EXERCISE 13.1

1. Given that E and F are events, such that P (E) = 0.6, P (F) = 0.3 and P (E ∩ F) = 0.2, find P (E|F) and P (F|E).

Solution:

Given P (E) = 0.6, P (F) = 0.3 and P (E ∩ F) = 0.2

We know that by the definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By substituting the values we get

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

And $\Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$

2. Compute P (A|B), if P (B) = 0.5 and P (A ∩ B) = 0.32

Solution:

Given: P (B) = 0.5 and P (A ∩ B) = 0.32

We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

3. If P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4, find
(i) P (A ∩ B)

(ii) P (A|B)

(iii) P (A ∪ B)

Solution:

Given P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4

(i) We know that by definition of conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow$ P (A ∩ B) = P (B|A) P (A)

$\Rightarrow$ P (A ∩ B) = 0.4 × 0.8

$\Rightarrow$ P (A ∩ B) = 0.32

(ii) We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now substituting the values we get

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

$\Rightarrow$ P (A|B) = 0.64

(iii) Now, ∵ P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

Substituting the values we get

$\Rightarrow$ P (A ∪ B) = 0.8 + 0.5 – 0.32 = 1.3 – 0.32

$\Rightarrow$ P (A ∪ B) = 0.98

4. Evaluate P (A ∪ B), if 2P (A) = P (B) = 5/13 and P (A|B) = 2/5.

Solution:

Given $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

$\Rightarrow P(B) = \frac{5}{13}, P(A) = \frac{5}{13\times2} = \frac{5}{26}, P(A|B) = \frac{2}{5}$ .... (i)

We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow$ P (A ∩ B) = P (A|B) P (B)

$$\Rightarrow \; P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13} \;\ldots\ldots\ldots \text{(ii)}$$

Now, ∵ P (A * B) = P (A) + P (B) – P (A ∩ B)

$$\Rightarrow P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5 + 10 - 4}{26} = \frac{15 - 4}{26}$$

$$\Rightarrow P(A \cup B) = \frac{11}{26}$$

5. If P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11, find
(i) P (A∩B)

(ii) P (A|B)

(iii) P (B|A)

Solution:

Given: $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}, P(A \cup B) = \frac{7}{11}$

(i) We know that P (A * B) = P (A) + P (B) – P (A ∩ B)

$\Rightarrow$ P (A ∩ B) = P (A) + P (B) – P (A ∪ B)

$$\Rightarrow P(A \cap B) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{11 - 7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

(ii) Now, by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{4/11}{5/11}$$

$$\Rightarrow P(A|B) = \frac{4}{5}$$

(iii) Again, by definition of conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{4/11}{6/11} = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow P(B|A) = \frac{2}{3}$$

Determine P (E|F) in Exercises 6 to 9.

6. A coin is tossed three times, where
(i) E : head on the third toss, F : heads on first two tosses
(ii) E : at least two heads, F : at most two heads
(iii) E : at most two tails, F : at least one tail

Solution:

The sample space of the given experiment will be:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(i) Here, E: head on third toss

And F: heads on first two tosses

⇒ E = {HHH, HTH, THH, TTH} and F = {HHH, HHT}

⇒ E ∩ F = {HHH}

So, $P(E) = \frac{4}{8} = \frac{1}{2}, P(F) = \frac{2}{8} = \frac{1}{4}, P(E \cap F) = \frac{1}{8}$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/8}{1/4} = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/8}{1/4} = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Here, E: at least two heads

And F: at most two heads

⇒ E = {HHH, HHT, HTH, THH} and F = {HHT, HTH, THH, HTT, THT, TTH, TTT}

⇒ E ∩ F = {HHT, HTH, THH}

So, $P(E) = \frac{4}{8} = \frac{1}{2}, P(F) = \frac{7}{8}, P(E \cap F) = \frac{3}{8}$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{3/8}{7/8} = \frac{3}{7}$$

