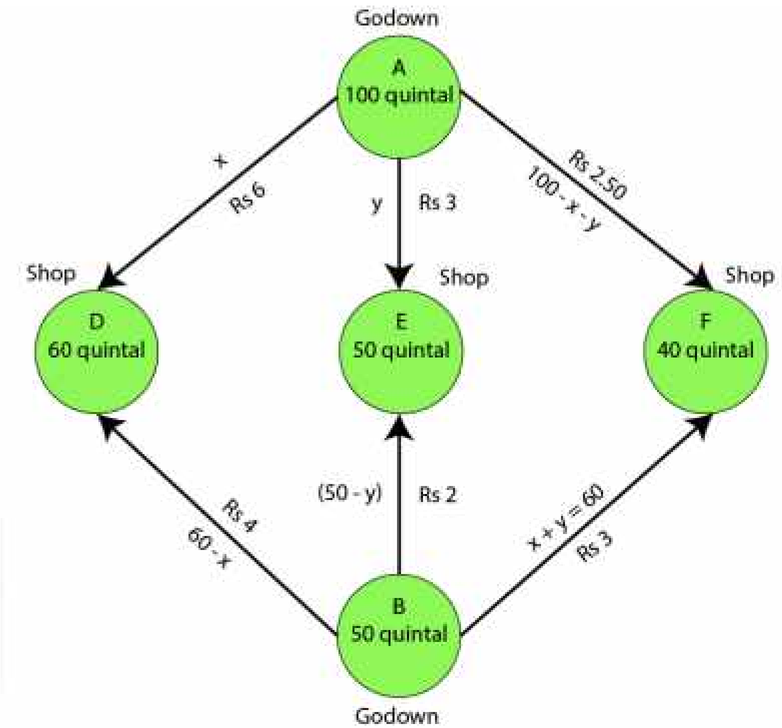
Let godown A supply x and y quintals of grain to shops D and E.

So, (100 – x – y) will be supplied to shop F.

Since x quintals are transported from godown A, so the requirement at shop D is 60 quintals. Hence, the remaining (60 – x) quintals will be transported from godown B.

Similarly, (50 – y) quintals and 40 – (100 – x – y) = (x + y – 60) quintals will be transported from godown B to shop E and F.

The given problem can be represented diagrammatically, as given below.



$x \geq 0$, $y \geq 0$, and $100 - x - y \geq 0$

Then, $x \geq 0$, $y \geq 0$, and $x + y \leq 100$

$60 - x \geq 0$, $50 - y \geq 0$, and $x + y - 60 \geq 0$

Then, $x \leq 60$, $y \leq 50$, and $x + y \geq 60$

Total transportation cost z is given by,

$z = 6x + 3y + 2.5\,(100 - x - y) + 4\,(60 - x) + 2\,(50 - y) + 3\,(x + y - 60)$

$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$

$= 2.5x + 1.5y + 410$

The given problem can be formulated as given below.

Minimise $z = 2.5x + 1.5y + 410$ …………. (i)

Subject to the constraints,

$x + y \leq 100$ ……….. (ii)

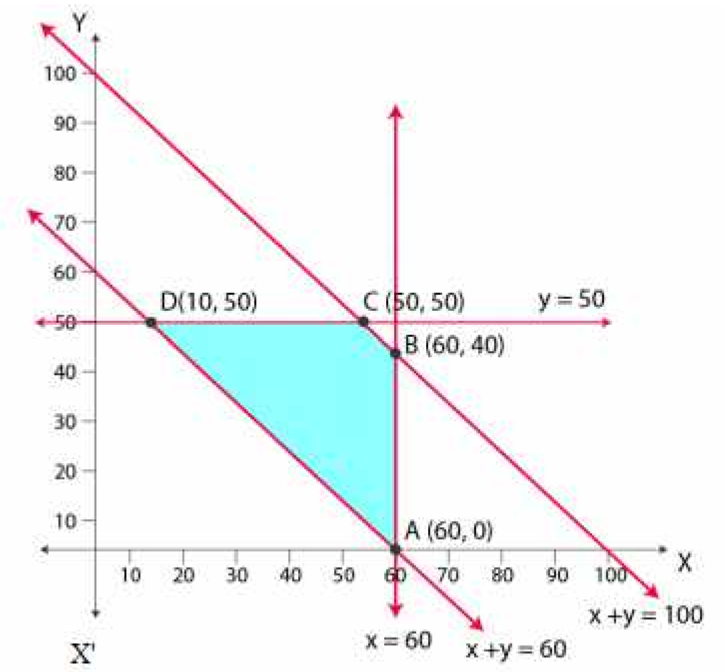
$x \leq 60$ ……….. (iii)

$y \leq 50$ ………. (iv)

$x + y \geq 60$ ……… (v)

x, y ≥ 0 …………..(vi)

The feasible region determined by the system of constraints is given below.



