1. Using divisibility tests, determine which of the following numbers are divisible by 2 ; by 3 ; by 4 ; by 5 ; by 6 ; by 8; by 9; by 10; by 11 (say, yes or no):

| Numbers |  |  |  | Divisible by |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 128 | Yes | No | Yes | No | No | Yes | No | No | No |
| 990 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 1586 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 275 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 6686 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 639210 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 429714 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 2856 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 3060 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 406839 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |

Solutions:

| Numbers |  |  | Divisible by |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 128 |  |  |  |  |  |  |  |  |  |
| 990 | Yes | No | Yes | No | No | Yes | No | No | No |
| 1586 | Yes | Yos | No | Yes | Yes | No | Yes | Yes | Yes |
| 275 | No | No | No | No | No | No | No | No | No |
| 6686 | Yes | No | No | No | No | No | No | No | Yes |
| 639210 | Yes | Yes | No | Yes | Yes | No | No | Yes | Yes |
| 429714 | Yes | Yes | No | No | Yes | No | Yes | No | No |
| 2856 | Yes | Yes | Yes | No | Yes | Yes | No | No | No |
| 3060 | Yes | Yes | Yes | Yes | Yes | No | Yes | Yes | No |
| 406839 | No | Yes | No | No | No | No | No | No | No |

2. Using divisibility tests, determine which of the following numbers are divisible by 4 ; by 8 :
(a) 572
(b) 726352
(c) $\mathbf{5 5 0 0}$
(d) 6000
(e) 12159
(f) $\mathbf{1 4 5 6 0}$
(g) 21084
(h) $\mathbf{3 1 7 9 5 0 7 2}$
(i) $\mathbf{1 7 0 0}$
(j) 2150

## Solutions:

(a) 572

72 are the last two digits. Since 72 is divisible by 4 . Hence, 572 is also divisible by 4 572 are the last three digits. Since 572 is not divisible by 8 . Hence, 572 is not divisible by 8
(b) 726352

52 are the last two digits. Since 52 is divisible by 4 . Hence, 726352 is divisible by 4 352 are the last three digits. Since 352 is divisible by 8 . Hence, 726352 is divisible by 8
(c) 5500

Since the last two digits are 00 . Hence 5500 is divisible by 4
500 are the last three digits. Since 500 is not divisible by 8 . Hence, 5500 is not divisible by 8
(d) 6000

Since the last two digits are 00 . Hence 6000 is divisible by 4
Since the last three digits are 000 . Hence, 6000 is divisible by 8
(e) 12159

59 are the last two digits. Since 59 is not divisible by 4 . Hence, 12159 is not divisible by 4 159 are the last three digits. Since 159 is not divisible by 8 . Hence, 12159 is not divisible by 8
(f) 14560

60 are the last two digits. Since 60 is divisible by 4 . Hence, 14560 is divisible by 4

560 are the last three digits. Since 560 is divisible by 8 . Hence, 14560 is divisible by 8
(g) 21084

84 are the last two digits. Since 84 is divisible by 4 . Hence, 21084 is divisible by 4 084 are the last three digits. Since 084 is not divisible by 8 . Hence, 21084 is not divisible by 8 (h) 31795072

72 are the last two digits. Since 72 is divisible by 4 . Hence, 31795072 is divisible by 4 072 are the last three digits. Since 072 is divisible by 8 . Hence, 31795072 is divisible by 8
(i) 1700

Since the last two digits are 00 . Hence, 1700 is divisible by 4
700 are the last three digits. Since 700 is not divisible by 8 . Hence, 1700 is not divisible by 8
(j) 2150

50 are the last two digits. Since 50 is not divisible by 4 . Hence, 2150 is not divisible by 4
150 are the last three digits. Since 150 is not divisible by 8 . Hence, 2150 is not divisible by 8
3. Using divisibility tests, determine which of the following numbers are divisible by 6:
(a) 297144
(b) 1258
(c) 4335
(d) 61233
(e) 901352
(f) $\mathbf{4 3 8 7 5 0}$
(g) 1790184
(h) 12583
(i) 639210
(j) 17852

## Solutions:

(a) 297144

Since the last digit of the number is 4 . Hence, the number is divisible by 2

By adding all the digits of the number, we get 27 , which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(b) 1258

