## EXERCISE 3.1

1. Write all the factors of the following numbers:
(a) 24
(b) 15
(c) 21
(d) 27
(e) 12
(f) $\mathbf{2 0}$
(g) 18
(h) 23
(i) 36

Solutions:
(a) 24
$24=1 \times 24$
$24=2 \times 12$
$24=3 \times 8$
$24=4 \times 6$
$24=6 \times 4$
Stop here since 4 and 6 have occurred earlier
Hence, the factors of 24 are $1,2,3,4,6,8,12$ and 24
(b) 15
$15=1 \times 15$
$15=3 \times 5$
$15=5 \times 3$
Stop here since 3 and 5 have occurred earlier
Hence, the factors of 15 are 1, 3, 5 and 15
(c) 21
$21=1 \times 21$
$21=3 \times 7$
$21=7 \times 3$
Stop here since 3 and 7 have occurred earlier
Hence, the factors of 21 are 1, 3, 7 and 21
(d) 27
$27=1 \times 27$
$27=3 \times 9$
$27=9 \times 3$
Stop here since 3 and 9 have occurred earlier
Hence, the factors of 27 are 1, 3, 9 and 27
(e) 12
$12=1 \times 12$
$12=2 \times 6$
$12=3 \times 4$
$12=4 \times 3$
Stop here since 3 and 4 have occurred earlier
Hence, the factors of 12 are 1, 2, 3, 4, 6 and 12
(f) 20
$20=1 \times 20$
$20=2 \times 10$
$20=4 \times 5$
$20=5 \times 4$
Stop here since 4 and 5 have occurred earlier
Hence, the factors of 20 are $1,2,4,5,10$ and 20
(g) 18
$18=1 \times 18$
$18=2 \times 9$
$18=3 \times 6$
$18=6 \times 3$
Stop here since 3 and 6 have occurred earlier
Hence, the factors of 18 are 1, 2, 3, 6, 9 and 18
(h) 23
$23=1 \times 23$
$23=23 \times 1$
Since 1 and 23 have occurred earlier
Hence, the factors of 23 are 1 and 23
(i) 36
$36=1 \times 36$
$36=2 \times 18$
$36=3 \times 12$
$36=4 \times 9$
$36=6 \times 6$
Stop here since both the factors (6) are the same. Thus the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36
2. Write the first five multiples of:
(a) 5
(b) 8
(c) 9

Solutions:
(a) The required multiples are:
$5 \times 1=5$
$5 \times 2=10$
$5 \times 3=15$
$5 \times 4=20$
$5 \times 5=25$
Hence, the first five multiples of 5 are 5, 10, 15, 20 and 25
(b) The required multiples are:
$8 \times 1=8$
$8 \times 2=16$
$8 \times 3=24$
$8 \times 4=32$
$8 \times 5=40$
Hence, the first five multiples of 8 are $8,16,24,32$ and 40
(c) The required multiples are:
$9 \times 1=9$
$9 \times 2=18$
$9 \times 3=27$
$9 \times 4=36$
$9 \times 5=45$
Hence, the first five multiples of 9 are $9,18,27,36$ and 45
3. Match the items in column 1 with the items in column 2.

Column 1
(i) 35
(ii) 15
(iii) 16
(iv) 20
(v) 25

## Column 2

(a) Multiple of 8
(b) Multiple of 7
(c) Multiple of 70
(d) Factor of 30
(e) Factor of 50
(f) Factor of 20

Solutions:
(i) 35 is a multiple of 7

Hence, option (b)
(ii) 15 is a factor of 30

Hence, option (d)
(iii) 16 is a multiple of 8

Hence, option (a)
(iv) 20 is a factor of 20

Hence, option (f)
(v) 25 is a factor of 50

Hence, option (e)
4. Find all the multiples of 9 up to 100 .

Solutions:
$9 \times 1=9$
$9 \times 2=18$
$9 \times 3=27$
$9 \times 4=36$
$9 \times 5=45$
$9 \times 6=54$
$9 \times 7=63$
$9 \times 8=72$
$9 \times 9=81$
$9 \times 10=90$
$9 \times 11=99$
$\therefore$ All the multiples of 9 up to 100 are $9,18,27,36,45,54,63,72,81,90$ and 99

## EXERCISE 3.2

1. What is the sum of any two (a) Odd numbers? (b) Even numbers?

## Solutions:

(a) The sum of any two odd numbers is an even number.

Examples: $5+3=8$
$15+13=28$
(b) The sum of any two even numbers is an even number

Examples: $2+8=10$
$12+28=40$
2. State whether the following statements are True or False:
(a) The sum of three odd numbers is even.
(b) The sum of two odd numbers and one even number is even.
(c) The product of three odd numbers is odd.
(d) If an even number is divided by 2 , the quotient is always odd.
(e) All prime numbers are odd.
(f) Prime numbers do not have any factors.
(g) Sum of two prime numbers is always even.
(h) 2 is the only even prime number.
(i) All even numbers are composite numbers.
(j) The product of two even numbers is always even.

## Solutions:

(a) False. The sum of three odd numbers is odd.

Example: $7+9+5=21$ i.e odd number
(b) True. The sum of two odd numbers and one even number is even.

Example: $3+5+8=16$ i.e., is an even number.
(c) True. The product of three odd numbers is odd.

Example: $3 \times 7 \times 9=189$ i.e., is an odd number.
(d) False. If an even number is divided by 2, the quotient is even.