A (60, 0), B (60, 40), C (50, 50) and D (10, 50) are the corner points.

The values of z at these corner points are given below.

Corner point	z = 2.5x + 1.5y + 410	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	Minimum

The minimum value of z is 510 at (10, 50)

Hence, the amount of grain transported from A to D, E and F is 10 quintals, 50 quintals and 40 quintals, respectively, and from B to D, E and F is 50 quintals, 0 quintals, 0 quintals, respectively

Thus, the minimum cost is Rs 510.

7. An oil company has two depots, A and B, with capacities of 7000 L and 4000 L, respectively. The company is to supply oil to three petrol pumps, D, E and F, whose requirements are 4500L, 3000L and 3500L, respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

Distance in (km)		
From / To	**A**	**B**
D	**7**	**3**
E	**6**	**4**
F	**3**	**2**

Assuming that the transportation cost of 10 litres of oil is Rs 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

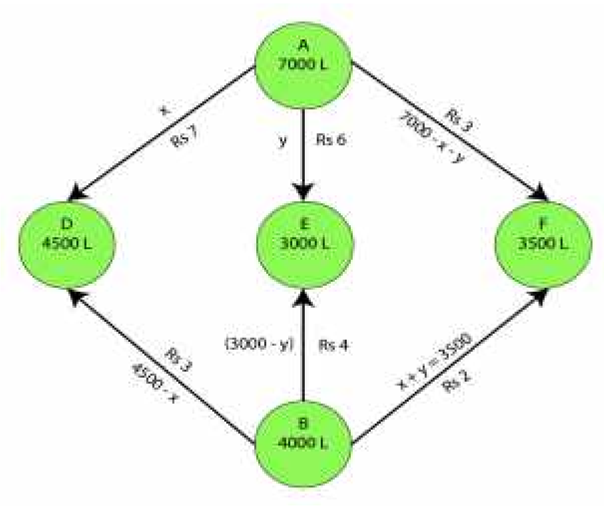
Solution:

Let x and y litres of oil be supplied from A to the petrol pumps, D and E. So, (7000 – x – y) will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500L. Since x L is transported from depot A, the remaining (4500 – x) L will be transported from petrol pump B.

Similarly, (3000 – y)L and 3500 – (7000 – x – y) = (x + y – 3500) L will be transported from depot B to petrol pumps E and F, respectively.

The given problem can be represented diagrammatically, as given below.



$x \geq 0$, $y \geq 0$, and $(7000 - x - y) \geq 0$

Then, $x \geq 0$, $y \geq 0$, and $x + y \leq 7000$

$4500 - x \geq 0$, $3000 - y \geq 0$, and $x + y - 3500 \geq 0$

Then, $x \leq 4500$, $y \leq 3000$, and $x + y \geq 3500$

Cost of transporting 10L of petrol = Rs 1

Cost of transporting 1L of petrol = Rs 1 / 10

Hence, the total transportation cost is given by

$z = (7 / 10)\, x + (6 / 10)\, y + 3 / 10\, (7000 - x - y) + 3 / 10\, (4500 - x) + 4 / 10\, (3000 - y) + 2 / 10\, (x + y - 3500)$

$= 0.3x + 0.1y + 3950$

The problem can be formulated as given below.

Minimise $z = 0.3x + 0.1y + 3950$ ………. (i)

Subject to constraints,

$x + y \leq 7000$ ………. (ii)

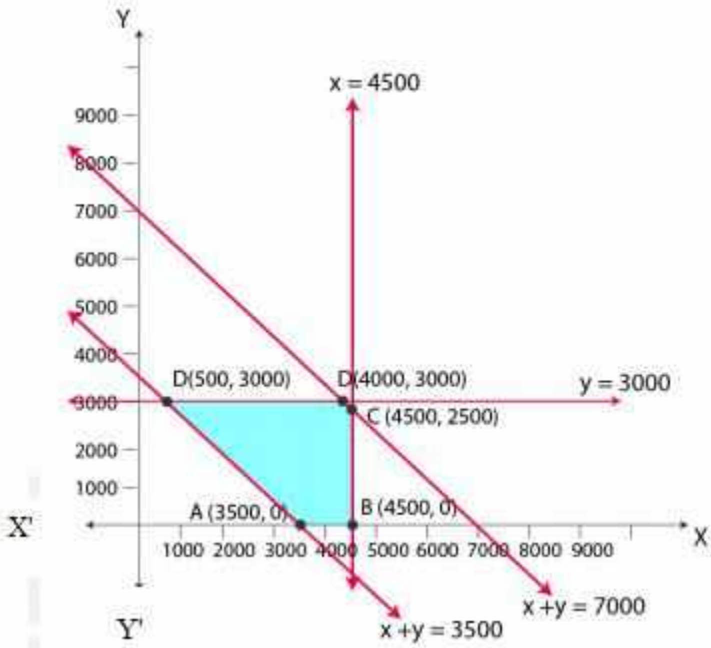
$x \leq 4500$ ……….. (iii)