(iii) Here, E: at most two tails

And F: at least one tail

⇒ E = {HHH, HHT, HTH, THH, HTT, THT, TTH}

And F = {HHT, HTH, THH, HTT, THT, TTH, TTT}

So, $P(E) = \frac{7}{8}, P(F) = \frac{7}{8}, P(E \cap F) = \frac{6}{8} = \frac{3}{4}$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{3/4}{7/8} = \frac{6}{7}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{3/4}{7/8} = \frac{6}{7}$$

7. Two coins are tossed once, where
(i) E: tail appears on one coin, F: one coin shows the head
(ii) E: no tail appears, F: no head appears

Solution:

The sample space of the given experiment is S = {HH, HT, TH, TT}

(i) Here, E: tail appears on one coin

And F: one coin shows head

⇒ E = {HT, TH} and F = {HT, TH}

⇒ E ∩ F = {HT, TH}

So, $P(E) = \frac{2}{4} = \frac{1}{2}, P(F) = \frac{2}{4} = \frac{1}{2}, P(E \cap F) = \frac{2}{4} = \frac{1}{2}$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/2}{1/2}$$

⇒ P (E|F) = 1

(ii) Here, E: no tail appears

And F: no head appears

⇒ E = {HH} and F = {TT}

⇒ E ∩ F = ϕ

So, $P(E) = \frac{1}{4}, P(F) = \frac{1}{4}, P(E \cap F) = \frac{0}{4} = 0$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Substituting the values we get

$$\Rightarrow P(E|F) = \frac{0}{1/4}$$

$\Rightarrow$ P (E|F) = 0

8. A die is thrown three times, E: 4 appears on the third toss, F: 6 and 5 appear, respectively, on the first two tosses.

Solution:

The sample space has 216 outcomes, where each element of the sample space has 3 entries and is of the form (x, y, z) where $1 \le x, y, z \le 6$.

Here, E: 4 appears on the third toss

$$\Rightarrow E = \left\{\begin{matrix} (1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4), \\ (2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4), \\ (3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4), \\ (4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4), \\ (5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4), \\ (6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4) \end{matrix}\right\}$$

Now, F: 6 and 5 appears respectively on first two tosses

$\Rightarrow F = \{(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)\}$

$\Rightarrow E \cap F = \{(6, 5, 4)\}$

So, $P(E) = \frac{36}{216}, P(F) = \frac{6}{216}, P(E \cap F) = \frac{1}{216}$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/216}{6/216} = \frac{1}{6}$$

$$\Rightarrow P(E|F) = \frac{1}{6}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Now by substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/216}{6/216} = \frac{1}{6}$$

$$\Rightarrow P(E|F) = \frac{1}{6}$$

9. Mother, father and son line up at random for a family picture
E: son on one end, F: father in the middle.

Solution:

Let M denotes mother, F denotes father, and S denotes son.

Then, the sample space for the given experiment will be

S = {MFS, SFM, FSM, MSF, SMF, FMS}

Here, E: Son on one end

And F: Father in the middle

⇒ E = {MFS, SFM, SMF, FMS} and F = {MFS, SFM}

⇒ E ∩ F = {MFS, SFM}

So, $P(E) = \frac{4}{6} = \frac{2}{3}, P(F) = \frac{2}{6} = \frac{1}{3}, P(E \cap F) = \frac{2}{6} = \frac{1}{3}$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Now by substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/3}{1/3} = 1$$

⇒ P (E | F) = 1

10. A black and a red dice are rolled.
(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution:

Let B denote black coloured die and R denote red coloured die.

