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1. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of
(i) 5 successes?

(ii) At least 5 successes?
(iii) At most 5 successes?

Solution:

We know that the repeated tosses of dice are known as the Bernoulli trials.

Let the number of successes of getting an odd number in an experiment of 6 trials be x.

Probability of getting an odd number in a single throw of a dice (p)

$$= \frac{\text{number of odd numbers on a dice}}{\text{total number of numbers on a dice}} = \frac{3}{6} = \frac{1}{2}$$

Thus, q = 1 – p = ½

Now, here x has a binomial distribution.

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2 …n

$= {}^6C_x (1/2)^{6-x} (1/2)^x$

$= {}^6C_x (1/2)^6$

(i) Probability of getting 5 successes = P(X = 5)

$= {}^6C_5 (1/2)^6$

= 6 ×1/64

= 3/32

(ii) Probability of getting at least 5 successes = $P(X \geq 5)$

= P(X = 5) + P(X = 6)

$= {}^6C_5 (1/2)^6 + {}^6C_5 (1/2)^6$

= 6 ×1/64 + 6 ×1/64

= 6/64 + 1/64

= 7/64

(iii) Probability of getting at most 5 successes = $P(X \leq 5)$

We can also write it as 1 – P(X>5)

$= 1 – P(X = 6)$

$= 1 – {}^6C_6 (1/2)^6$

$= 1 – 1/64$

$= 63/64$

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Solution:

We know that the repeated tosses of a pair of dice are known as Bernoulli trials.

Let the number of times of getting doublets in an experiment of throwing two dice simultaneously four times be x.

$$\text{Probability of getting doublets in a single throw of a a pair of dice}(p) = \frac{\text{number of doublets possible in a pair of dice}}{\text{total number of posible pairs when two dice thrown}} = \frac{6}{36} = \frac{1}{6}$$

Thus, $q = 1 – p = 1 – 1/6 = 5/6$

Now, here x has a binomial distribution, where $n = 4$, $p = 1/6$, $q = 5/6$

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where $x = 0, 1, 2, \ldots n$

$= {}^4C_x (5/6)^{4-x} (1/6)^x$

$= {}^4C_x (5^{4-x}/6^6)$

Hence, probability of getting 2 successes = $P(X = 2)$

$= {}^4C_2 (5^{4-2}/6^4)$

$= 6 (5^2/6^4)$

$= 6 \times (25/1296)$

$= 25/216$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution:

Let there be x number of defective items in a sample of ten items drawn successively.

Now, as we can see that the drawing of the items is done with replacement. Thus, the trials are Bernoulli trials.

Now, the probability of getting a defective item, p = 5/100 = 1/20

Thus, q = 1 – 1/20 = 19/20

∴ We can say that x has a binomial distribution, where n = 10 and p = 1/20

Thus, P(X = x) = $^nC_x q^{n-x} p^x$, where x = 0, 1, 2 …n

$$= {}^{10}C_x \left(\frac{19}{20}\right)^{10-x} \left(\frac{1}{20}\right)^x$$

Probability of getting not more than one defective item = P(X ≤1)

= P(X = 0) + P(X = 1)

= $^{10}C_0$ $(19/20)^{10}(1/20)^0$ +$^{10}C_1$ $(19/20)^9(1/20)^1$

$$= \left(\frac{19}{20}\right)^{10} + 10 \times \left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right)^1$$

$$= \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{19}{20}\right)^9 \times \left(\frac{29}{20}\right)$$

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that
(i) All five cards are spades?
(ii) Only 3 cards are spades?
(iii) None is a spade?

Solution:

Let the number of spade cards among the five drawn cards be x.

As we can observe that the drawing of cards is with replacement; thus, the trials will be Bernoulli trials.

Now, we know that in a deck of 52 cards, there are total of 13 spade cards.

Thus, the probability of drawing a spade from a deck of 52 cards

= 13/52 = ¼

q = 1 – ¼ = 3/4

Thus, x has a binomial distribution with n = 5 and p = ¼

Thus, P(X = x) = $^nC_x q^{n-x} p^x$, where x = 0, 1, 2, …n

$$= {}^5C_x\left(\frac{3}{4}\right)^{5-x}\left(\frac{1}{4}\right)^x$$

(i) Probability of drawing all five cards as spades = P(X = 5)

$$= {}^5C_5\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5$$

$$= 1\times\frac{1}{1024}$$

$$= \frac{1}{1024}$$

= 1/1024

(ii) Probability of drawing three out five cards as spades = P(X = 3)

$$= {}^5C_3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^3$$

$$= 10\times\frac{9}{16}\times\frac{1}{64}$$

$$= \frac{45}{512}$$

(iii) Probability of drawing all five cards as non-spades = P(X = 0)

$$= {}^5C_0\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$= 1\times\frac{243}{1024}$$

$$= \frac{243}{1024}$$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(i) none
(ii) not more than one
(iii) more than one
(iv) at least one will fuse after 150 days of use.