$y \leq 3000$ ……. (iv)

$x + y \geq 3500$ ……… (v)

$x, y \geq 0$ ………… (vi)

The feasible region determined by the constraints is given below.



A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000) and E (500, 3000) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 0.3x + 0.1y + 3950	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	Minimum

The minimum value of z is 4400 at (500, 3000).

Hence, the oil supplied from depot A is 500 L, 3000 L and 3500 L and from depot B is 4000 L, 0 L and 0 L to petrol pumps D, E and F, respectively.

Therefore, the minimum transportation cost is Rs 4400.

8. A fruit grower can use two types of fertiliser in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table, Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added to the garden?

Kg per bag		
	Brand P	**Brand Q**
Nitrogen	**3**	**3.5**
Phosphoric acid	**1**	**2**
Potash	**3**	**1.5**
Chlorine	**1.5**	**2**

Solution:

Let the fruit grower use x bags of brand P and y bags of brand Q, respectively.

The problem can be formulated as given below.

Minimise $z = 3x + 3.5y$ …………. (i)

Subject to the constraints,

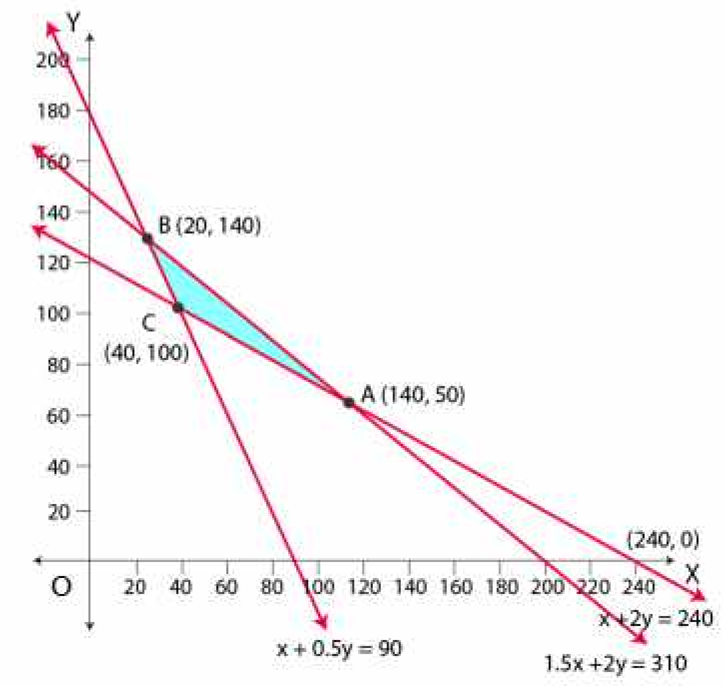
$\mathbf{x + 2y} \geq 240$ ……….. (ii)

$x + 0.5y \geq 90$ …….. (iii)

$1.5x + 2y \leq 310$ ……….. (iv)

$x, y \geq 0$ …………. (v)

The feasible region determined by the system of constraints is given below.



A (240, 0), B (140, 50) and C (20, 140) are the corner points of the feasible region.

The value of z at these corner points is given below.

Corner point	z = 3x + 3.5y	
A (140, 50)	595	
B (20, 140)	550	
C (40, 100)	470	Minimum

The maximum value of z is 470 at (40, 100).

Therefore, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimise the amount of nitrogen.

Hence, the minimum amount of nitrogen added to the garden is 470 kg.

9. Refer to Question 8. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

Solution:

Let the fruit grower use x bags of brand P and y bags of brand Q, respectively.

The problem can be formulated as given below.