Then, the sample space for the given experiment will be:

$$S = \begin{Bmatrix} (B1,R1),(B1,R2),(B1,R3),(B1,R4),(B1,R5),(B1,R6), \\ (B2,R1),(B2,R2),(B2,R3),(B2,R4),(B2,R5),(B2,R6), \\ (B3,R1),(B3,R2),(B3,R3),(B3,R4),(B3,R5),(B3,R6), \\ (B4,R1),(B4,R2),(B4,R3),(B4,R4),(B4,R5),(B4,R6), \\ (B5,R1),(B5,R2),(B5,R3),(B5,R4),(B5,R5),(B5,R6), \\ (B6,R1),(B6,R2),(B6,R3),(B6,R4),(B6,R5),(B6,R6) \end{Bmatrix}$$

(a) Let A be the event of 'obtaining a sum greater than 9' and B be the event of 'getting a 5 on black die'.

Then, A = {(B4, R6), (B5, R5), (B5, R6), (B6, R4), (B6, R5), (B6, R6)}

And B = {(B5, R1), (B5, R2), (B5, R3), (B5, R4), (B5, R5), (B5, R6)}

⇒ A ∩ B = {(B5, R5), (B5, R6)}

So, $P(A) = \frac{6}{36} = \frac{1}{6}, P(B) = \frac{6}{36} = \frac{1}{6}, P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Now, we know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{1/18}{1/6} = \frac{6}{18} = \frac{1}{3}$$

$$\Rightarrow P(A|B) = \frac{1}{3}$$

(b) Let A be the event of 'obtaining a sum 8' and B be the event of 'getting a number less than 4 on red die'.

Then, A = {(B2, R6), (B3, R5), (B4, R4), (B5, R3), (B6, R2)}

And $B = \begin{Bmatrix} (B1,R1)(B2,R1),(B3,R1),(B4,R1),(B5,R1),(B6,R1), \\ (B1,R2),(B2,R2),(B3,R2),(B4,R2),(B5,R2),(B6,R2), \\ (B1,R3),(B2,R3),(B3,R3),(B4,R3),(B5,R3),(B6,R3) \end{Bmatrix}$

⇒ A ∩ B = {(B5, R3), (B6, R2)}

So, $P(A) = \frac{5}{36}, P(B) = \frac{18}{36} = \frac{1}{2}, P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Now, we know that

By definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{1/18}{1/2} = \frac{2}{18} = \frac{1}{9}$$

$$\Rightarrow P(A|B) = \frac{1}{9}$$

11. A fair die is rolled. Consider events E = {1,3,5}, F = {2,3} and G = {2,3,4,5}
Find
(i) P (E|F) and P (F|E)
(ii) P (E|G) and P (G|E)
(iii) P ((E ∪ F)|G) and P ((E ∩ F)|G)

Solution:

The sample space for the given experiment is S = {1, 2, 3, 4, 5, 6}

Here, E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5} .... (i)

$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}, P(F) = \frac{2}{6} = \frac{1}{3}, P(G) = \frac{4}{6} = \frac{2}{3}$ .... (ii)

Now, E ∩ F = {3}, F ∩ G = {2, 3}, E ∩ G = {3, 5} .... (iii)

$\Rightarrow P(E \cap F) = \frac{1}{6}, P(F \cap G) = \frac{2}{6} = \frac{1}{3}, P(E \cap G) = \frac{2}{6} = \frac{1}{3}$ .... (iv)

(i) We know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/6}{1/3} = \frac{3}{6} = \frac{1}{2}$$ [Using (II) and (IV)]

$$\Rightarrow P(E|F) = \frac{1}{2}$$

Similarly, we have

$$P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3}$$ [Using (ii) and (iv)]

$$\Rightarrow P(F|E) = \frac{1}{3}$$

(ii) We know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{1/3}{2/3} = \frac{1}{2}$$

$$\Rightarrow P(E|G) = \frac{1}{2}$$

Similarly, we have

$$P(G|E) = \frac{P(G \cap E)}{P(E)} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$\Rightarrow P(G|E) = \frac{2}{3}$$

