EXERCISE 9.5

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1. Use a suitable identity to get each of the following products.

(i) (x + 3) (x + 3)

(ii) (2y + 5) (2y + 5)

(iii) (2a – 7) (2a – 7)

(iv) (3a – 1/2)(3a – 1/2)

(v) (1.1m – 0.4) (1.1m + 0.4)

(vi) $(a^2+ b^2) (- a^2+ b^2)$

(vii) (6x – 7) (6x + 7)

(viii) (- a + c) (- a + c)

(ix) (1/2x + 3/4y) (1/2x + 3/4y)

(x) (7a – 9b) (7a – 9b)

Solution:

(i) $(x + 3) (x + 3) = (x + 3)^2$

$= x^2 + 6x + 9$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

ii) $(2y + 5) (2y + 5) = (2y + 5)^2$

$= 4y^2 + 20y + 25$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

iii) $(2a – 7) (2a – 7) = (2a – 7)^2$

$= 4a^2 – 28a + 49$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

iv) $(3a – 1/2)(3a – 1/2) = (3a – 1/2)^2$

$= 9a^2 -3a+(1/4)$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

v) (1.1m – 0.4) (1.1m + 0.4)

$= 1.21m^2 – 0.16$

Using $(a – b)(a + b) = a^2 – b^2$

vi) $(a^2 + b^2) (– a^2 + b^2)$

$= (b^2 + a^2) (b^2 – a^2)$

$= -a^4 + b^4$

Using $(a – b)(a + b) = a^2 – b^2$

vii) $(6x – 7) (6x + 7)$

$=36x^2 – 49$

Using $(a – b)(a + b) = a^2 – b^2$

viii) $(– a + c) (– a + c) = (– a + c)^2$

$= c^2 + a^2 – 2ac$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

ix) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$

$= (x^2/4) + (9y^2/16) + (3xy/4)$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

x) $(7a – 9b) (7a – 9b) = (7a – 9b)^2$

$= 49a^2 – 126ab + 81b^2$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

2. Use the identity $(x + a) (x + b) = x^2 + (a + b) x + ab$ to find the following products.

(i) $(x + 3) (x + 7)$

(ii) $(4x + 5) (4x + 1)$

(iii) $(4x – 5) (4x – 1)$

(iv) $(4x + 5) (4x – 1)$

(v) $(2x + 5y) (2x + 3y)$

(vi) $(2a^2 + 9) (2a^2 + 5)$

(vii) (xyz – 4) (xyz – 2)

Solution:

(i)(x + 3) (x + 7)

= x^2 + (3+7)x + 21

= x^2 + 10x + 21

ii) (4x + 5) (4x + 1)

= $16x^2$ + 4x + 20x + 5

= $16x^2$ + 24x + 5

iii) (4x – 5) (4x – 1)

= $16x^2$ – 4x – 20x + 5

= $16x^2$ – 24x + 5

iv) (4x + 5) (4x – 1)

= $16x^2$ + (5-1)4x – 5

= $16x^2$ +16x – 5

v) (2x + 5y) (2x + 3y)

= $4x^2$ + (5y + 3y)2x + $15y^2$

= $4x^2$ + 16xy + $15y^2$

vi) ($2a^2$+ 9) ($2a^2$+ 5)

= $4a^4$ + (9+5)$2a^2$ + 45

= $4a^4$ + $28a^2$ + 45

vii) (xyz – 4) (xyz – 2)

= $x^2y^2z^2$ + (-4 -2)xyz + 8

= $x^2y^2z^2$ – 6xyz + 8

3. Find the following squares by using the identities.

(i) $(b – 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 – 5y)^2$

(iv) $[(2m/3) + (3n/2)]^2$

(v) $(0.4p – 0.5q)^2$

(vi) $(2xy + 5y)^2$

Solution:

Using identities:

$(a – b)^2 = a^2 + b^2 – 2ab$ $(a + b)^2 = a^2 + b^2 + 2ab$

(i) $(b – 7)^2 = b^2 – 14b + 49$

(ii) $(xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 – 5y)^2 = 36x^4 – 60x^2y + 25y^2$

(iv) $[(2m/3\}) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$

(v) $(0.4p – 0.5q)^2 = 0.16p^2 – 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = 4x^2y^2 + 20xy^2 + 25y^2$

4. Simplify.

(i) $(a^2 – b^2)^2$

(ii) $(2x + 5)^2 – (2x – 5)^2$

(iii) $(7m – 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$

(vi) $(ab + bc)^2 – 2ab^2c$

(vii) $(m^2 – n^2m)^2 + 2m^3n^2$

Solution:

i) $(a^2 – b^2)^2 = a^4 + b^4 – 2a^2b^2$

ii) $(2x + 5)^2 – (2x – 5)^2$
$= 4x^2 + 20x + 25 – (4x^2 – 20x + 25) = 4x^2 + 20x + 25 – 4x^2 + 20x – 25 = 40x$

iii) $(7m – 8n)^2 + (7m + 8n)^2$
$= 49m^2 – 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$

iv) $(4m + 5n)^2 + (5m + 4n)^2$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$

v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
$= 6.25p^2 – 7.5pq + 2.25q^2 – 2.25p^2 + 7.5pq – 6.25q^2$
$= 4p^2 – 4q^2$

vi) $(ab + bc)^2 – 2ab^2c = a^2b^2 + 2ab^2c + b^2c^2 – 2ab^2c = a^2b^2 + b^2c^2$

vii) $(m^2 – n^2m)^2 + 2m^3n^2$
$= m^4 – 2m^3n^2 + m^2n^4 + 2m^3n^2$
$= m^4 + m^2n^4$