Since the last digit of the number is 8 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 16 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(c) 4335

Since the last digit of the number is 5 , which is not divisible by 2 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 15 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(d) 61233

Since the last digit of the number is 3 , which is not divisible by 2 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 15 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(e) 901352

Since the last digit of the number is 2 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 20 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(f) 438750

Since the last digit of the number is 0 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 27 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(g) 1790184

Since the last digit of the number is 4 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 30 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(h) 12583

Since the last digit of the number is 3 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 19 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(i) 639210

Since the last digit of the number is 0 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 21 , which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(j) 17852

Since the last digit of the number is 2 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 23 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
4. Using divisibility tests, determine which of the following numbers are divisible by 11:
(a) 5445
(b) 10824
(c) 7138965
(d) 70169308
(e) 10000001
(f) 901153

## Solutions:

(a) 5445

Sum of the digits at odd places $=5+4$
$=9$
Sum of the digits at even places $=4+5$
$=9$
Difference $=9-9=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 5445 is divisible by 11
(b) 10824

Sum of digits at odd places $=4+8+1$
$=13$
Sum of digits at even places $=2+0$
$=2$

Difference $=13-2=11$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 11 , which is divisible by 11 . Hence, 10824 is divisible by 11
(c) 7138965

Sum of digits at odd places $=5+9+3+7=24$
Sum of digits at even places $=6+8+1=15$
Difference $=24-15=9$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 9 , which is not divisible by 11 . Hence, 7138965 is not divisible by 11
(d) 70169308

Sum of digits at odd places $=8+3+6+0$
$=17$
Sum of digits at even places $=0+9+1+7$
$=17$
Difference $=17-17=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 70169308 is divisible by 11
(e) 10000001

Sum of digits at odd places $=1$
Sum of digits at even places $=1$
Difference $=1-1=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 10000001 is divisible by 11
(f) 901153

Sum of digits at odd places $=3+1+0$
$=4$
Sum of digits at even places $=5+1+9$
$=15$
Difference $=15-4=11$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 11 , which is divisible by 11 . Hence, 901153 is divisible by 11
5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :
(a) $\qquad$ 6724
(b) 4765 $\qquad$

## Solutions:

(a) _ 6724

The sum of the given digits $=19$
The sum of its digit should be divisible by 3 to make the number divisible by 3
Since 21 is the smallest multiple of 3 , which comes after 19
So, smallest number $=21-19$
$=2$

Now $2+3+3=8$
But $2+3+3+3=11$

Now, if we put 8 , the sum of digits will be 27 , which is divisible by 3
Therefore the number will be divisible by 3
Hence, the largest number is 8
(b) 4765 $\qquad$ 2

Sum of the given digits $=24$
Sum of its digits should be divisible by 3 to make the number divisible by 3
Since, 24 is already divisible by 3 . Hence, the smallest number that can be replaced is 0
Now, $0+3=3$
$3+3=6$
$3+3+3=9$
$3+3+3+3=12$
If we put 9 , the sum of its digits becomes 33 . Since 33 is divisible by 3 .
Therefore the number will be divisible by 3
Hence, the largest number is 9
6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11:
(a) 92 389
(b) 8 $\qquad$ 9484

## Solutions:

(a) 92 $\qquad$
Let ' $a$ ' be placed here
Sum of its digits at odd places $=9+3+2$
$=14$
Sum of its digits at even places $=8+a+9$
$=17+\mathrm{a}$
Difference $=17+\mathrm{a}-14$
$=3+\mathrm{a}$
The difference should be 0 or a multiple of 11 , then the number is divisible by 11
If $3+\mathrm{a}=0$
$\mathrm{a}=-3$
But it cannot be a negative
Taking the closest multiple of 11 which is near 3
It is 11 which is near 3
Now, $3+\mathrm{a}=11$
$a=11-3$
$\mathrm{a}=8$

Therefore the required digit is 8
(b) 8 $\qquad$ 9484

Let ' $a$ ' be placed here
Sum of its digits at odd places $=4+4+\mathrm{a}$
$=8+\mathrm{a}$
Sum of its digits at even places $=8+9+8$
$=25$
Difference $=25-(8+a)$
$=17-\mathrm{a}$
The difference should be 0 or a multiple of 11 , then the number is divisible by 11
If $17-\mathrm{a}=0$
$\mathrm{a}=17$ (which is not possible)
Now, take a multiple of 11.
Let's take 11
$17-\mathrm{a}=11$
$a=17-11$
$a=6$