Solution:

Let us assume that the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials be x.

As we can see that the trial is made with replacement; thus, the trials will be Bernoulli trials.

It is already mentioned in the question that p = 0.05

Thus, q = 1 – p = 1 – 0.05 = 0.95

Here, we can clearly observe that x has a binomial representation with n = 5 and p = 0.05

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2… n

$= {}^5C_x (0.95)^{5-x}(0.05)^x$

(i) Probability of no such bulb in a random drawing of 5 bulbs = P(X = 0)

$= {}^5C_0 (0.95)^{5-0}(0.05)^0$

$= 1\times 0.95^5$

$= (0.95)^5$

(ii) Probability of not more than one such bulb in a random drawing of 5 bulbs = P (X≤ 1)

= P(X = 0) + P(X = 1)

$= {}^5C_0 (0.95)^{5-0}(0.05)^0 + {}^5C_1(0.95)^{5-1}(0.05)^1$

$= 1\times 0.95^5 + 5 \times (0.95)^4 \times 0.05$

$= (0.95)^4 (0.95 + 0.25)$

$= (0.95)^4 \times 1.2$

(iii) Probability of more than one such bulb in a random drawing of 5 bulbs = P (X>1)

= 1 – P(X ≤ 1)

$= 1 – [(0.95)^4 \times 1.2]$

(iv) Probability of at least one such bulb in a random drawing of 5 bulbs = P (X ≥ 1)

= 1 – P(X < 1)

= 1 – P(X = 0)

$= 1 – (0.95)^5$

6. A bag consists of 10 balls, each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Solution:

Let us assume that number of balls with a digit marked as zero among the experiment of 4 balls drawn simultaneously is x.

As we can see that the balls are drawn with replacement; thus, the trial is a Bernoulli trial.

Probability of a ball drawn from the bag to be marked as digit 0 = 1/10

It can be clearly observed that X has a binomial distribution with n = 4 and p = 1/10

Thus, q = 1 – p = 1 – 1/10 = 9/10

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2, …n

$$= {}^4C_x\left(\frac{9}{10}\right)^{4-x}\left(\frac{1}{10}\right)^x$$

Probability of no ball marked with zero among the 4 balls = P(X = 0)

$$= {}^4C_0\left(\frac{9}{10}\right)^{4-0}\left(\frac{1}{10}\right)^0$$

$$= {}^4C_x\left(\frac{9}{10}\right)^{4}\left(\frac{1}{10}\right)^0$$

$$= 1 \times \left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^4$$

7. In an examination, 20 questions of the true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution:

Let us assume that the number of correctly answered questions out of twenty questions be x.

'Head' on the coin shows the true answer, and the 'tail' on the coin shows the false answer. Thus, the repeated tosses or the correctly answered questions are Bernoulli trails.

Thus, p = ½ and q = 1 – p = 1 – ½ = ½

Here, it can be clearly observed that x has a binomial distribution, where n = 20 and p = ½

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2 … n

$$= {}^{20}C_x\left(\frac{1}{2}\right)^{20-x}\left(\frac{1}{2}\right)^{x}$$

$$= {}^{20}C_x\left(\frac{1}{2}\right)^{20}$$

Probability of at least 12 questions answered correctly = p(X ≥ 12)

= P (X = 12) + P (X = 13) +...+ P (X = 20)

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{20} + \cdots + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{20}\right)^{20}\left({}^{20}C_{12} + {}^{20}C_{13} + \cdots + {}^{20}C_{20}\right)$$

8. Suppose X has a binomial distribution B (6, ½). Show that X = 3 is the most likely outcome.
(Hint: P(X = 3) is the maximum among all P(xi), xi = 0,1,2,3,4,5,6)

Solution:

Given, X is any random variable whose binomial distribution is B (6, ½)

Thus, n = 6 and p = ½

q = 1 – p = 1 – ½ = ½

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2 …n

$$= {}^{6}C_x\left(\frac{1}{2}\right)^{6-x}\left(\frac{1}{2}\right)^{x}$$

$$= {}^{6}C_x\left(\frac{1}{2}\right)^{6}$$

It can be clearly observed that P(X = x) will be the maximum if ${}^6c_x$ will be the maximum.