Example: $8 \div 2=4$
(e) False, All prime numbers are not odd.

Example: 2 is a prime number, but it is also an even number.
(f) False. Since 1 and the number itself are factors of the number
(g) False. The sum of two prime numbers may also be an odd number

Example: $2+5=7$ i.e., an odd number.
(h) True. 2 is the only even prime number.
(i) False. Since 2 is a prime number.
(j) True. The product of two even numbers is always even.

Example: $2 \times 4=8$ i.e., even number.
3. The numbers 13 and 31 are prime numbers. Both these numbers have the same digits, 1 and 3 . Find such pairs of prime numbers up to 100 .

## Solutions:

The prime numbers with the same digits up to 100 are as follows:
17 and 71
37 and 73
79 and 97
4. Write down separately the prime and composite numbers less than 20.

## Solutions:

$2,3,5,7,11,13,17$ and 19 are the prime numbers less than 20
$4,6,8,9,10,12,14,15,16$ and 18 are the composite numbers less than 20
5. What is the greatest prime number between 1 and 10 ?

## Solutions:

$2,3,5$ and 7 are the prime numbers between 1 and 10.7 is the greatest prime number among them.
6. Express the following as the sum of two odd primes.
(a) 44
(b) 36
(c) 24
(d) 18

Solutions:
(a) $3+41=44$
(b) $5+31=36$
(c) $5+19=24$
(d) $5+13=18$
7. Give three pairs of prime numbers whose difference is 2 . [Remark: Two prime numbers whose difference is 2 are called twin primes].

Solutions:
The three pairs of prime numbers whose difference is 2 are
3, 5
5, 7
11,13
8. Which of the following numbers is prime?
(a) 23
(b) 51
(c) 37
(d) 26

Solutions:
(a) 23
$1 \times 23=23$
$23 \times 1=23$
Therefore 23 has only two factors 1 and 23 . Hence, it is a prime number.
(b) 51
$1 \times 51=51$
$3 \times 17=51$
Therefore 51 has four factors $1,3,17$ and 51 . Hence, it is not a prime number, it is a composite number.
(c) 37
$1 \times 37=37$
$37 \times 1=37$
Therefore 37 has two factors 1 and 37 . Hence, it is a prime number.
(d) 26
$1 \times 26=26$
$2 \times 13=26$
Therefore 26 has four factors 1,2,13 and 26. Hence, it is not a prime number, it is a composite number.
9. Write seven consecutive composite numbers less than 100 so there is no prime number between them.

## Solutions:

Seven composite numbers between 89 and 97 , both of which are prime numbers, are $90,91,92,93,94,95$ and 96
Numbers
Factors
90
$1,2,3,5,6,9,10,15,18,30,45,90$
91
92
93
94
95
96
1, 7, 13, 91
1, 2, 4, 23, 46, 92
1,3,31, 93
1, 2, 47, 94
1, 5, 19, 95
$1,2,3,4,6,8,12,16,24,32,48,96$
10. Express each of the following numbers as the sum of three odd primes:
(a) 21
(b) 31
(c) 53
(d) 61

## Solutions:

(a) $3+5+13=21$
(b) $3+5+23=31$
(c) $13+17+23=53$
(d) $7+13+41=61$
11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5 . (Hint: $3+7=10$ )

Solutions:
The five pairs of prime numbers less than 20 whose sum is divisible by 5 are
$2+3=5$
$2+13=15$
$3+17=20$
$7+13=20$
$19+11=30$
12. Fill in the blanks:
(a) A number with only two factors is called a $\qquad$ .
(b) A number with more than two factors is called a $\qquad$ .
(c) $\mathbf{1}$ is neither $\qquad$ nor $\qquad$ .
(d) The smallest prime number is $\qquad$ .
(e) The smallest composite number is $\qquad$ .
(f) The smallest even number is $\qquad$ .

## Solutions:

(a) A number with only two factors is called a prime number.
(b) A number with more than two factors is called a composite number.
(c) 1 is neither a prime number nor a composite number.
(d) The smallest prime number is 2
(e) The smallest composite number is 4
(f) The smallest even number is 2 .

1. Using divisibility tests, determine which of the following numbers are divisible by 2 ; by 3 ; by 4 ; by 5 ; by 6 ; by 8; by 9; by 10; by 11 (say, yes or no):

| Numbers |  |  |  | Divisible by |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 128 | Yes | No | Yes | No | No | Yes | No | No | No |
| 990 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 1586 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 275 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 6686 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 639210 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 429714 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 2856 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 3060 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |
| 406839 | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... | ....... |

Solutions:

| Numbers |  |  |  | Divisible by |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 128 |  |  |  |  |  |  |  |  |  |
| 990 | Yes | No | Yes | No | No | Yes | No | No | No |
| 1586 | Yes | Yo | No | Yes | Yes | No | Yes | Yes | Yes |
| 275 | No | No | No | No | No | No | No | No | No |
| 6686 | Yes | No | No | No | No | No | No | No | No |
| 639210 | Yes | Yes | No | Yes | Yes | No | No | Yes | Yes |
| 429714 | Yes | Yes | No | No | Yes | No | Yes | No | No |
| 2856 | Yes | Yes | Yes | No | Yes | Yes | No | No | No |
| 3060 | Yes | Yes | Yes | Yes | Yes | No | Yes | Yes | No |
| 406839 | No | Yes | No | No | No | No | No | No | No |

