EXERCISE 9.2

In each of the Exercises 1 to 10, verify that the given functions (explicit or implicit) are a solution of the corresponding differential equation:

1. y = ex + 1 : y″ – y′ = 0

Solution:-

From the question, it is given that $y = e^x + 1$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) \quad \text{... [Equation (i)]}$$

Now, differentiating equation (i) both sides with respect to x, we have,

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$

$\Rightarrow y'' = e^x$

Then,

Substituting the values of y' and y" in the given differential equations, we get,

$y'' – y' = e^x – e^x$ = RHS.

Therefore, the given function is a solution of the given differential equation.

2. $y = x^2 + 2x + C$: y′ – 2x – 2 = 0

Solution:-

From the question, it is given that $y = x^2 + 2x + C$

Differentiating both sides with respect to x, we get

$$y' = \frac{d}{dx}(x^2 + 2x + C)$$

$y' = 2x + 2$

Then,

Substituting the values of y' in the given differential equations, we get

$= y' – 2x - 2$

= 2x + 2 – 2x – 2

= 0

= RHS

Therefore, the given function is a solution of the given differential equation.

3. y = cos x + C : y′ + sin x = 0

Solution:-

From the question, it is given that y = cos x + C

Differentiating both sides with respect to x, we get

$$y' = \frac{d}{dx}(\cos x + C)$$

y’ = -sinx

Then,

Substituting the values of y’ in the given differential equations, we get

= y’ + sinx

= – sinx + sinx

= 0

= RHS

Therefore, the given function is a solution of the given differential equation.

4. $y = \sqrt{(1 + x^2)}$: $y' = ((xy)/(1 + x^2))$

Solution:-

From the question it is given that y = $\sqrt{1+x^2}$

Differentiating both sides with respect to x, we get,

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$\Rightarrow y' = \frac{1}{2\sqrt{1+x^2}}.\frac{d}{dx}(1+x^2)$$

By differentiating $(1 + x^2)$ we get,

$$\Rightarrow y' = \frac{2x}{2\sqrt{1+x^2}}$$

On simplifying we get,

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}}$$

By multiplying and dividing $\sqrt{(1 + x^2)}$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

Substituting the value of $\sqrt{(1 + x^2)}$

$$\Rightarrow y' = \frac{x}{1+x^2}.y$$

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$$\Rightarrow y' = \frac{x}{1+x^2}.y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

Therefore, LHS = RHS

Therefore, the given function is a solution of the given differential equation.

5. $y = Ax : xy' = y\ (x \neq 0)$

Solution:-

From the question, it is given that y = Ax

Differentiating both sides with respect to x, we get

$$y' = \frac{d}{dx}(Ax)$$

y' = A

Then,

Substituting the values of y' in the given differential equations, we get

= xy'

= x × A

= Ax

= Y … [from the question]

= RHS

Therefore, the given function is a solution of the given differential equation.

6. y = x sinx: xy' = y + x ($\sqrt{(x^2 - y^2)}$) (x ≠ 0 and x>y or x< – y)

Solution:-

From the question it is given that y = xsinx

Differentiating both sides with respect to x, we get,

$$y' = \frac{d}{dx}(x\sin x)$$

$$\Rightarrow y' = \sin x\frac{d}{dx}(x) + x.\frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x\cos x$$

Then,

Substituting the values of y' in the given differential equations, we get, Then,

Substituting the values of y' in the given differential equations, we get,

LHS = xy' = x(sinx + xcosx)

$= x\sin x + x^2\cos x$

From the question substitute y instead of xsinx, we get,

$= y + x^2.\sqrt{1 - \sin^2 x}$

$= y + x^2\sqrt{1 - \left(\frac{y}{x}\right)^2}$

$= y + x\sqrt{(y)^2 - (x)^2}$

= RHS

Therefore, the given function is a solution of the given differential equation

7. xy = logy + C : $y' = \dfrac{y^2}{1 - xy}\ (xy \neq 1)$

Solution:-

From the question it is given that xy = logy + C

Differentiating both sides with respect to x, we get,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y.\frac{d}{dx}(x) + x.\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$

On simplifying, we get.

$$\Rightarrow y + xy' = \frac{1}{y}\frac{dy}{dx}$$

By cross multiplication,

$$\Rightarrow y^2 + xyy' = y'$$

$$\Rightarrow (xy - 1)y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

By comparing LHS and RHS

LHS = RHS

Therefore, the given function is the solution of the corresponding differential equation.

8. y – cos y = x : (y sin y + cos y + x) y′ = y

Solution:-

From the question it is given that y – cosy = x

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} - \frac{d}{dx}\cos y = \frac{d}{dx}(x)$$

$$\Rightarrow y' + \sin y . y' = 1$$

$$\Rightarrow y' (1 + \sin y) = 1$$

$$\Rightarrow y' = \frac{1}{1+\sin y}$$

Then,

Substituting the values of y' in the given differential equations, we get,

Consider LHS = (ysiny + cosy + x)y'

$$= (y\sin y + \cos y + y - \cos y) \times \frac{1}{1 + \sin y}$$

$$= y(1 + \sin y) \times \frac{1}{1 + \sin y}$$

On simplifying we get,

= y

= RHS

Therefore, the given function is the solution of the corresponding differential equation.

9. $x + y = \tan^{-1}y : y^2 y' + y^2 + 1 = 0$

Solution:-

From the question it is given that $x + y = \tan^{-1}y$

Differentiating both sides with respect to x, we get,

$$\frac{d}{dx}(x+y) = \frac{d}{dx}(\tan^{-1}y)$$

$$\Rightarrow 1 + y' = \left[\frac{1}{1+y^2}\right]y'$$

By transposing y' to RHS and it becomes – y' and take out y' as common for both, we get,

$$\Rightarrow y'\left[\frac{1}{1+y^2} - 1\right] = 1$$

On simplifying,

$$\Rightarrow y'\left[\frac{1-(1+y^2)}{1+y^2}\right] = 1$$

$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right] = 1$$

$$\Rightarrow y' = \frac{-(1+y^2)}{y^2}$$

Then,

Substituting the values of y' in the given differential equations, we get,

Consider, LHS = $y^2y' + y^2 + 1$

$$= y^2\left[\frac{-(1+y^2)}{y^2}\right] + y^2 + 1$$

$= -1 - y^2 + y^2 + 1$

$= 0$

= RHS

Therefore, the given function is the solution of the corresponding differential equation.

10. $y = \sqrt{a^2 - x^2}\ x \in (-a, a):\quad x + y\dfrac{dy}{dx} = 0\ (y \neq 0)$

Solution:-

From the question it is given that $y = \sqrt{a^2 - x^2}$

Differentiating both sides with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{a^2 - x^2})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \cdot \frac{d}{dx}(a^2 - x^2)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

Then,

Substituting the values of y' in the given differential equations, we get,

Consider LHS = $x + y\frac{dy}{dx}$

$$= x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}}$$

On simplifying, we get,

= x −x

= 0

By comparing LHS and RHS

LHS = RHS.

Therefore, the given function is the solution of the corresponding differential equation.

11. The number of arbitrary constants in the general solution of a differential equation of fourth order is:

(A) 0 (B) 2 (C) 3 (D) 4

Solution:-

(D) 4

The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.

12. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(A) 3 (B) 2 (C) 1 (D) 0

Solution:-

(D) 0

The solution free from arbitrary constants, i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants, is called a particular solution of the differential equation.