(iii) Clearly, from (i), we have

E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}

⇒ E ∪ F = {1, 2, 3, 5}

⇒ (E ∪ F) ∩ G = {2, 3, 5}

(iii) Clearly, from (i), we have

E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}

⇒ E ∪ F = {1, 2, 3, 5}

⇒ (E ∪ F) ∩ G = {2, 3, 5}

$\Rightarrow P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow P((E \cup F) \cap G) = \frac{1}{2}$ .... (v)

Now, we know that

By definition of conditional probability,

$P(E|F) = \frac{P(E \cap F)}{P(F)}$

$\Rightarrow P((E \cup F)|G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{1/2}{2/3} = \frac{3}{4}$ [Using (ii) and (v)]

$\Rightarrow P((E \cup F)|G) = \frac{3}{4}$

Similarly, we have $E \cap F = \{3\}$ [Using (iii)]

And $G = \{2, 3, 4, 5\}$ [Using (i)]

$\Rightarrow (E \cap F) \cap G = \{3\}$

$\Rightarrow P((E \cap F) \cap G) = \frac{1}{6}$ .... (vi)

So,

$P((E \cap F)|G) = \frac{P((E \cap F) \cap G)}{P(G)} = \frac{1/6}{2/3} = \frac{1}{4}$ [Using (ii) and (vi)]

$\Rightarrow P((E \cap F)|G) = \frac{1}{4}$

12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

Solution:

Let B denote boy and G denote girl.

Then, the sample space of the given experiment is S = {GG, GB, BG, BB}

Let E be the event that 'both are girls'.

$\Rightarrow$ E = {GG}

$$\Rightarrow P(E) = \frac{1}{4}$$

(i) Let F be the event that 'the youngest is a girl'.

$\Rightarrow$ F = {GG, BG}

$\Rightarrow P(F) = \frac{2}{4} = \frac{1}{2}$ .... (i)

Now, E ∩ F = {GG}

$\Rightarrow P(E \cap F) = \frac{1}{4}$ .... (ii)

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\Rightarrow P(E|F) = \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$ [Using (i) and (ii)]

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Let H be the event that 'at least one is a girl'.

$\Rightarrow$ H = {GG, GB, BG}

$\Rightarrow P(H) = \frac{3}{4}$ .... (iii)

Now, E ∩ H = {GG}

$\Rightarrow P(E \cap H) = \frac{1}{4}$ .... (iv)

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

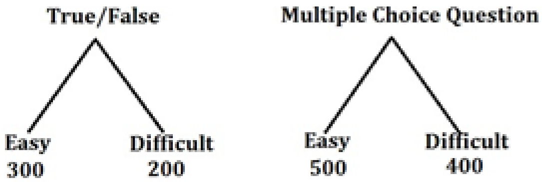
$\Rightarrow P(E|H) = \frac{P(E \cap H)}{P(H)} = \frac{1/4}{3/4} = \frac{1}{3}$ [Using (iii) and (iv)]

$\Rightarrow P(E|H) = \frac{1}{3}$

13. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question, given that it is a multiple-choice question?

Solution:

Here, there are two types of questions, True/False or Multiple Choice Questions (T/F or MCQ), and each of them is divided into Easy and Difficult type, as shown below in the tree diagram.



So, in all, there are, 500 T/F questions and 900 MCQs.

Also, there are 800 Easy questions and 600 difficult questions.

⇒ the sample space of this experiment has 500 + 900 = 1400 outcomes.

Now, let E be the event of 'getting an Easy question' and F be the event of 'getting an MCQ'.

$\Rightarrow P(E) = \frac{800}{1400} = \frac{8}{14}$ And $P(F) = \frac{900}{1400} = \frac{9}{14}$ .... (i)

Now, E ∩ F is the event of getting an MCQ which is Easy.

Clearly, from the diagram, we know that there are 500 MCQs that are easy.