2. Using divisibility tests, determine which of the following numbers are divisible by 4 ; by 8 :
(a) 572
(b) 726352
(c) 5500
(d) 6000
(e) 12159
(f) $\mathbf{1 4 5 6 0}$
(g) 21084
(h) $\mathbf{3 1 7 9 5 0 7 2}$
(i) $\mathbf{1 7 0 0}$
(j) 2150

## Solutions:

(a) 572

72 are the last two digits. Since 72 is divisible by 4 . Hence, 572 is also divisible by 4 572 are the last three digits. Since 572 is not divisible by 8 . Hence, 572 is not divisible by 8
(b) 726352

52 are the last two digits. Since 52 is divisible by 4 . Hence, 726352 is divisible by 4 352 are the last three digits. Since 352 is divisible by 8 . Hence, 726352 is divisible by 8
(c) 5500

Since the last two digits are 00 . Hence 5500 is divisible by 4
500 are the last three digits. Since 500 is not divisible by 8 . Hence, 5500 is not divisible by 8
(d) 6000

Since the last two digits are 00 . Hence 6000 is divisible by 4
Since the last three digits are 000 . Hence, 6000 is divisible by 8
(e) 12159

59 are the last two digits. Since 59 is not divisible by 4 . Hence, 12159 is not divisible by 4 159 are the last three digits. Since 159 is not divisible by 8 . Hence, 12159 is not divisible by 8
(f) 14560

60 are the last two digits. Since 60 is divisible by 4 . Hence, 14560 is divisible by 4

560 are the last three digits. Since 560 is divisible by 8 . Hence, 14560 is divisible by 8
(g) 21084

84 are the last two digits. Since 84 is divisible by 4 . Hence, 21084 is divisible by 4 084 are the last three digits. Since 084 is not divisible by 8 . Hence, 21084 is not divisible by 8 (h) 31795072

72 are the last two digits. Since 72 is divisible by 4 . Hence, 31795072 is divisible by 4 072 are the last three digits. Since 072 is divisible by 8 . Hence, 31795072 is divisible by 8
(i) 1700

Since the last two digits are 00 . Hence, 1700 is divisible by 4
700 are the last three digits. Since 700 is not divisible by 8 . Hence, 1700 is not divisible by 8
(j) 2150

50 are the last two digits. Since 50 is not divisible by 4 . Hence, 2150 is not divisible by 4
150 are the last three digits. Since 150 is not divisible by 8 . Hence, 2150 is not divisible by 8
3. Using divisibility tests, determine which of the following numbers are divisible by 6:
(a) 297144
(b) 1258
(c) 4335
(d) 61233
(e) 901352
(f) $\mathbf{4 3 8 7 5 0}$
(g) 1790184
(h) 12583
(i) 639210
(j) 17852

## Solutions:

(a) 297144

Since the last digit of the number is 4 . Hence, the number is divisible by 2

By adding all the digits of the number, we get 27 , which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(b) 1258

Since the last digit of the number is 8 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 16 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(c) 4335

Since the last digit of the number is 5 , which is not divisible by 2 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 15 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(d) 61233

Since the last digit of the number is 3 , which is not divisible by 2 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 15 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(e) 901352

Since the last digit of the number is 2 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 20 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(f) 438750

Since the last digit of the number is 0 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 27 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(g) 1790184

Since the last digit of the number is 4 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 30 which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(h) 12583

Since the last digit of the number is 3 . Hence, the number is not divisible by 2
By adding all the digits of the number, we get 19 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
(i) 639210

Since the last digit of the number is 0 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 21 , which is divisible by 3 . Hence, the number is divisible by 3
$\therefore$ The number is divisible by both 2 and 3 . Hence, the number is divisible by 6
(j) 17852

Since the last digit of the number is 2 . Hence, the number is divisible by 2
By adding all the digits of the number, we get 23 which is not divisible by 3 . Hence, the number is not divisible by 3
$\therefore$ The number is not divisible by both 2 and 3 . Hence, the number is not divisible by 6
4. Using divisibility tests, determine which of the following numbers are divisible by 11:
(a) 5445
(b) 10824
(c) 7138965
(d) 70169308
(e) 10000001
(f) 901153

## Solutions:

(a) 5445

Sum of the digits at odd places $=5+4$
$=9$
Sum of the digits at even places $=4+5$
$=9$
Difference $=9-9=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 5445 is divisible by 11
(b) 10824

Sum of digits at odd places $=4+8+1$
$=13$
Sum of digits at even places $=2+0$
$=2$

Difference $=13-2=11$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 11 , which is divisible by 11 . Hence, 10824 is divisible by 11
(c) 7138965

Sum of digits at odd places $=5+9+3+7=24$
Sum of digits at even places $=6+8+1=15$
Difference $=24-15=9$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 9 , which is not divisible by 11 . Hence, 7138965 is not divisible by 11
(d) 70169308

Sum of digits at odd places $=8+3+6+0$
$=17$
Sum of digits at even places $=0+9+1+7$
$=17$
Difference $=17-17=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 70169308 is divisible by 11
(e) 10000001

Sum of digits at odd places $=1$
Sum of digits at even places $=1$
Difference $=1-1=0$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 0 . Hence, 10000001 is divisible by 11
(f) 901153