5. Show that.

(i) $(3x + 7)^2 – 84x = (3x – 7)^2$

(ii) $(9p – 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $(4/3m – 3/4n)^2 + 2mn = 16/9\ m^2 + 9/16\ n^2$

(iv) $(4pq + 3q)^2 – (4pq – 3q)^2 = 48pq^2$

(v) $(a – b)(a + b) + (b – c)(b + c) + (c – a)(c + a) = 0$

Solution:

i) LHS = $(3x + 7)^2 – 84x$

$= 9x^2 + 42x + 49 – 84x$
$= 9x^2 – 42x + 49$
= RHS

LHS = RHS

ii) LHS = $(9p – 5q)^2 + 180pq$
$= 81p^2 – 90pq + 25q^2 + 180pq$
$= 81p^2 + 90pq + 25q^2$
RHS = $(9p + 5q)^2$
$= 81p^2 + 90pq + 25q^2$
LHS = RHS

(iii) $LHS = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$
$= \frac{16}{9}m^2 + \frac{9}{16}n^2$
= RHS

LHS = RHS

iv) LHS = $(4pq + 3q)^2 – (4pq – 3q)^2$

= $16p^2q^2 + 24pq^2 + 9q^2 – 16p^2q^2 + 24pq^2 – 9q^2$

= $48pq^2$

= RHS

LHS = RHS

v) LHS = (a – b) (a + b) + (b – c) (b + c) + (c – a) (c + a)

= $a^2 – b^2 + b^2 – c^2 + c^2 – a^2$

= 0

= RHS

6. Using identities, evaluate.

(i) 71^2

(ii) 99^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(vi) 297 x 303

(vii) 78 x 82

(viii) 8.9^2

(ix) 10.5 x 9.5

Solution:

i) 71^2

= $(70+1)^2$

= $70^2 + 140 + 1^2$

= 4900 + 140 +1

= 5041

ii) 99^2

= $(100 -1)^2$

$= 100^2 – 200 + 1^2$

$= 10000 – 200 + 1$

$= 9801$

iii) 102^2

$= (100 + 2)^2$

$= 100^2 + 400 + 2^2$

$= 10000 + 400 + 4 = 10404$

iv) 998^2

$= (1000 – 2)^2$

$= 1000^2 – 4000 + 2^2$

$= 1000000 – 4000 + 4$

$= 996004$

v) 5.2^2

$= (5 + 0.2)^2$

$= 5^2 + 2 + 0.2^2$

$= 25 + 2 + 0.04 = 27.04$

vi) 297 x 303

$= (300 – 3)(300 + 3)$

$= 300^2 – 3^2$

$= 90000 – 9$

$= 89991$

vii) 78 x 82

$= (80 – 2)(80 + 2)$

$= 80^2 – 2^2$

$= 6400 – 4$

$= 6396$

viii) 8.9^2

$= (9 – 0.1)^2$

$= 9^2 – 1.8 + 0.1^2$

$= 81 – 1.8 + 0.01$

$= 79.21$

ix) 10.5 x 9.5

$= (10 + 0.5)(10 – 0.5)$

$= 10^2 – 0.5^2$

$= 100 – 0.25$

$= 99.75$

7. Using $a^2 – b^2 = (a + b) (a – b)$, find

(i) $51^2 – 49^2$

(ii) $(1.02)^2 – (0.98)^2$

(iii) $153^2 – 147^2$

(iv) $12.1^2 – 7.9^2$

Solution:

i) $51^2 – 49^2$

$= (51 + 49)(51 – 49) = 100 \times 2 = 200$

ii) $(1.02)^2 – (0.98)^2$

$= (1.02 + 0.98)(1.02 – 0.98) = 2 \times 0.04 = 0.08$

iii) $153^2 – 147^2$

$= (153 + 147)(153 – 147) = 300 \times 6 = 1800$

iv) $12.1^2 – 7.9^2$

$= (12.1 + 7.9)(12.1 – 7.9) = 20 \times 4.2 = 84$

8. Using $(x + a) (x + b) = x^2 + (a + b) x + ab$, find

(i) 103 x 104

(ii) 5.1 x 5.2

(iii) 103 x 98

(iv) 9.7 x 9.8

Solution:

i) 103 x 104

= (100 + 3)(100 + 4)

= 100^2 + (3 + 4)100 + 12

= 10000 + 700 + 12

= 10712

ii) 5.1 x 5.2

= (5 + 0.1)(5 + 0.2)

= 5^2 + (0.1 + 0.2)5 + 0.1 x 0.2

= 25 + 1.5 + 0.02

= 26.52

iii) 103 x 98

= (100 + 3)(100 – 2)

= 100^2 + (3-2)100 – 6

= 10000 + 100 – 6

= 10094

iv) 9.7 x 9.8

= (9 + 0.7)(9 + 0.8)

= 9^2 + (0.7 + 0.8)9 + 0.56

= 81 + 13.5 + 0.56

= 95.06