1. A park, in the shape of a quadrilateral ABCD , has $\mathrm{C}=90^{\circ}, \mathrm{AB}=9 \mathrm{~m}, \mathrm{BC}=12 \mathrm{~m}, \mathrm{CD}=5 \mathrm{~m}$ and $\mathrm{AD}=\mathbf{8} \mathrm{m}$. How much area does it occupy?

## Solution:

First, construct a quadrilateral ABCD and join BD .
We know that
$\mathrm{C}=90^{\circ}, \mathrm{AB}=9 \mathrm{~m}, \mathrm{BC}=12 \mathrm{~m}, \mathrm{CD}=5 \mathrm{~m}$ and $\mathrm{AD}=8 \mathrm{~m}$
The diagram is:


Now, apply Pythagoras theorem in $\triangle B C D$
$\mathrm{BD}^{2}=\mathrm{BC}^{2}+\mathrm{CD}^{2}$
$\Rightarrow \mathrm{BD}^{2}=12^{2}+5^{2}$
$\Rightarrow \mathrm{BD}^{2}=169$
$\Rightarrow \mathrm{BD}=13 \mathrm{~m}$
Now, the area of $\Delta \mathrm{BCD}=(1 / 2 \times 12 \times 5)=30 \mathrm{~m}^{2}$
The semi perimeter of $\triangle A B D$
$(\mathrm{s})=($ perimeter $/ 2)$
$=(8+9+13) / 2 \mathrm{~m}$
$=30 / 2 \mathrm{~m}=15 \mathrm{~m}$
Using Heron's formula,
Area of $\triangle A B D$

$$
\begin{aligned}
& \sqrt{s(s-a)(s-b)(s-c)} \\
& =\sqrt{15(15-13)(15-9)(15-8)} \mathrm{m}^{2} \\
& =\sqrt{15 \times 2 \times 6 \times 7} \mathrm{~m}^{2}
\end{aligned}
$$

$=6 \sqrt{ } 35 \mathrm{~m}^{2}=35.5 \mathrm{~m}^{2}$ (approximately)
$\therefore$ The area of quadrilateral $A B C D=$ Area of $\triangle B C D+$ Area of $\triangle A B D$
$=30 \mathrm{~m}^{2}+35.5 \mathrm{~m}^{2}=65.5 \mathrm{~m}^{2}$
2. Find the area of a quadrilateral ABCD in which $\mathrm{AB}=3 \mathrm{~cm}, \mathrm{BC}=4 \mathrm{~cm}, \mathrm{CD}=4 \mathrm{~cm}, \mathrm{DA}=5 \mathrm{~cm}$ and $\mathrm{AC}=5$ cm.

## Solution:

First, construct a diagram with the given parameter.


Now, apply Pythagorean theorem in $\triangle \mathrm{ABC}$,
$\mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}$
$\Rightarrow 5^{2}=3^{2}+4^{2}$
$\Rightarrow 25=25$

Thus, it can be concluded that $\Delta \mathrm{ABC}$ is a right angled at B .
So, area of $\triangle \mathrm{BCD}=(1 / 2 \times 3 \times 4)=6 \mathrm{~cm}^{2}$
The semi perimeter of $\triangle \mathrm{ACD}(\mathrm{s})=($ perimeter $/ 2)=(5+5+4) / 2 \mathrm{~cm}=14 / 2 \mathrm{~cm}=7 \mathrm{~m}$
Now, using Heron's formula,
Area of $\triangle \mathrm{ACD}$

$$
\begin{aligned}
& \sqrt{s(s-a)(s-b)(s-c)} \\
= & {[\sqrt{7(7-5)(7-5)(7-4)}] \mathrm{cm}^{2} } \\
= & (\sqrt{7 \times 2 \times 2 \times 3}) \mathrm{cm}^{2}
\end{aligned}
$$

$=2 \sqrt{21} \mathrm{~cm}^{2}=9.17 \mathrm{~cm}^{2}$ (approximately)
Area of quadrilateral $\mathrm{ABCD}=$ Area of $\triangle \mathrm{ABC}+$ Area of $\triangle \mathrm{ACD}=6 \mathrm{~cm}^{2}+9.17 \mathrm{~cm}^{2}=15.17 \mathrm{~cm}^{2}$
3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15. Find the total area of the paper used.