Sum of digits at odd places $=3+1+0$
$=4$
Sum of digits at even places $=5+1+9$
$=15$
Difference $=15-4=11$
Since the difference between the sum of digits at odd places and the sum of digits at even places is 11 , which is divisible by 11 . Hence, 901153 is divisible by 11
5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :
(a) $\qquad$ 6724
(b) 4765 $\qquad$

## Solutions:

(a) _ 6724

The sum of the given digits $=19$
The sum of its digit should be divisible by 3 to make the number divisible by 3
Since 21 is the smallest multiple of 3 , which comes after 19
So, smallest number $=21-19$
$=2$

Now $2+3+3=8$
But $2+3+3+3=11$
Now, if we put 8 , the sum of digits will be 27 , which is divisible by 3
Therefore the number will be divisible by 3
Hence, the largest number is 8
(b) 4765 $\qquad$ 2

Sum of the given digits $=24$
Sum of its digits should be divisible by 3 to make the number divisible by 3
Since, 24 is already divisible by 3 . Hence, the smallest number that can be replaced is 0
Now, $0+3=3$
$3+3=6$
$3+3+3=9$
$3+3+3+3=12$
If we put 9 , the sum of its digits becomes 33 . Since 33 is divisible by 3 .
Therefore the number will be divisible by 3
Hence, the largest number is 9
6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11:
(a) 92 389
(b) 8 $\qquad$ 9484

## Solutions:

(a) 92 $\qquad$
Let ' $a$ ' be placed here
Sum of its digits at odd places $=9+3+2$
$=14$
Sum of its digits at even places $=8+a+9$
$=17+\mathrm{a}$
Difference $=17+\mathrm{a}-14$
$=3+\mathrm{a}$
The difference should be 0 or a multiple of 11 , then the number is divisible by 11
If $3+\mathrm{a}=0$
$\mathrm{a}=-3$
But it cannot be a negative
Taking the closest multiple of 11 which is near 3
It is 11 which is near 3
Now, $3+\mathrm{a}=11$
$a=11-3$
$\mathrm{a}=8$

Therefore the required digit is 8
(b) 8 $\qquad$ 9484

Let ' $a$ ' be placed here
Sum of its digits at odd places $=4+4+\mathrm{a}$
$=8+\mathrm{a}$
Sum of its digits at even places $=8+9+8$
$=25$
Difference $=25-(8+a)$
$=17-\mathrm{a}$
The difference should be 0 or a multiple of 11 , then the number is divisible by 11
If $17-\mathrm{a}=0$
$\mathrm{a}=17$ (which is not possible)
Now, take a multiple of 11.
Let's take 11
$17-\mathrm{a}=11$
$a=17-11$
$a=6$

Therefore the required digit is 6

## EXERCISE 3.4

1. Find the common factors of:
(a) 20 and 28
(b) 15 and 25
(c) 35 and 50
(d) 56 and 120

## Solutions:

(a) 20 and 28
$1,2,4,5,10$ and 20 are factors of 20
$1,2,4,7,14$ and 28 are factors of 28
Common factors $=1,2,4$
(b) 15 and 25
$1,3,5$ and 15 are factors of 15
1,5 and 25 are factors of 25
Common factors $=1,5$
(c) 35 and 50
$1,5,7$ and 35 are factors of 35
$1,2,5,10,25$ and 50 are factors of 50
Common factors $=1,5$
(d) 56 and 120
$1,2,4,7,8,14,28$ and 56 are factors of 56
$1,2,3,4,5,6,8,10,12,15,20,24,30,40,60$ and 120 are factors of 120
Common factors $=1,2,4,8$
2. Find the common factors of:
(a) 4, 8 and 12
(b) 5, 15 and 25

## Solutions:

(a) 4, 8 and 12
$1,2,4$ are factors of 4
$1,2,4,8$ are factors of 8
$1,2,3,4,6,12$ are factors of 12
Common factors $=1,2,4$
(b) 5, 15 and 25

1,5 are factors of 5
$1,3,5,15$ are factors of 15
$1,5,25$ are factors of 25
Common factors $=1,5$
3. Find the first three common multiples of:
(a) 6 and 8
(b) 12 and 18

Solutions:
(a) 6 and 8
$6,12,18,24,30$ are multiples of 6
$8,16,24,32$ are multiples of 8
Three common multiples are $24,48,72$
(b) 12 and 18

12, 24, 36, 48 are multiples of 12
$18,36,54,72$ are multiples of 18
Three common factors are 36, 72, 108
4. Write all the numbers less than 100 , which are common multiples of 3 and 4.

## Solutions:

Multiples of 3 are 3, 6, 9, 12, 15
Multiples of 4 are 4, 8, 12, 16, 20

Common multiples are $12,24,36,48,60,72,84$ and 96
5. Which of the following numbers are co-prime?
(a) 18 and 35
(b) 15 and 37
(c) 30 and 415
(d) 17 and 68
(e) 216 and 215
(f) 81 and 16

## Solutions:

(a) 18 and 35

Factors of 18 are 1, 2, 3, 6, 9, 18
Factors of 35 are 1, 5, 7, 35
Common factor $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(b) 15 and 37

Factors of 15 are 1, 3, 5, 15
Factors of 37 are 1, 37

Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(c) 30 and 415

Factors of 30 are $1,2,3,5,6,10,15,30$
Factors of 415 are 1, 5, 83, 415
Common factors $=1,5$
Since their common factor is other than 1 . Hence, the given two numbers are not co-prime
(d) 17 and 68

Factors of 17 are 1, 17
Factors of 68 are 1,2,4,17,34, 68

Common factors $=1,17$
Since their common factor is other than 1. Hence, the given two numbers are not co-prime
(e) 216 and 215

Factors of 216 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216
Factors of 215 are 1, 5, 43, 215
Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
(f) 81 and 16

Factors of 81 are 1, 3, 9, 27, 81
Factors of 16 are 1, 2, 4, 8, 16
Common factors $=1$
Since their common factor is 1 . Hence, the given two numbers are co-prime
6. A number is divisible by both 5 and 12 . By which other number will that number always be divisible?

