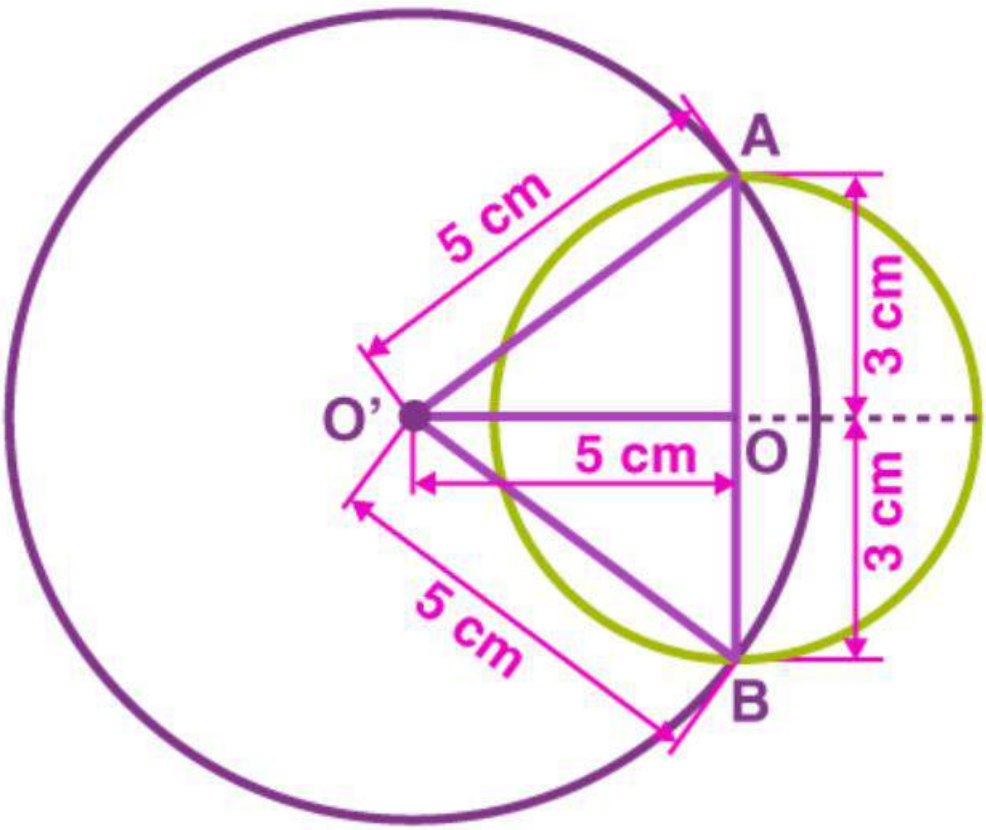
EXERCISE: 10.4 (PAGE NO: 179)

1. Two circles of radii 5 cm and 3 cm intersect at two points, and the distance between their centres is 4 cm. Find the length of the common chord.

Solution:

The perpendicular bisector of the common chord passes through the centres of both circles.



As the circles intersect at two points, we can construct the above figure.

Consider AB as the common chord and O and O' as the centres of the circles.

O'A = 5 cm

OA = 3 cm

OO' = 4 cm [Distance between centres is 4 cm.]

As the radius of the bigger circle is more than the distance between the two centres, we know that the centre of the smaller circle lies inside the bigger circle.

The perpendicular bisector of AB is OO'.

OA = OB = 3 cm

As O is the midpoint of AB

AB = 3 cm + 3 cm = 6 cm

The length of the common chord is 6 cm.

It is clear that the common chord is the diameter of the smaller circle.

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution:

Let AB and CD be two equal cords (i.e., AB = CD). In the above question, it is given that AB and CD intersect at a point, say, E.

It is now to be proven that the line segments AE = DE and CE = BE

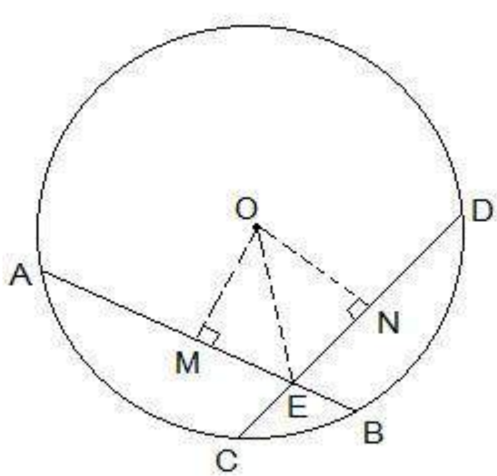
Construction Steps

Step 1: From the centre of the circle, draw a perpendicular to AB, i.e., OM ⊥ AB.

