EXERCISE 14.2

1. Factorise the following expressions.

(i) $a^2+8a+16$

(ii) $p^2-10p+25$

(iii) $25m^2+30m+9$

(iv) $49y^2+84yz+36z^2$

(v) $4x^2-8x+4$

(vi) $121b^2-88bc+16c^2$

(vii) $(l+m)^2-4lm$ **(Hint: Expand** $(l+m)^2$ **first)**

(viii) $a^4+2a^2b^2+b^4$

Solution:

(i) $a^2+8a+16$

$= a^2+2\times4\times a+4^2$

$= (a+4)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(ii) $p^2-10p+25$

$= p^2-2\times5\times p+5^2$

$= (p-5)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(iii) $25m^2+30m+9$

$= (5m)^2+2\times5m\times3+3^2$

$= (5m+3)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(iv) $49y^2+84yz+36z^2$

$=(7y)^2+2\times7y\times6z+(6z)^2$

$= (7y+6z)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(v) $4x^2–8x+4$

$= (2x)^2-2×4x+2^2$

$= (2x-2)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(vi) $121b^2-88bc+16c^2$

$= (11b)^2-2×11b×4c+(4c)^2$

$= (11b-4c)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(vii) $(l+m)^2-4lm$ (Hint: Expand $(l+m)^2$ first)

Expand $(l+m)^2$ using the identity $(x+y)^2 = x^2+2xy+y^2$

$(l+m)^2-4lm = l^2+m^2+2lm-4lm$

$= l^2+m^2-2lm$

$= (l-m)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(viii) $a^4+2a^2b^2+b^4$

$= (a^2)^2+2×a^{2×}b^2+(b^2)^2$

$= (a^2+b^2)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

2. Factorise.

(i) $4p^2–9q^2$

(ii) $63a^2–112b^2$

(iii) $49x^2–36$

(iv) $16x^5–144x^3$ differ

(v) $(l+m)^2-(l-m)^2$

(vi) $9x^2y^2–16$

(vii) $(x^2–2xy+y^2)–z^2$

(viii) $25a^2–4b^2+28bc–49c^2$

Solution:

(i) $4p^2–9q^2$

$= (2p)^2-(3q)^2$

$= (2p-3q)(2p+3q)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(ii) $63a^2–112b^2$

$= 7(9a^2 –16b^2)$

$= 7((3a)^2–(4b)^2)$

$= 7(3a+4b)(3a-4b)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(iii) $49x^2–36$

$= (7x)^2 -6^2$

$= (7x+6)(7x–6)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(iv) $16x^5–144x^3$

$= 16x^3(x^2–9)$

$= 16x^3(x^2–9)$

$= 16x^3(x–3)(x+3)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(v) $(l+m)^2-(l-m)^2$

$= \{(l+m)-(l–m)\}\{(l +m)+(l–m)\}$

Using the identity $x^2-y^2 = (x+y)(x-y)$

$= (l+m–l+m)(l+m+l–m)$

$= (2m)(2l)$

$= 4\ ml$

(vi) $9x^2y^2–16$

$= (3xy)^2 - 4^2$

$= (3xy-4)(3xy+4)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(vii) $(x^2-2xy+y^2)-z^2$

$= (x-y)^2-z^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

$= \{(x-y)-z\}\{(x-y)+z\}$

$= (x-y-z)(x-y+z)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(viii) $25a^2-4b^2+28bc-49c^2$

$= 25a^2-(4b^2-28bc+49c^2)$

$= (5a)^2-\{(2b)^2-2(2b)(7c)+(7c)^2\}$

$= (5a)^2-(2b-7c)^2$

Using the identity $x^2-y^2 = (x+y)(x-y)$, we have

$= (5a+2b-7c)(5a-2b+7c)$

3. Factorise the expressions.

(i) ax^2+bx

(ii) $7p^2+21q^2$

(iii) $2x^3+2xy^2+2xz^2$

(iv) $am^2+bm^2+bn^2+an^2$

(v) $(lm+l)+m+1$

(vi) $y(y+z)+9(y+z)$

(vii) $5y^2-20y-8z+2yz$

(viii) $10ab+4a+5b+2$

(ix)$6xy-4y+6-9x$

Solution:

(i) $ax^2+bx = x(ax+b)$

(ii) $7p^2+21q^2 = 7(p^2+3q^2)$

(iii) $2x^3+2xy^2+2xz^2 = 2x(x^2+y^2+z^2)$

(iv) $am^2+bm^2+bn^2+an^2 = m^2(a+b)+n^2(a+b) = (a+b)(m^2+n^2)$

(v) $(lm+l)+m+1 = lm+m+l+1 = m(l+1)+(l+1) = (m+1)(l+1)$

(vi) $y(y+z)+9(y+z) = (y+9)(y+z)$

(vii) $5y^2–20y–8z+2yz = 5y(y–4)+2z(y–4) = (y–4)(5y+2z)$

(viii) $10ab+4a+5b+2 = 5b(2a+1)+2(2a+1) = (2a+1)(5b+2)$

(ix) $6xy–4y+6–9x = 6xy–9x–4y+6 = 3x(2y–3)–2(2y–3) = (2y–3)(3x–2)$

4.Factorise.

(i) $a^4–b^4$

(ii) $p^4–81$

(iii) $x^4–(y+z)^4$

(iv) $x^4–(x–z)^4$

(v) $a^4–2a^2b^2+b^4$

Solution:

(i) $a^4–b^4$

$= (a^2)^2-(b^2)^2$

$= (a^2-b^2)\ (a^2+b^2)$

$= (a – b)(a + b)(a^2+b^2)$

(ii) $p^4–81$

$= (p^2)^2-(9)^2$

$= (p^2-9)(p^2+9)$

$= (p^2-3^2)(p^2+9)$

$=(p-3)(p+3)(p^2+9)$

(iii) $x^4–(y+z)^4 = (x^2)^2-[(y+z)^2]^2$

$= \{x^2-(y+z)^2\}\{ x^2+(y+z)^2\}$

$= \{(x –(y+z)(x+(y+z)\}\{x^2+(y+z)^2\}$

$= (x-y-z)(x+y+z)\{x^2+(y+z)^2\}$

(iv) $x^4-(x-z)^4 = (x^2)^2-\{(x-z)^2\}^2$

$= \{x^2-(x-z)^2\}\{x^2+(x-z)^2\}$

$= \{x-(x-z)\}\{x+(x-z)\}\{x^2+(x-z)^2\}$

$= z(2x-z)(x^2+x^2-2xz+z^2)$

$= z(2x-z)(2x^2-2xz+z^2)$

(v) $a^4-2a^2b^2+b^4 = (a^2)^2-2a^2b^2+(b^2)^2$

$= (a^2-b^2)^2$

$= ((a-b)(a+b))^2$

$= (a-b)^2(a+b)^2$

5. Factorise the following expressions.

(i) p^2+6p+8

(ii) $q^2-10q+21$

(iii) $p^2+6p-16$

Solution:

(i) p^2+6p+8

We observed that $8 = 4\times2$ and $4+2 = 6$

p^2+6p+8 can be written as $p^2+2p+4p+8$

Taking Common terms, we get

$p^2+6p+8 = p^2+2p+4p+8 = p(p+2)+4(p+2)$

Again, $p+2$ is common in both the terms.

$= (p+2)(p+4)$

This implies that $p^2+6p+8 = (p+2)(p+4)$

(ii) $q^2-10q+21$

We observed that $21 = -7\times-3$ and $-7+(-3) = -10$

$q^2-10q+21 = q^2-3q-7q+21$

$= q(q-3)-7(q-3)$

$= (q-7)(q-3)$

This implies that $q^2-10q+21 = (q-7)(q-3)$

(iii) $p^2+6p-16$

We observed that $-16 = -2\times8$ and $8+(-2) = 6$

$p^2+6p-16 = p^2-2p+8p-16$

$= p(p-2)+8(p-2)$

$= (p+8)(p-2)$

So, $p^2+6p-16 = (p+8)(p-2)$