## Solutions:

Factors of 5 are 1,5
Factors of 12 are 1, 2, 3, 4, 6, 12
Their common factor $=1$
Since their common factor is 1 . The given two numbers are co-prime and are also divisible by their product 60
Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
7. A number is divisible by $\mathbf{1 2}$. By what other numbers will that number be divisible?

## Solutions:

Since the number is divisible by 12 . Hence, it also divisible by its factors i.e., $1,2,3,4,6,12$
Therefore $1,2,3,4$, and 6 are the numbers other than 12 by which this number is also divisible

## EXERCISE 3.5

1. Which of the following statements is true?
(a) If a number is divisible by 3 , it must be divisible by 9 .
(b) If a number is divisible by 9 , it must be divisible by 3 .
(c) A number is divisible by 18 , if it is divisible by both 3 and 6 .
(d) If a number is divisible by 9 and 10 , then it must be divisible by 90 .
(e) If two numbers are co-primes, at least one of them must be prime.
(f) All numbers which are divisible by 4 must also be divisible by 8 .
(g) All numbers which are divisible by 8 must also be divisible by 4 .
(h) If a number exactly divides two numbers separately, it must exactly divide their sum.
(i) If a number exactly divides the sum of two numbers, it must exactly divide the two numbers separately.

## Solutions:

(a) False, 6 is divisible by 3 but is not divisible by 9
(b) True, as $9=3 \times 3$. Hence, if a number is divisible by 9 , it will also be divisible by 3
(c) False. Since 30 is divisible by both 3 and 6 but is not divisible by 18
(d) True, as $9 \times 10=90$. Hence, if a number is divisible by both 9 and 10 then it is divisible by 90
(e) False. Since 15 and 32 are co-primes and also composite numbers
(f) False, as 12 is divisible by 4 but is not divisible by 8
(g) True, as $2 \times 4=8$. Hence, if a number is divisible by 8 , it will also be divisible by 2 and 4
(h) True, as 2 divides 4 and 8 , and it also divides $12(4+8=12)$
(i) False, since 2 divides 12 but it does not divide 7 and 5
2. Here are two different factor trees for $\mathbf{6 0}$. Write the missing numbers.
(a)

(b)


Solutions:
(a) Since $6=2 \times 3$ and $10=5 \times 2$

(b) Since $60=30 \times 2$
$30=10 \times 3$
$10=5 \times 2$

3. Which factors are not included in the prime factorisation of a composite number?

## Solutions:

1 and the number itself are not included in the prime factorisation of a composite number.
4. Write the greatest 4-digit number and express it in terms of its prime factors.

## Solutions:

The greatest four-digit number is 9999
Therefore $9999=3 \times 3 \times 11 \times 101$

5. Write the smallest 5-digit number and express it in the form of its prime factors.

## Solutions:

The smallest five-digit number $=10000$

$10000=2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
6. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any, between two consecutive prime factors.

Solutions:

| 7 | 1729 |
| :---: | :---: |
| 13 | 247 |
| 19 | 19 |
|  | 1 |
|  |  |

$13-7=6$
$19-13=6$
Hence, the difference between two consecutive prime factors is 6 .
7. The product of three consecutive numbers is always divisible by 6 . Verify this statement with the help of some examples.

## Solutions:

(i) $2 \times 3 \times 4=24$ which is divisible by 6
(ii) $5 \times 6 \times 7=210$ which is divisible by 6
8. The sum of two consecutive odd numbers is divisible by 4 . Verify this statement with the help of some examples.

## Solutions:

(i) $5+3=8$ which is divisible by 4
(ii) $7+9=16$ which is divisible by 4
(iii) $13+15=28$ which is divisible by 4
9. In which of the following expressions has prime factorisation been done?
(a) $24=2 \times 3 \times 4$
(b) $56=7 \times 2 \times 2 \times 2$
(c) $70=2 \times 5 \times 7$
(d) $54=2 \times 3 \times 9$

Solutions:
(a) $24=2 \times 3 \times 4$

Since 4 is composite. Hence, prime factorisation has not been done
(b) $56=7 \times 2 \times 2 \times 2$

Since all the factors are prime. Hence, prime factorisation has been done
(c) $70=2 \times 5 \times 7$

Since all the factors are prime. Hence, prime factorisation has been done
(d) $54=2 \times 3 \times 9$

Since 9 is composite. Hence prime factorisation has not been done
10. Determine if $\mathbf{2 5 1 1 0}$ is divisible by 45 . [Hint: 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].