Step 2: Similarly, draw ON ⊥ CD.

Step 3: Join OE.

Now, the diagram is as follows:



Proof:

From the diagram, it is seen that OM bisects AB, and so OM ⊥ AB

Similarly, ON bisects CD, and so ON ⊥ CD.

It is known that AB = CD. So,

AM = ND — (i)

and MB = CN — (ii)

Now, triangles ΔOME and ΔONE are similar by RHS congruency, since

∠OME = ∠ONE (They are perpendiculars.)

OE = OE (It is the common side.)

OM = ON (AB and CD are equal, and so they are equidistant from the centre.)

∴ ΔOME ≅ ΔONE

ME = EN (by CPCT) — (iii)

Now, from equations (i) and (ii), we get

AM+ME = ND+EN

So, AE = ED

Now from equations (ii) and (iii), we get

MB-ME = CN-EN

So, EB = CE (Hence, proved)

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution:

From the question, we know the following:

(i) AB and CD are 2 chords which are intersecting at point E.

(ii) PQ is the diameter of the circle.

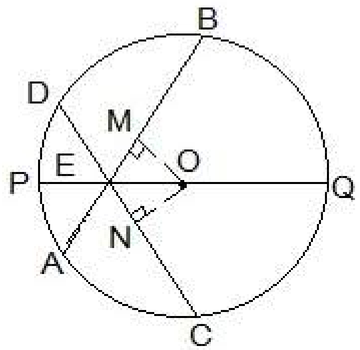
(iii) AB = CD.

Now, we will have to prove that **∠BEQ = ∠CEQ**

For this, the following construction has to be done.

Construction:

Draw two perpendiculars are drawn as OM ⊥ AB and ON ⊥ D. Now, join OE. The constructed diagram will look as follows:



Now, consider the triangles ΔOEM and ΔOEN.

Here,

(i) OM = ON [The equal chords are always equidistant from the centre.]

(ii) OE = OE [It is the common side.]

(iii) ∠OME = ∠ONE [These are the perpendiculars.]

So, by RHS congruency criterion, ΔOEM ≅ ΔOEN.

Hence, by the CPCT rule, ∠MEO = ∠NEO

∴ ∠BEQ = ∠CEQ (Hence, proved)

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 10.25).

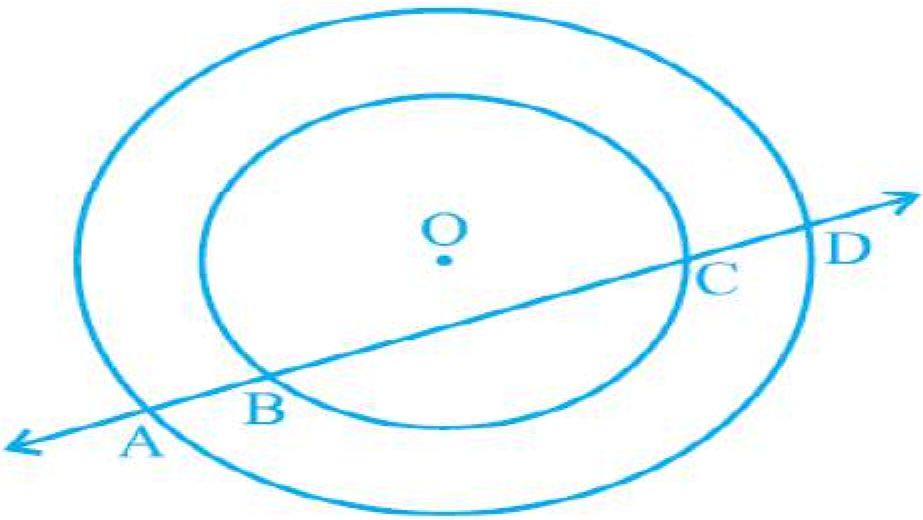


Fig. 10.25

Solution:

The given image is as follows:

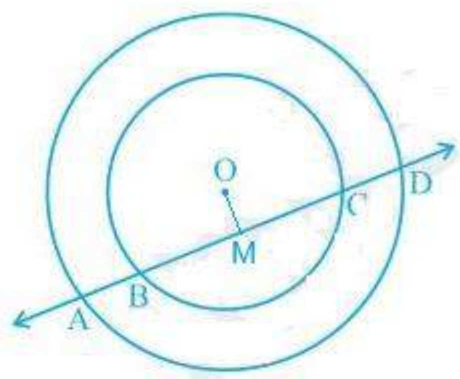


Fig. 10.25

First, draw a line segment from O to AD, such that OM ⊥ AD.