Maximise z = 3x + 3.5y ……….. (i)

Subject to the constraints,

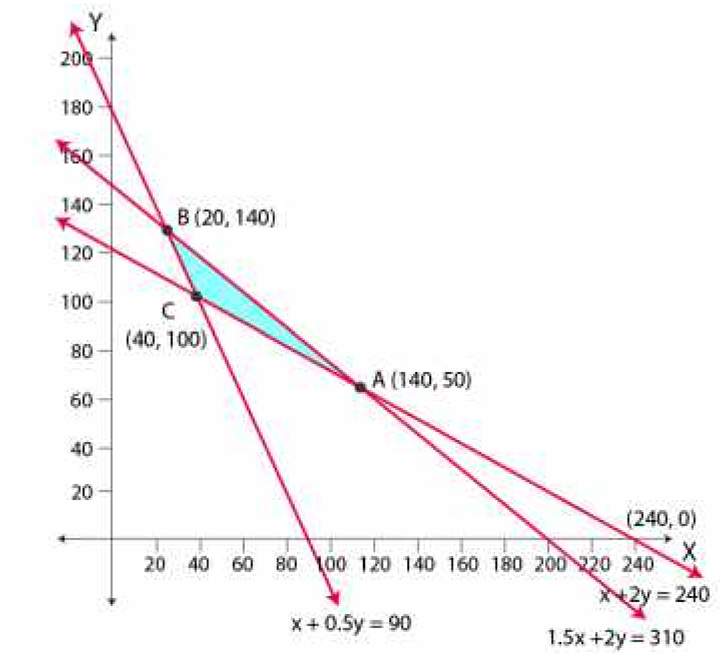
x + 2y ≥ 240 ……….. (ii)

x + 0.5y ≥ 90 ……….. (iii)

1.5x + 2y ≤ 310 ……….. (iv)

x, y ≥ 0 …………. (v)

The feasible region determined by the system of constraints is given below.



A (140, 50), B (20, 140) and C (40, 100) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 3x + 3.5y	
A (140, 50)	595	Maximum

B (20, 140)	550	
C (40, 100)	470	

The maximum value of z is 595 at (140, 50).

Hence, 140 bags of brand P and 50 bags of brand Q should be used to maximise the amount of nitrogen.

Thus, the maximum amount of nitrogen added to the garden is 595 kg.

10. A toy company manufactures two types of dolls, A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls per week, and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll, respectively, on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

Solution:

Let x and y be the number of dolls of type A and B, respectively, that are produced in a week.

The given problem can be formulated as given below.

Maximise $z = 12x + 16y$ ……….. (i)

Subject to the constraints,

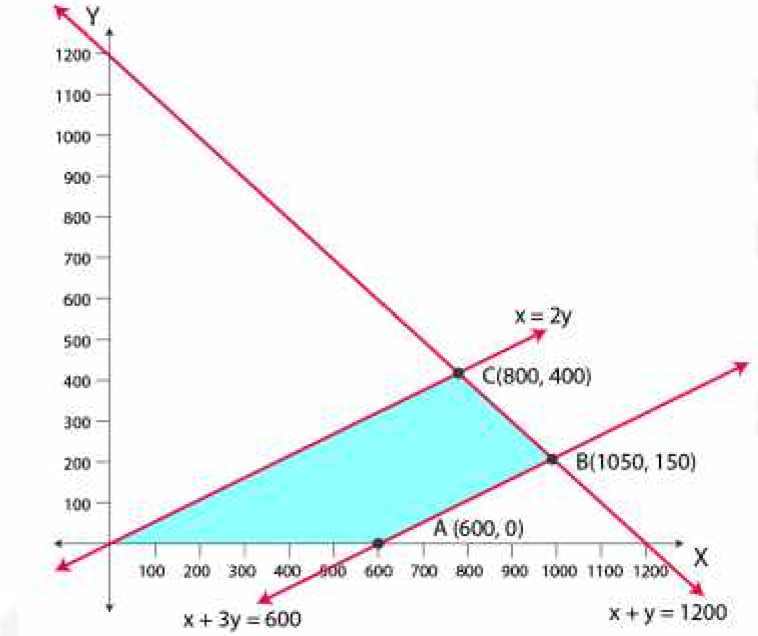
$\mathbf{x + y} \leq 1200$ ………… (ii)

$y \leq x / 2$ or $x \geq 2y$ ………. (iii)

$x - 3y \leq 600$ …………. (iv)

$x, y \geq 0$ …………… (v)

The feasible region determined by the system of constraints is given below.



A (600, 0), B (1050, 150) and C (800, 400) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 12x + 16y	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	Maximum

The maximum value of z is 16000 at (800, 400).

Hence, 800 and 400 dolls of type A and type B should be produced, respectively, to get the maximum profit of Rs 16000.