## Solutions:

$45=5 \times 9$
1,5 are factors of 5
$1,3,9$ are factors of 9
Hence, 5 and 9 are co-prime numbers
The last digit of 25110 is 0 . Hence, it is divisible by 5
Sum of digits 25110
$2+5+1+1+0$
$=9$
Since the sum of digits of 25110 is divisible by 9 . Hence, 25110 is divisible by 9
Since the number is divisible by both 5 and 9
Therefore 25110 is divisible by 45
11. 18 is divisible by both 2 and 3 . It is also divisible by $2 \times 3=6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6=24$ ? If not, give an example to justify your answer.

## Solutions:

No, since 12 and 36 are both divisible by 4 and 6 . But 12 and 36 are not divisible by 24
12. I am the smallest number, having four different prime factors. Can you find me?

Solutions:
Since it is the smallest number. Therefore it will be the product of 4 smallest prime numbers
$2 \times 3 \times 5 \times 7=210$

## EXERCISE 3.6

1. Find the HCF of the following numbers:
(a) 18, 48
(b) 30, 42
(c) 18,60
(d) 27, 63
(e) 36, 84
(f) $\mathbf{3 4}, 102$
(g) 70, 105, 175
(h) 91, 112, 49
(i) $18,54,81$
(j) 12, 45, 75

Solutions:
(a) 18,48

| 2 | 18 |
| :---: | :---: |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 2 | 48 |
| :---: | :---: |
| 2 | 24 |
| 2 | 12 |
| 2 | 6 |
| 3 | 3 |
|  | 1 |

$$
\begin{aligned}
& 18=2 \times 3 \times 3 \\
& 48=2 \times 2 \times 2 \times 2 \times 3 \\
& \mathrm{HCF}=2 \times 3=6
\end{aligned}
$$

Therefore the HCF of 18,48 is 6
(b) 30, 42

| 2 | 30 |
| :---: | :---: |
| 3 | 15 |
| 5 | 5 |
|  | 1 |


| 2 | 42 |
| :---: | :---: |
| 3 | 21 |
| 7 | 7 |
|  | 1 |

$30=2 \times 3 \times 5$
$42=2 \times 3 \times 7$
$\mathrm{HCF}=2 \times 3=6$
Therefore the HCF of 30,42 is 6
(c) 18,60

| 2 | 18 |
| :---: | :---: |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 2 | 60 |
| :---: | :---: |
| 2 | 30 |
| 3 | 15 |
| 5 | 5 |
|  | 1 |

$18=2 \times 3 \times 3$
$60=2 \times 2 \times 3 \times 5$
$\mathrm{HCF}=2 \times 3=6$
Therefore the HCF of 18,60 is 6
(d) 27, 63

| 3 | 27 |
| :---: | :---: |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 3 | 63 |
| :---: | :---: |
| 3 | 21 |
| 7 | 7 |
|  | 1 |

$27=3 \times 3 \times 3$
$63=3 \times 3 \times 7$
$\mathrm{HCF}=3 \times 3=9$
Therefore the HCF of 27, 63 is 9
(e) 36,84

| 2 | 36 |
| :---: | :---: |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 2 | 84 |
| :---: | :---: |
| 2 | 42 |
| 3 | 21 |
| 7 | 7 |
|  | 1 |

$36=2 \times 2 \times 3 \times 3$
$84=2 \times 2 \times 3 \times 7$
$\mathrm{HCF}=2 \times 2 \times 3=12$
Therefore the HCF of 36,84 is 12
(f) 34,102

| 2 | 34 |
| :---: | :---: |
| 17 | 17 |
|  | 1 |


| 2 | 102 |
| :---: | :---: |
| 3 | 51 |
| 17 | 17 |
|  | 1 |

$34=2 \times 17$
$102=2 \times 3 \times 17$
$\mathrm{HCF}=2 \times 17=34$
Therefore the HCF of 34,102 is 34
(g) $70,105,175$

| 2 | 70 |
| :---: | :---: |
| 5 | 35 |
| 7 | 7 |
|  | 1 |


| 3 | 105 |
| :---: | :---: |
| 5 | 35 |
| 7 | 7 |
|  | 1 |


| 5 | 175 |
| :---: | :---: |
| 5 | 35 |
| 7 | 7 |
|  | 1 |

$$
\begin{aligned}
& 70=2 \times 5 \times 7 \\
& 105=3 \times 5 \times 7 \\
& 175=5 \times 5 \times 7 \\
& \mathrm{HCF}=5 \times 7=35
\end{aligned}
$$

Therefore the HCF of $70,105,175$ is 35
(h) $91,112,49$

| 7 | 91 |
| :---: | :---: |
| 13 | 13 |
|  | 1 |


| 2 | 112 |
| :---: | :---: |
| 2 | 56 |
| 2 | 28 |
| 2 | 14 |
| 7 | 7 |
|  | 1 |


| 7 | 49 |
| :---: | :---: |
| 7 | 7 |
|  | 1 |

$91=7 \times 13$
$112=2 \times 2 \times 2 \times 2 \times 7$
$49=7 \times 7$
$\mathrm{HCF}=7$
Therefore the HCF of 91, 112, 49 is 7
(i) $18,54,81$

| 2 | 18 |
| :---: | :---: |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 2 | 54 |
| :---: | :---: |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
|  | 1 |


| 3 | 81 |
| :---: | :---: |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
|  | 1 |

$18=2 \times 3 \times 3$
$54=2 \times 3 \times 3 \times 3$
$81=3 \times 3 \times 3 \times 3$
$\mathrm{HCF}=3 \times 3=9$
Therefore the HCF of $18,54,81$ is 9
(j) $12,45,75$

| 2 | 12 |
| :---: | :---: |
| 2 | 6 |
| 3 | 3 |
|  | 1 |


| 3 | 45 |
| :---: | :---: |
| 3 | 15 |
| 5 | 5 |
|  | 1 |


| 3 | 75 |
| :---: | :---: |
| 5 | 25 |
| 5 | 5 |
|  | 1 |

$$
\begin{aligned}
& 12=2 \times 2 \times 3 \\
& 45=3 \times 3 \times 5 \\
& 75=3 \times 5 \times 5 \\
& \mathrm{HCF}=3
\end{aligned}
$$

Therefore the HCF of 12,45 , and 75 is 3
2. What is the HCF of two consecutive
(a) numbers?
(b) even numbers?
(c) odd numbers?