So, now OM is bisecting AD since OM ⊥ AD.

Therefore, AM = MD — (i)

Also, since OM ⊥ BC, OM bisects BC.

Therefore, BM = MC — (ii)

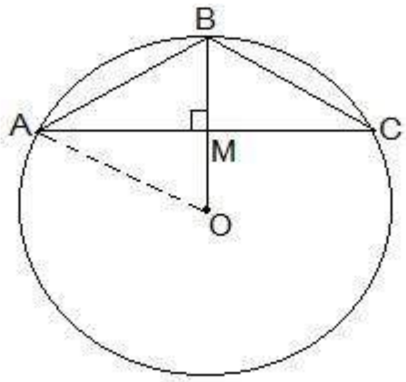
From equation (i) and equation (ii),

AM-BM = MD-MC

∴ AB = CD

5. Three girls, Reshma, Salma and Mandip, are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, and Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution:



Let the positions of Reshma, Salma and Mandip be represented as A, B and C, respectively.

From the question, we know that AB = BC = 6cm

So, the radius of the circle, i.e., OA = 5cm

Now, draw a perpendicular BM ⊥ AC.

Since AB = BC, ABC can be considered an isosceles triangle. M is the mid-point of AC. BM is the perpendicular bisector of AC, and thus it passes through the centre of the circle.

Now,

let AM = y and

OM = x

So, BM will be = (5-x).

By applying the Pythagorean theorem in ΔOAM, we get

$OA^2 = OM^2 + AM^2$

$\Rightarrow 5^2 = x^2 + y^2$ — (i)

Again, by applying the Pythagorean theorem in ΔAMB,

$AB^2 = BM^2 + AM^2$

$\Rightarrow 6^2 = (5-x)^2 + y^2$ — (ii)

Subtracting equation (i) from equation (ii), we get

$36-25 = (5-x)^2 + y^2 - x^2 - y^2$

Now, solving this equation, we get the value of x as

x = 7/5

Substituting the value of x in equation (i), we get

$y^2 + (49/25) = 25$

$\Rightarrow y^2 = 25 – (49/25)$

Solving it, we get the value of y as

$y = 24/5$

Thus,

AC = 2×AM

= 2×y

= 2×(24/5) m

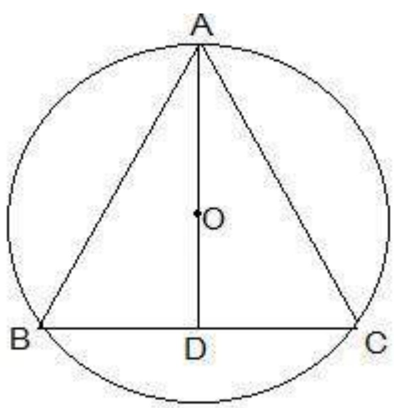
AC = 9.6 m

So, the distance between Reshma and Mandip is 9.6 m.

6. A circular park of radius 20m is situated in a colony. Three boys, Ankur, Syed and David, are sitting at equal distances on its boundary, each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Solution:

First, draw a diagram according to the given statements. The diagram will look as follows:



Here, the positions of Ankur, Syed and David are represented as A, B and C, respectively. Since they are sitting at equal distances, the triangle ABC will form an equilateral triangle.

AD ⊥ BC is drawn. Now, AD is the median of ΔABC, and it passes through the centre O.

Also, O is the centroid of the ΔABC. OA is the radius of the triangle.

OA = 2/3 AD

Let the side of a triangle a metres, then BD = a/2 m.

Applying Pythagoras' theorem in ΔABD,

$AB^2 = BD^2 + AD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow AD^2 = a^2 - (a/2)^2$

$\Rightarrow AD^2 = 3a^2/4$

$\Rightarrow AD = \sqrt{3}a/2$

OA = 2/3 AD

$20 \text{ m} = 2/3 \times \sqrt{3}a/2$

$a = 20\sqrt{3}$ m

So, the length of the string of the toy is $20\sqrt{3}$ m.