EXERCISE 9.1

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Determine the order and degree (if defined) of differential equations given in Exercises 1 to 10.

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

Solution:

The given differential equation is,

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

⇒ y”” + sin (y’’’) = 0

The highest order derivative present in the differential equation is y’’’’, so its order is three. Hence, the given differential equation is not a polynomial equation in its derivatives, so its degree is not defined.

2. y’ + 5y = 0

Solution:

The given differential equation is y’ + 5y = 0

The highest order derivative present in the differential equation is y’, so its order is one.

Therefore, the given differential equation is a polynomial equation in its derivatives.

So, its degree is one.

3. $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

Solution:-

The given differential equation is,

$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

The highest order derivative present in the differential equation is $\frac{d^2s}{dt^2}$.

The order is two. Therefore, the given differential equation is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$.

So, its degree is one.

4. $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

Solution:-

The given differential equation is,

$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.

The order is two. Therefore, the given differential equation is not a polynomial.

So, its degree is not defined.

5. $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$

Solution:-

The given differential equation is,

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} - \cos 3x - \sin 3x = 0$$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$.

The order is two. Therefore, the given differential equation is a polynomial equation in $\frac{d^2y}{dx^2}$ and the power is 1.

Therefore, its degree is one.

6. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Solution:

The given differential equation is, $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

The highest order derivative present in the differential equation is y'''.

The order is three. Therefore, the given differential equation is a polynomial equation in y''', y'' and y'.

Then, the power raised to y''' is 2.

Therefore, its degree is two.

7. $y''' + 2y'' + y' = 0$

Solution:

The given differential equation is, $y''' + 2y'' + y' = 0$

The highest order derivative present in the differential equation is y'''.

The order is three. Therefore, the given differential equation is a polynomial equation in y''', y'' and y'.

Then, the power raised to y''' is 1.

Therefore, its degree is one.

8. $y' + y = e^x$

Solution:

The given differential equation is $y' + y = e^x$

$= y' + y - e^x = 0$

The highest order derivative present in the differential equation is y'.

The order is one. Therefore, the given differential equation is a polynomial equation in y'.

Then, the power raised to y' is 1.

Therefore, its degree is one.

9. $y''' + (y')^2 + 2y = 0$

Solution:

The given differential equation is, $y''' + (y')^2 + 2y = 0$

The highest order derivative present in the differential equation is y''.

The order is two. Therefore, the given differential equation is a polynomial equation in y'' and y'.

Then, the power raised to y'' is 1.

Therefore, its degree is one.

10. $y''' + 2y' + \sin y = 0$

Solution:-

The given differential equation is, $y''' + 2y' + \sin y = 0$

The highest order derivative present in the differential equation is y''.

The order is two. Therefore, the given differential equation is a polynomial equation in y'' and y'.

Then the power raised to y'' is 1.

Therefore, its degree is one.

11. The degree of the differential equation.

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is}$$

(A) 3 (B) 2 (C) 1 (D) not defined

Solution:-

(D) not defined

The given differential equation is,

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0 \text{ is}$$

The highest order derivative present in the differential equation is

$\frac{d^2y}{dx^2}$

The order is three. Therefore, the given differential equation is not a polynomial.

Therefore, its degree is not defined.

12. The order of the differential equation

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0 \text{ is}$$

(A) 2 (B) 1 (C) 0 (D) not defined

Solution:-

(A) 2

The given differential equation is,

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$$

The highest order derivative present in the differential equation is

$\frac{d^2y}{dx^2}$

Therefore, its order is two.