So $P(E \cap F) = \frac{500}{1400} = \frac{5}{14}$.......... (ii)

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\Rightarrow P(E|F) = \frac{5/14}{9/14} = \frac{5}{9}$ [Using (i) and (ii)]

$$\Rightarrow P(E|F) = \frac{5}{9}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\Rightarrow P(E|F) = \frac{5/14}{9/14} = \frac{5}{9}$ [Using (i) and (ii)]

$$\Rightarrow P(E|F) = \frac{5}{9}$$

14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Solution:

The sample space of the given experiment is given below

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Let E be the event that 'the sum of numbers on the dice is 4' and F be the event that 'the two numbers appearing on throwing the two dice are different'.

$\Rightarrow$ E = {(1, 3), (2, 2), (3, 1)}

$$F = \begin{Bmatrix} (1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5) \end{Bmatrix}$$

And

$\Rightarrow$ E ∩ F = {(1, 3), (3, 1)}

$$\Rightarrow P(E) = \frac{3}{36} = \frac{1}{12}, P(F) = \frac{30}{36} = \frac{5}{6}, P(E \cap F) = \frac{2}{36} = \frac{1}{18} \text{ .... (i)}$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

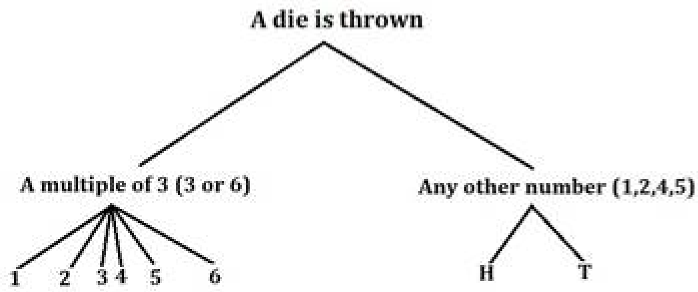
$$\Rightarrow P(E|F) = \frac{1/18}{5/6} = \frac{1}{15}$$

$$\Rightarrow P(E|F) = \frac{1}{15}$$

15. Consider the experiment of throwing a die; if a multiple of 3 comes up, throw the die again and if any other number comes up, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Solution:

The experiment is explained below in the tree diagram:



The sample space of the given experiment is given below

$$S = \begin{Bmatrix} (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \\ 1H, 2H, 4H, 5H, 1T, 2T, 4T, 5T \end{Bmatrix}$$

Let E be the event that 'the coin shows a tail' and F be the event that 'at least one die shows a 3'.

$\Rightarrow$ E = {1T, 2T, 4T, 5T} and F = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)}

$\Rightarrow E \cap F = \varphi \Rightarrow P(E \cap F) = 0$ .... (i)

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\Rightarrow P(E|F) = \frac{0}{P(F)} = 0$ [Using (i)]

$\Rightarrow P(E|F) = 0$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\Rightarrow P(E|F) = \frac{0}{P(F)} = 0$ [Using (i)]

$\Rightarrow P(E|F) = 0$

16. If P (A) = 1/2, P (B) = 0, then P (A|B) is
A. 0
B. ½
C. not defined
D. 1

Solution:

C. Not defined

Explanation:

We know that by definition of conditional probability,

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ .... (i)

Given: P (A) = ½

And P (B) = 0

$\Rightarrow$ Using (i), we have

$P(A|B) = \frac{P(A \cap B)}{0} = (A \cap B) \times \frac{1}{0}$, which is not defined.

17. If A and B are events such that P (A|B) = P (B|A), then
A. A ⊂ B but A ≠ B
B. A = B
C. A ∩ B = φ
D. P (A) = P (B)

Solution:

D. P (A) = P (B)

Explanation:

Given: P (A|B) = P (B|A) .... (i)

Now, we know that by definition of conditional probability,

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ .... (ii)

$P(B|A) = \frac{P(A \cap B)}{P(A)}$ .... (iii)

Using (i), we have

P (A|B) = P (B|A)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

⇒ P (A) = P (B)