

EXERCISE 3.7

PAGE NO: 67

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight, which can measure the weight of the fertiliser exact number of times.

Solutions:

Given weight of two bags of fertiliser = 75 kg and 69 kg

Maximum weight = HCF of two bags weight i.e., (75, 69)

3	75
5	25
5	5
	1

3	69
23	23
	1

$$75 = 3 \times 5 \times 5$$

$$69 = 3 \times 23$$

$$\text{HCF} = 3$$

Hence, 3 kg is the maximum value of weight which can measure the weight of the fertiliser exact number of times.

2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm, respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

Solutions:

The first boy's steps measure = 63 cm

The second boy's steps measure = 70 cm

Third boy's steps measure = 77 cm

LCM of 63, 70, 77

2	63	70	77
3	63	35	77
3	21	35	77
5	7	35	77
7	7	7	77
11	1	1	11
	1	1	1

$$\text{LCM} = 2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930$$

Hence, 6930 cm is the distance each should cover so that all can cover the distance in complete steps.

3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm, respectively. Find the longest tape that can measure the room's three dimensions exactly.

Solutions:

Given the length of a room = 825 cm

The breadth of a room = 675 cm

Height of a room = 450 cm

3	825
5	275
5	55
11	11
	1

3	675
3	225
3	75
5	25
5	5
	1

2	450
3	225
3	75
5	25
5	5
	1

$$825 = 3 \times 5 \times 5 \times 11$$

$$675 = 3 \times 3 \times 3 \times 5 \times 5$$

$$450 = 2 \times 3 \times 3 \times 5 \times 5$$

$$\text{HCF} = 3 \times 5 \times 5 = 75 \text{ cm}$$

Hence longest tape is 75 cm which can measure the three dimensions of the room exactly.

4. Determine the smallest 3-digit number, which is exactly divisible by 6, 8 and 12.

Solutions:

LCM of 6, 8, 12 = smallest number

2	6	8	12
2	3	4	6
2	3	2	3
3	3	1	3
	1	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 3 = 24$$

Now we need to find the smallest 3-digit multiple of 24

We know that $24 \times 4 = 96$ and $24 \times 5 = 120$

Hence, 120 is the smallest 3-digit number which is exactly divisible by 6, 8 and 12

5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

Solutions:

LCM of 8, 10 and 12

2	8	10	12
2	4	5	6
2	2	5	3
3	1	5	3
5	1	5	1
	1	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

Now we need to find the greatest 3-digit multiple of 120

We may find $120 \times 8 = 960$ and $120 \times 9 = 1080$

Hence, 960 is the greatest 3-digit number exactly divisible by 8, 10 and 12

6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds, respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

Solutions:

LCM of 48, 72, 108 = time period after which these lights change

2	48	72	108
2	24	36	54
2	12	18	27
2	6	9	27
3	3	9	27
3	1	3	9
3	1	1	3
	1	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$$

Hence, lights will change together after every 432 seconds

Therefore the lights will change simultaneously at 7 minutes and 12 seconds.

7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel, respectively. Find the maximum capacity of a container that can measure the diesel of the three containers the exact number of times.

Solutions:

HCF of 403, 434, 465 = Maximum capacity of tanker required

$$403 = 13 \times 31$$

$$434 = 2 \times 7 \times 31$$

$$465 = 3 \times 5 \times 31$$

$$\text{HCF} = 31$$

Hence, a container of 31 litres can measure the diesel of the three containers the exact number of times.

8. Find the least number, which, when divided by 6, 15 and 18 leave the remainder 5 in each case.

Solutions:

LCM of 6, 15, 18

2	6	15	18
3	3	15	9
3	1	5	3
5	1	5	1
	1	1	1

$$\text{LCM} = 2 \times 3 \times 3 \times 5 = 90$$

$$\text{Required number} = 90 + 5$$

$$= 95$$

Hence, 95 is the required number.

9. Find the smallest 4-digit number, which is divisible by 18, 24 and 32.

Solutions:

LCM of 18, 24, 32

2	18	24	32
2	9	12	16
2	9	6	8
2	9	3	4
2	9	3	2
3	9	3	1
3	3	1	1
	1	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$$

Here, we need to find the smallest 4-digit multiple of 288

We find $288 \times 3 = 864$ and $288 \times 4 = 1152$

Hence, 1152 is the smallest 4-digit number which is divisible by 18, 24 and 32

10. Find the LCM of the following numbers:

(a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?

Solutions:

(a) LCM of 9, 4

2	9	4
2	9	2
3	9	1
3	3	1
	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 3 = 36$$

(b) LCM of 12, 5

2	12	5
2	6	5
3	3	5
5	1	5
	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$$

(c) LCM of 6, 5

2	6	5
3	3	5
5	1	5
	1	1

$$\text{LCM} = 2 \times 3 \times 5 = 30$$

(d) LCM of 15, 4

2	15	4
2	15	2
3	15	1
5	5	1
	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$$

Yes in each case the LCM of given numbers is the product of these numbers.

11. Find the LCM of the following numbers in which one number is the factor of the other.

(a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

What do you observe in the results obtained?

Solutions:

(a) 5, 20

2	5	20
2	5	10
5	5	5
	1	1

$$\text{LCM} = 2 \times 2 \times 5 = 20$$

(b) 6, 18

2	6	18
3	3	9
3	1	3
	1	1

$$\text{LCM} = 2 \times 3 \times 3 = 18$$

(c) 12, 48

2	12	48
2	6	24
2	3	12
2	3	6
3	3	3
	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

(d) 9, 45

3	9	45
3	3	15
5	1	5
	1	1

$$\text{LCM} = 3 \times 3 \times 5 = 45$$

∴ Hence, in each case the LCM of given numbers is the larger number. When a number is a factor of other number then their LCM will be the larger number.

Disclaimer:

Dropped Topic – 3.6 Some more divisibility rules.