$\therefore {}^6c_x = {}^6c_6 = 1$

${}^6c_1 = {}^6c_5 = 6$

${}^6C_2 = {}^6C_4 = 15$

${}^6C_3 = 20$

Hence, we can clearly see that ${}^6C_3$ is the maximum.

∴ for x = 3, P(X = x) is maximum.

Hence, proved that the most likely outcome is x = 3.

9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution:

In this question, we have the repeated correct answer guessing from the given multiple choice questions, using the Bernoulli trials.

Let us now assume that X represents the number of correct answers by guessing in the multiple-choice set.

Now, the probability of getting a correct answer, p = 1/3

Thus, q = 1 – p = 1 – 1/3 = 2/3

Clearly, we X is a binomial distribution, where n = 5 and P = 1/3

$$\therefore P(X = x) = {}^nC_x\, q^{n-x} p^x$$

$$= {}^5C_x \left(\frac{2}{3}\right)^{5-x} . \left(\frac{1}{3}\right)^x$$

Hence, probability of guessing more than 4 correct answer = P (X ≥ 4)

= P (X = 4) + P (X = 5)

$$= {}^5C_4 \left(\frac{2}{3}\right) . \left(\frac{1}{3}\right)^4 + {}^5C_5 \left(\frac{1}{3}\right)^5$$

$$= 5 . \frac{2}{3} . \frac{1}{81} + 1 . \frac{1}{243}$$

$$= \frac{11}{243}$$

10. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100. What is the probability that he will win a prize
(a) At least once
(b) Exactly once
(c) At least twice?

Solution:

(a) Let X represents the number of prizes won in 50 lotteries, and the trials are Bernoulli trials.

Here, X is a binomial distribution where n = 50 and p = 1/100

Thus, q = 1 – p = 1 – 1/100 = 99/100

$$\therefore P(X = x) = {}^{n}C_{x}\, q^{n-x}\, p^{x}$$

$$= {}^{50}C_{x}\left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^{x}$$

Hence, probability of winning in lottery at least once = P (X ≥ 1)

= 1 – P (X <1)

= 1 – P (X = 0)

$$= 1 - {}^{50}C_{0}\left(\frac{99}{100}\right)^{50}$$

$$= 1 - 1.\left(\frac{99}{100}\right)^{50}$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

(b) Probability of winning in lottery exactly once = P (X = 1)

$$= {}^{50}C_{1}\left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) Probability of winning in lottery at least twice = P (X ≥ 2)

= 1 – P (X < 2)

$= 1 - P(X \leq 1)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= [1 - P(X = 0)] - P(X = 1)$

$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}.\left(\frac{99}{100}\right)^{49}$

$= 1 - \left(\frac{99}{100}\right)^{49}.\left[\frac{99}{100} + \frac{1}{2}\right]$

$= 1 - \left(\frac{99}{100}\right)^{49}.\left(\frac{149}{100}\right)$

$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

$= 1 - \left(\frac{99}{100}\right)^{49}.\left(\frac{149}{100}\right)$

$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

11. Find the probability of getting 5 exactly twice in 7 throws of a die.

Solution:

Let us assume X represent the number of times of getting 5 in 7 throws of the die.

Also, the repeated tossing of a die is the Bernoulli trials.

Thus, the probability of getting 5 in a single throw, p = 1/6

And, q = 1 – p

= 1 – 1/6

= 5/6

Clearly, X has the binomial distribution where n = 7 and p = 1/6

$\therefore P\ (X\ =\ x) = {}^{n}C_{x}\, q^{n-x} p^{x}$

$= {}^{7}C_{x}\left(\frac{5}{6}\right)^{7-x}.\left(\frac{1}{6}\right)^{x}$

Hence, probability of getting 5 exactly twice in a die = P (X = 2)

$= {}^{7}C_{2}\left(\frac{5}{6}\right)^{5}.\left(\frac{1}{6}\right)^{2}$

$=\ 21\,.\left(\frac{5}{6}\right)^{5}.\frac{1}{36}$

$=\ \left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^{5}$

12. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution:

Let us assume X represent the number of times of getting sixes in 6 throws of a die.