Fig. 12.15
Solution:
For the triangle I section:


It is an isosceles triangle and the sides are $5 \mathrm{~cm}, 1 \mathrm{~cm}$ and 5 cm

Perimeter $=5+5+1=11 \mathrm{~cm}$

So, semi perimeter $=11 / 2 \mathrm{~cm}=5.5 \mathrm{~cm}$

Using Heron's formula,
Area $=\sqrt{ }[\mathrm{s}(\mathrm{s}-\mathrm{a})(\mathrm{s}-\mathrm{b})(\mathrm{s}-\mathrm{c})]$
$=\sqrt{ }[5.5(5.5-5)(5.5-5)(5.5-1)] \mathrm{cm}^{2}$
$=\sqrt{ }[5.5 \times 0.5 \times 0.5 \times 4.5] \mathrm{cm}^{2}$
$=0.75 \sqrt{ } 11 \mathrm{~cm}^{2}$
$=0.75 \times 3.317 \mathrm{~cm}^{2}$
$=2.488 \mathrm{~cm}^{2}$ (approx)

For the quadrilateral II section:
This quadrilateral is a rectangle with length and breadth as 6.5 cm and 1 cm respectively.
$\therefore$ Area $=6.5 \times 1 \mathrm{~cm}^{2}=6.5 \mathrm{~cm}^{2}$

For the quadrilateral III section:
It is a trapezoid with 2 sides as 1 cm each and the third side as 2 cm .
Area of the trapezoid = Area of the parallelogram + Area of the equilateral triangle

The perpendicular height of the parallelogram will be

$$
\begin{aligned}
& \left(\sqrt{1^{2}-(0.5)^{2}}\right) \\
= & 0.86 \mathrm{~cm}
\end{aligned}
$$

And, the area of the equilateral triangle will be $\left(\sqrt{ } 3 / 4 \times \mathrm{a}^{2}\right)=0.43$
$\therefore$ Area of the trapezoid $=0.86+0.43=1.3 \mathrm{~cm}^{2}($ approximately $)$.
For triangle IV and V:
These triangles are 2 congruent right angled triangles having the base as 6 cm and height 1.5 cm
Area triangles IV and V $=2 \times(1 / 2 \times 6 \times 1.5) \mathrm{cm}^{2}=9 \mathrm{~cm}^{2}$
So, the total area of the paper used $=(2.488+6.5+1.3+9) \mathrm{cm}^{2}=19.3 \mathrm{~cm}^{2}$
4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are $26 \mathrm{~cm}, 28$ cm and 30 cm , and the parallelogram stands on the base 28 cm , find the height of the parallelogram.

## Solution:

Given,
It is given that the parallelogram and triangle have equal areas.
The sides of the triangle are given as $26 \mathrm{~cm}, 28 \mathrm{~cm}$ and 30 cm .
So, the perimeter $=26+28+30=84 \mathrm{~cm}$
And its semi perimeter $=84 / 2 \mathrm{~cm}=42 \mathrm{~cm}$
Now, by using Heron's formula, area of the triangle $=$
$\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{ }[42(42-26)(42-28)(42-30)] \mathrm{cm}^{2}$
$=\sqrt{ }[42 \times 16 \times 14 \times 12] \mathrm{cm}^{2}$
$=336 \mathrm{~cm}^{2}$
Now, let the height of parallelogram be $h$.
As the area of parallelogram $=$ area of the triangle,
$28 \mathrm{~cm} \times \mathrm{h}=336 \mathrm{~cm}^{2}$
$\therefore \mathrm{h}=336 / 28 \mathrm{~cm}$
So, the height of the parallelogram is 12 cm .
5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is $\mathbf{3 0} \mathbf{~ m}$ and its longer diagonal is 48 m , how much area of grass field will each cow be getting?

## Solution:

Draw a rhombus-shaped field first with the vertices as ABCD . The diagonal AC divides the rhombus into two congruent triangles which are having equal areas. The diagram is as follows.


Consider the triangle BCD,
Its semi-perimeter $=(48+30+30) / 2 \mathrm{~m}=54 \mathrm{~m}$
Using Heron's formula,
Area of the $\triangle \mathrm{BCD}=$

$$
\begin{aligned}
& =\sqrt{s(s-a)(s-b)(s-c)} \\
& =(\sqrt{54(54-48)(54-30)(54-30)}) \mathrm{m}^{2} \\
& =(\sqrt{54 \times 6 \times 24 \times 24}) \mathrm{m}^{2}
\end{aligned}
$$

$=432 \mathrm{~m}^{2}$
$\therefore$ Area of field $=2 \times$ area of the $\triangle B C D=(2 \times 432) \mathrm{m}^{2}=864 \mathrm{~m}^{2}$
Thus, the area of the grass field that each cow will be getting $=(864 / 18) \mathrm{m}^{2}=48 \mathrm{~m}^{2}$
6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.12.16), each piece measuring $20 \mathrm{~cm}, 50 \mathrm{~cm}$ and 50 cm . How much cloth of each colour is required for the umbrella?