Solutions:
(a) The HCF of two consecutive numbers is 1

Example: The HCF of 2 and 3 is 1
(b) The HCF of two consecutive even numbers is 2

Example: The HCF of 2 and 4 is 2
(c) The HCF of two consecutive odd numbers is 1

Example: The HCF of 3 and 5 is 1
3. HCF of co-prime numbers 4 and 15 was found as follows by factorisation:
$4=2 \times 2$ and $15=3 \times 5$ since there is no common prime factor, so HCF of 4 and 15 is 0 . Is the answer correct? If not, what is the correct HCF?

Solutions:
No. The answer is not correct. The correct answer is 1 .

## EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg . Find the maximum value of weight, which can measure the weight of the fertiliser exact number of times.

## Solutions:

Given weight of two bags of fertiliser $=75 \mathrm{~kg}$ and 69 kg
Maximum weight $=\mathrm{HCF}$ of two bags weight i.e., $(75,69)$

| 3 | 75 |
| :---: | :---: |
| 5 | 25 |
| 5 | 5 |
|  | 1 |


| 3 | 69 |
| :---: | :---: |
| 23 | 23 |
|  | 1 |

$75=3 \times 5 \times 5$
$69=3 \times 23$
$\mathrm{HCF}=3$
Hence, 3 kg is the maximum value of weight which can measure the weight of the fertiliser exact number of times.
2. Three boys step off together from the same spot. Their steps measure $63 \mathrm{~cm}, 70 \mathrm{~cm}$ and 77 cm , respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

## Solutions:

The first boy's steps measure $=63 \mathrm{~cm}$
The second boy's steps measure $=70 \mathrm{~cm}$
Third boy's steps measure $=77 \mathrm{~cm}$

LCM of 63, 70, 77

| 2 | 63 | 70 | 77 |
| :---: | :---: | :---: | :---: |
| 3 | 63 | 35 | 77 |
| 3 | 21 | 35 | 77 |
| 5 | 7 | 35 | 77 |
| 7 | 7 | 7 | 77 |
| 11 | 1 | 1 | 11 |
|  | 1 | 1 | 1 |

$\mathrm{LCM}=2 \times 3 \times 3 \times 5 \times 7 \times 11=6930$
Hence, 6930 cm is the distance each should cover so that all can cover the distance in complete steps.
3. The length, breadth and height of a room are $825 \mathrm{~cm}, 675 \mathrm{~cm}$ and 450 cm , respectively. Find the longest tape that can measure the room's three dimensions exactly.

## Solutions:

Given the length of a room $=825 \mathrm{~cm}$
The breadth of a room $=675 \mathrm{~cm}$
Height of a room $=450 \mathrm{~cm}$

| 3 | 825 |
| :---: | :---: |
| 5 | 275 |
| 5 | 55 |
| 11 | 11 |
|  | 1 |


| 3 | 675 |
| :---: | :---: |
| 3 | 225 |
| 3 | 75 |
| 5 | 25 |
| 5 | 5 |
|  | 1 |


| 2 | 450 |
| :---: | :---: |
| 3 | 225 |
| 3 | 75 |
| 5 | 25 |
| 5 | 5 |
|  | 1 |

$825=3 \times 5 \times 5 \times 11$
$675=3 \times 3 \times 3 \times 5 \times 5$
$450=2 \times 3 \times 3 \times 5 \times 5$
$\mathrm{HCF}=3 \times 5 \times 5=75 \mathrm{~cm}$
Hence longest tape is 75 cm which can measure the three dimensions of the room exactly.
4. Determine the smallest 3-digit number, which is exactly divisible by 6,8 and 12 .

## Solutions:

LCM of $6,8,12=$ smallest number

| 2 | 6 | 8 | 12 |
| :---: | :---: | :---: | :---: |
| 2 | 3 | 4 | 6 |
| 2 | 3 | 2 | 3 |
| 3 | 3 | 1 | 3 |
|  | 1 | 1 | 1 |

$\mathrm{LCM}=2 \times 2 \times 2 \times 3=24$
Now we need to find the smallest 3-digit multiple of 24
We know that $24 \times 4=96$ and $24 \times 5=120$
Hence, 120 is the smallest 3-digit number which is exactly divisible by 6,8 and 12
5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12 .