Also, the repeated tossing of die selection is the Bernoulli trials

Thus, the probability of getting six in a single throw of the die, p = 1/6

Clearly, X has the binomial distribution, where n = 6 and p = 1/6

And, q = 1 – p = 1 – 1/6 = 5/6

$\therefore P\ (X\ =\ x) = {}^{n}C_x\, q^{n-x} p^x$

$= {}^{6}C_x \left(\frac{5}{6}\right)^{6-x} \left(\frac{1}{6}\right)^{x}$

Hence, probability of throwing at most 2 sixes = P (X ≤2)

= P (X = 0) + p (X = 1) + P (X = 2)

$= {}^{6}C_0.\left(\frac{5}{6}\right)^{6} + {}^{6}C_1.\left(\frac{5}{6}\right)65\,.\left(\frac{1}{6}\right) + {}^{6}C_2\left(\frac{5^4}{6}\right).\left(\frac{1}{6}\right)^{2}$

$= \ 1.\left(\frac{5}{6}\right)^{6} + \ 6\,.\frac{1}{6}.\left(\frac{5}{6}\right)^{5} + \ 15\,.\frac{1}{36}.\left(\frac{5}{6}\right)^{4}$

$= \left(\frac{5}{6}\right)^{6} + \left(\frac{5}{6}\right)^{5} + \frac{5}{12}.\left(\frac{5}{6}\right)^{4}$

$= \left(\frac{5}{6}\right)^{4}\left[\left(\frac{5}{6}\right)^{2} + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right)\right]$

$= \left(\frac{5}{6}\right)^{4}.\left[\frac{25+30+15}{36}\right]$

$= \frac{70}{36}.\left(\frac{5}{6}\right)^{4}$

$= \frac{35}{18}.\left(\frac{5}{6}\right)^{4}$

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Solution:

Let us assume X represent the number of times selecting defective articles in a random sample space of 12 given articles.

Also, the repeated articles in a random sample space are the Bernoulli trials.

Clearly, X has the binomial distribution where n = 12 and p = 10% = 1/10

And, q = 1 – p = 1 – 1/10 = 9/10

$\therefore P\ (X\ =\ x) = {}^nC_x\, q^{n-x} p^x$

$= {}^{12}C_x \left(\frac{9}{10}\right)^{12-x} . \left(\frac{1}{10}\right)^x$

Hence, probability of selecting 9 defective articles $= {}^{12}C_9 \left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right)^9$

$= 220 . \frac{9^3}{10^3} . \frac{1}{10^9}$

$= \frac{22 \times 9^3}{10^{11}}$

14. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is
A. 10^{-1}
B. $(1/2)^5$
C. $(9/10)^5$
D. 9/10

Solution:

C. $(9/10)^5$

Explanation:

Let us assume X represent the number of times selecting defective bulbs in a random sample of given 5 bulbs.

Also, the repeated selection of defective bulbs from a box is the Bernoulli trials.

Clearly, X has the binomial distribution where n = 5 and p = 1/10

And, q = 1 – p = 1 – 1/10

$\therefore P\ (X\ =\ x) = {}^nC_x\, q^{n-x} p^x$

$= {}^5C_x \left(\frac{9}{10}\right)^{5-x} \left(\frac{1}{10}\right)^x$

Hence, probability that none bulb is defective = P (X = 0)

$= {}^5C_0 . \left(\frac{9}{10}\right)^5$

$$= 1.\left(\frac{9}{10}\right)^5$$

$$= \left(\frac{9}{10}\right)^5$$

∴ Option C is correct

15. The probability that a student is not a swimmer is 1/5. Then the probability that out of five students, four are swimmers is
A. $^5C_4$ 1/5 $(4/5)^4$
B. $(4/5)^4$ (1/5)
C. $^5C_1$ 1/5 $(4/5)^4$
D. None of these

Solution:

A. $^5C_4$ 1/5 $(4/5)^4$

Explanation:

Let us assume X represent the number of students out of 5 who are swimmers.

Also, the repeated selection of students who are swimmers is the Bernoulli trials.

Thus, the probability of students who are not swimmers = q = 1/5

Clearly, we have X has the binomial distribution where n = 5

And, p = 1 – q

= 1 – 1/5

= 4/5

$$\therefore P\ (X\ =\ x) = {}^nC_x\ q^{n-x}p^x$$

$$= {}^5C_x\left(\frac{1}{5}\right)^{5-x}.\left(\frac{4}{5}\right)^x$$

Hence, probability that four students are swimmers = P (X = 4)

$$= {}^5C_4\ \left(\frac{1}{5}\right).\left(\frac{4}{5}\right)^4$$

∴ Option A is correct