Fig. 12.16

## Solution:

For each triangular piece, The semi perimeter will be
$\mathrm{s}=(50+50+20) / 2 \mathrm{~cm}=120 / 2 \mathrm{~cm}=60 \mathrm{~cm}$

Using Heron's formula,

Area of the triangular piece
$=$
$\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{ }[60(60-50)(60-50)(60-20)] \mathrm{cm}^{2}$
$=\sqrt{ }[60 \times 10 \times 10 \times 40] \mathrm{cm}^{2}$
$=200 \sqrt{6} \mathrm{~cm}^{2}$
$\therefore$ The area of all the triangular pieces $=5 \times 200 \sqrt{ } 6 \mathrm{~cm}^{2}=1000 \sqrt{6} \mathrm{~cm}^{2}$
7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base $\mathbf{8 c m}$ and sides 6 cm each is to be made of three different shades as shown in Fig. 12.17. How much paper of each shade has been used in it?


Fig. 12.17

## Solution:

As the kite is in the shape of a square, its area will be
$\mathrm{A}=(1 / 2) \times(\text { diagonal })^{2}$
Area of the kite $=(1 / 2) \times 32 \times 32=512 \mathrm{~cm}^{2}$
The area of shade $\mathrm{I}=$ Area of shade II
$512 / 2 \mathrm{~cm}^{2}=256 \mathrm{~cm}^{2}$
So, the total area of the paper that is required in each shade $=256 \mathrm{~cm}^{2}$
For the triangle section (III),
The sides are given as $6 \mathrm{~cm}, 6 \mathrm{~cm}$ and 8 cm
Now, the semi perimeter of this isosceles triangle $=(6+6+8) / 2 \mathrm{~cm}=10 \mathrm{~cm}$
By using Heron's formula, the area of the III triangular piece will be
$=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{ }[10(10-6)(10-6)(10-8)] \mathrm{cm}^{2}$
$=\sqrt{ }(10 \times 4 \times 4 \times 2) \mathrm{cm}^{2}$
$=8 \sqrt{5 \mathrm{~cm}^{2}}=17.92 \mathrm{~cm}^{2}$ (approx.)
8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being $9 \mathrm{~cm}, 28$ cm and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50 p per $\mathrm{cm}^{2}$.


Fig. 12.18

## Solution:

The semi perimeter of the each triangular shape $=(28+9+35) / 2 \mathrm{~cm}=36 \mathrm{~cm}$
By using Heron's formula,
The area of each triangular shape will be
$=\sqrt{s(s-a)(s-b)(s-c)}$
$=(\sqrt{36 \times(36-35) \times(36-28) \times(36-9)})$
$=(\sqrt{36 \times 1 \times 8 \times 27}) \mathrm{cm}^{2}$
$=36 \sqrt{6} \mathrm{~cm}^{2}=88.2 \mathrm{~cm}^{2}$
Now, the total area of 16 tiles $=16 \times 88.2 \mathrm{~cm}^{2}=1411.2 \mathrm{~cm}^{2}$
It is given that the polishing cost of tiles $=50$ paise $/ \mathrm{cm}^{2}$
$\therefore$ The total polishing cost of the tiles $=$ Rs. $(1411.2 \times 0.5)=$ Rs. 705.6
9. A field is in the shape of a trapezium whose parallel sides are $\mathbf{2 5} \mathrm{m}$ and 10 m . The non-parallel sides are $\mathbf{1 4} \mathrm{m}$ and 13 m . Find the area of the field.

## Solution:

First, draw a line segment BE parallel to the line AD . Then, from B, draw a perpendicular on the line segment CD .


Now, it can be seen that the quadrilateral ABED is a parallelogram. So,
$\mathrm{AB}=\mathrm{ED}=10 \mathrm{~m}$
$\mathrm{AD}=\mathrm{BE}=13 \mathrm{~m}$
$\mathrm{EC}=25-\mathrm{ED}=25-10=15 \mathrm{~m}$
Now, consider the triangle BEC,
Its semi perimeter $(\mathrm{s})=(13+14+15) / 2=21 \mathrm{~m}$
By using Heron's formula,
Area of $\triangle \mathrm{BEC}=$
$=\sqrt{s(s-a)(s-b)(s-c)}$
$=(\sqrt{21 \times(21-13) \times(21-14) \times(21-15)}) m^{2}$
$=(\sqrt{21 \times 8 \times 7 \times 6}) m^{2}$
$=84 \mathrm{~m}^{2}$

We also know that the area of $\Delta \mathrm{BEC}=(1 / 2) \times \mathrm{CE} \times \mathrm{BF}$
$84 \mathrm{~cm}^{2}=(1 / 2) \times 15 \times$ BF
$\mathrm{BF}=(168 / 15) \mathrm{cm}=11.2 \mathrm{~cm}$
So, the total area of ABED will be $\mathrm{BF} \times \mathrm{DE}$ i.e. $11.2 \times 10=112 \mathrm{~m}^{2}$
$\therefore$ Area of the field $=84+112=196 \mathrm{~m}^{2}$