Therefore the required digit is 6

## EXERCISE 3.4

1. Find the common factors of:
(a) 20 and 28
(b) 15 and 25
(c) 35 and 50
(d) 56 and 120

## Solutions:

(a) 20 and 28
$1,2,4,5,10$ and 20 are factors of 20
$1,2,4,7,14$ and 28 are factors of 28
Common factors $=1,2,4$
(b) 15 and 25
$1,3,5$ and 15 are factors of 15
1,5 and 25 are factors of 25
Common factors $=1,5$
(c) 35 and 50
$1,5,7$ and 35 are factors of 35
$1,2,5,10,25$ and 50 are factors of 50
Common factors $=1,5$
(d) 56 and 120
$1,2,4,7,8,14,28$ and 56 are factors of 56
$1,2,3,4,5,6,8,10,12,15,20,24,30,40,60$ and 120 are factors of 120
Common factors $=1,2,4,8$
2. Find the common factors of:
(a) 4, 8 and 12
(b) 5, 15 and 25

## Solutions:

(a) 4, 8 and 12
$1,2,4$ are factors of 4
$1,2,4,8$ are factors of 8
$1,2,3,4,6,12$ are factors of 12
Common factors $=1,2,4$
(b) 5, 15 and 25

1,5 are factors of 5
$1,3,5,15$ are factors of 15
$1,5,25$ are factors of 25
Common factors $=1,5$
3. Find the first three common multiples of:
(a) 6 and 8
(b) 12 and 18

Solutions:
(a) 6 and 8
$6,12,18,24,30$ are multiples of 6
$8,16,24,32$ are multiples of 8
Three common multiples are $24,48,72$
(b) 12 and 18

12, 24, 36, 48 are multiples of 12
$18,36,54,72$ are multiples of 18
Three common factors are 36, 72, 108
4. Write all the numbers less than 100 , which are common multiples of 3 and 4.

## Solutions:

Multiples of 3 are 3, 6, 9, 12, 15
Multiples of 4 are 4, 8, 12, 16, 20

Common multiples are $12,24,36,48,60,72,84$ and 96
5. Which of the following numbers are co-prime?
(a) 18 and 35
(b) 15 and 37
(c) 30 and 415
(d) 17 and 68
(e) 216 and 215
(f) 81 and 16

## Solutions:

(a) 18 and 35

Factors of 18 are 1, 2, 3, 6, 9, 18
Factors of 35 are 1, 5, 7, 35
Common factor $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(b) 15 and 37

Factors of 15 are 1, 3, 5, 15
Factors of 37 are 1, 37

Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(c) 30 and 415

Factors of 30 are $1,2,3,5,6,10,15,30$
Factors of 415 are 1, 5, 83, 415
Common factors $=1,5$
Since their common factor is other than 1 . Hence, the given two numbers are not co-prime
(d) 17 and 68

Factors of 17 are 1, 17
Factors of 68 are 1, 2, 4, 17, 34, 68

Common factors $=1,17$
Since their common factor is other than 1. Hence, the given two numbers are not co-prime
(e) 216 and 215

Factors of 216 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216
Factors of 215 are 1, 5, 43, 215
Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(f) 81 and 16

Factors of 81 are 1, 3, 9, 27, 81
Factors of 16 are 1, 2, 4, 8, 16
Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
6. A number is divisible by both 5 and 12 . By which other number will that number always be divisible?

## Solutions:

Factors of 5 are 1,5
Factors of 12 are 1, 2, 3, 4, 6, 12
Their common factor $=1$
Since their common factor is 1 . The given two numbers are co-prime and are also divisible by their product 60
Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
7. A number is divisible by $\mathbf{1 2}$. By what other numbers will that number be divisible?

## Solutions:

Since the number is divisible by 12 . Hence, it also divisible by its factors i.e., $1,2,3,4,6,12$
Therefore $1,2,3,4$, and 6 are the numbers other than 12 by which this number is also divisible