Solutions:
LCM of 8,10 and 12

| 2 | 8 | 10 | 12 |
| :---: | :---: | :---: | :---: |
| 2 | 4 | 5 | 6 |
| 2 | 2 | 5 | 3 |
| 3 | 1 | 5 | 3 |
| 5 | 1 | 5 | 1 |
|  | 1 | 1 | 1 |

$\mathrm{LCM}=2 \times 2 \times 2 \times 3 \times 5=120$
Now we need to find the greatest 3-digit multiple of 120
We may find $120 \times 8=960$ and $120 \times 9=1080$
Hence, 960 is the greatest 3-digit number exactly divisible by 8,10 and 12
6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds, respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

Solutions:
LCM of $48,72,108=$ time period after which these lights change

| 2 | 48 | 72 | 108 |
| :---: | :---: | :---: | :---: |
| 2 | 24 | 36 | 54 |
| 2 | 12 | 18 | 27 |
| 2 | 6 | 9 | 27 |
| 3 | 3 | 9 | 27 |
| 3 | 1 | 3 | 9 |
| 3 | 1 | 1 | 3 |
|  | 1 | 1 | 1 |

LCM $=2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3=432$
Hence, lights will change together after every 432 seconds
Therefore the lights will change simultaneously at 7 minutes and 12 seconds.
7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel, respectively. Find the maximum capacity of a container that can measure the diesel of the three containers the exact number of times.

Solutions:
HCF of $403,434,465=$ Maximum capacity of tanker required
$403=13 \times 31$
$434=2 \times 7 \times 31$
$465=3 \times 5 \times 31$
$\mathrm{HCF}=31$

Hence, a container of 31 litres can measure the diesel of the three containers the exact number of times.
8. Find the least number, which, when divided by 6,15 and 18 leave the remainder 5 in each case.

Solutions:
LCM of 6, 15, 18

| 2 | 6 | 15 | 18 |
| :---: | :---: | :---: | :---: |
| 3 | 3 | 15 | 9 |
| 3 | 1 | 5 | 3 |
| 5 | 1 | 5 | 1 |
|  | 1 | 1 | 1 |

LCM $=2 \times 3 \times 3 \times 5=90$
Required number $=90+5$
$=95$

Hence, 95 is the required number.
9. Find the smallest 4-digit number, which is divisible by 18,24 and 32 .

## Solutions:

LCM of 18, 24, 32

| 2 | 18 | 24 | 32 |
| :---: | :---: | :---: | :---: |
| 2 | 9 | 12 | 16 |
| 2 | 9 | 6 | 8 |
| 2 | 9 | 3 | 4 |
| 2 | 9 | 3 | 2 |
| 3 | 9 | 3 | 1 |
| 3 | 3 | 1 | 1 |
|  | 1 | 1 | 1 |

LCM $=2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3=288$
Here, we need to find the smallest 4-digit multiple of 288
We find $288 \times 3=864$ and $288 \times 4=1152$
Hence, 1152 is the smallest 4-digit number which is divisible by 18,24 and 32
10. Find the LCM of the following numbers:
(a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?
Solutions:
(a) LCM of 9,4

| 2 | 9 | 4 |
| :--- | :--- | :--- |
| 2 | 9 | 2 |
| 3 | 9 | 1 |
| 3 | 3 | 1 |
|  | 1 | 1 |

LCM $=2 \times 2 \times 3 \times 3=36$
(b) LCM of 12,5

| 2 | 12 | 5 |
| :---: | :---: | :---: |
| 2 | 6 | 5 |
| 3 | 3 | 5 |
| 5 | 1 | 5 |
|  | 1 | 1 |

LCM $=2 \times 2 \times 3 \times 5=60$
(c) LCM of 6,5

| 2 | 6 | 5 |
| :--- | :--- | :--- |
| 3 | 3 | 5 |
| 5 | 1 | 5 |
|  | 1 | 1 |

$\mathrm{LCM}=2 \times 3 \times 5=30$
(d) LCM of 15,4

| 2 | 15 | 4 |
| :---: | :---: | :---: |
| 2 | 15 | 2 |
| 3 | 15 | 1 |
| 5 | 5 | 1 |
|  | 1 | 1 |

LCM $=2 \times 2 \times 3 \times 5=60$
Yes in each case the LCM of given numbers is the product of these numbers.
11. Find the LCM of the following numbers in which one number is the factor of the other.
(a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

What do you observe in the results obtained?
Solutions:
(a) 5,20

| 2 | 5 | 20 |
| :---: | :---: | :---: |
| 2 | 5 | 10 |
| 5 | 5 | 5 |
|  | 1 | 1 |

$\mathrm{LCM}=2 \times 2 \times 5=20$
(b) 6, 18

| 2 | 6 | 18 |
| :---: | :---: | :---: |
| 3 | 3 | 9 |
| 3 | 1 | 3 |
|  | 1 | 1 |

$\mathrm{LCM}=2 \times 3 \times 3=18$
(c) 12,48

| 2 | 12 | 48 |
| :---: | :---: | :---: |
| 2 | 6 | 24 |
| 2 | 3 | 12 |
| 2 | 3 | 6 |
| 3 | 3 | 3 |
|  | 1 | 1 |

LCM $=2 \times 2 \times 2 \times 2 \times 3=48$
(d) 9,45

| 3 | 9 | 45 |
| :---: | :---: | :---: |
| 3 | 3 | 15 |
| 5 | 1 | 5 |
|  | 1 | 1 |

LCM $=3 \times 3 \times 5=45$
$\therefore$ Hence, in each case the LCM of given numbers is the larger number. When a number is a factor of other number then their LCM will be the larger number.

## Disclaimer:

Dropped Topic - 3.6 Some more divisibility rules.

