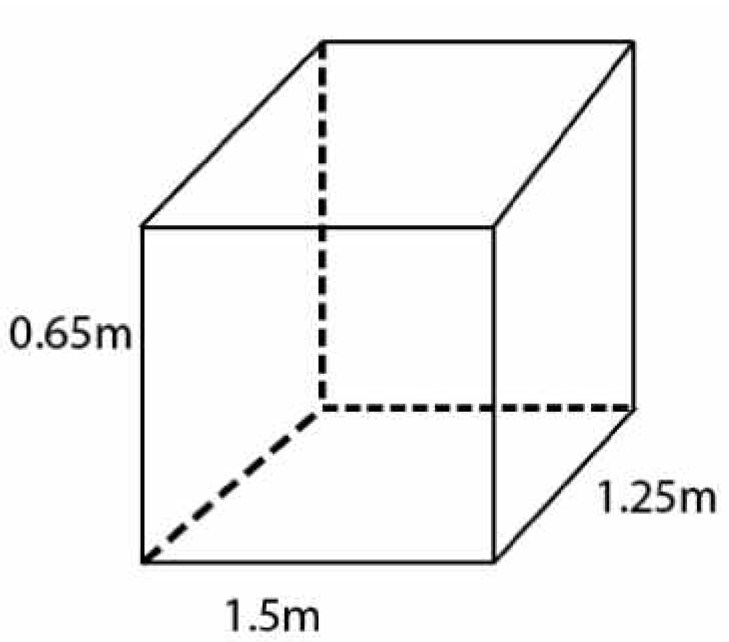
EXERCISE 13.1 PAGE NO: 213

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine

(I) The area of the sheet required for making the box.

(ii) The cost of the sheet for it, if a sheet measuring 1m² costs Rs. 20.

Solution:



Given: length (l) of box = 1.5m

Breadth (b) of box = 1.25 m

Depth (h) of box = 0.65m

(i) Box is to be open at the top.

Area of sheet required.

= 2lh+2bh+lb

= [2×1.5×0.65+2×1.25×0.65+1.5×1.25]m²

= (1.95+1.625+1.875) m² = 5.45 m²

(ii) Cost of sheet per m² area = Rs.20

Cost of sheet of 5.45 m² area = Rs (5.45×20)

= Rs.109.

2. The length, breadth and height of a room are 5 m, 4 m and 3 m, respectively. Find the cost of whitewashing the walls of the room and ceiling at the rate of Rs 7.50 per m^2.

Solution:

Length (l) of room = 5m

Breadth (b) of room = 4m

Height (h) of room = 3m

It can be observed that four walls and the ceiling of the room are to be whitewashed.

Total area to be whitewashed = Area of walls + Area of the ceiling of the room

= 2lh+2bh+lb

= [2×5×3+2×4×3+5×4]

= (30+24+20)

= 74

Area = 74 m^2

Also,

Cost of whitewash per m^2 area = Rs.7.50 (Given)

Cost of whitewashing 74 m^2 area = Rs. (74×7.50)

= Rs. 555

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs.15,000, find the height of the hall.

[Hint: Area of the four walls = Lateral surface area.]

Solution:

Let the length, breadth, and height of the rectangular hall be l, b, and h, respectively.

Area of four walls = 2lh+2bh

= 2(l+b)h

Perimeter of the floor of hall = 2(l+b)

= 250 m

Area of four walls = 2(l+b) h = 250h m^2

Cost of painting per square metre area = Rs.10

Cost of painting 250h square metre area = Rs (250h×10) = Rs.2500h

However, it is given that the cost of painting the walls is Rs. 15,000.

15000 = 2500h

Or h = 6

Therefore, the height of the hall is 6 m.

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm×10 cm×7.5 cm can be painted out of this container?

Solution:

Total surface area of one brick = 2(lb +bh+lb)

= [2(22.5×10+10×7.5+22.5×7.5)] cm^2

= 2(225+75+168.75) cm^2

= (2×468.75) cm^2

= 937.5 cm^2

Let n bricks can be painted out by the paint of the container.

Area of n bricks = (n×937.5) cm^2 = 937.5n cm^2

As per the given instructions, the area that can be painted by the paint of the container = 9.375 m^2 = 93750 cm^2

So, we have 93750 = 937.5n

n = 100

Therefore, 100 bricks can be painted out by the paint of the container.

5. A cubical box has each edge 10 cm, and another cuboidal box is 12.5cm long, 10 cm wide, and 8 cm high.

(i) Which box has the greater lateral surface area, and by how much?

(ii) Which box has the smaller total surface area, and by how much?

Solution:

From the question statement, we have

Edge of a cube = 10cm

Length, l = 12.5 cm

Breadth, b = 10cm

Height, h = 8 cm

(i) Find the lateral surface area for both figures.

Lateral surface area of cubical box = 4 $(\text{edge})^2$

$= 4(10)^2$

$= 400 \text{ cm}^2$ …(1)

Lateral surface area of cuboidal box = 2[lh+bh]

$= [2(12.5\times8+10\times8)]$

$= (2\times180) = 360$

Therefore, the lateral surface area of the cuboidal box is 360 cm^2. …(2)

From (1) and (2), the lateral surface area of the cubical box is more than the lateral surface area of the cuboidal box. The difference between both lateral surfaces is 40 cm^2.

(Lateral surface area of the cubical box – Lateral surface area of cuboidal box=400cm^2–360cm^2 = 40 cm^2)

(ii) Find the total surface area for both figures.

The total surface area of the cubical box = $6(\text{edge})^2 = 6(10 \text{ cm})^2 = 600 \text{ cm}^2$…(3)

The total surface area of the cuboidal box

= 2[lh+bh+lb]

$= [2(12.5\times8+10\times8+12.5\times100)]$

= 610

This implies that the total surface area of the cuboidal box is 610 cm^2..(4)

From (3) and (4), the total surface area of the cubical box is smaller than that of the cuboidal box. And their difference is 10cm^2.

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2

6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including the base) held together with tape. It is 30cm long, 25 cm wide, and 25 cm high.

(i) What is the area of the glass?

(ii)How much tape is needed for all 12 edges?

Solution:

Length of the greenhouse, say l = 30cm

Breadth of the greenhouse, say b = 25 cm

Height of greenhouse, say h = 25 cm

(i) Total surface area of greenhouse = Area of the glass = 2[lb+lh+bh]

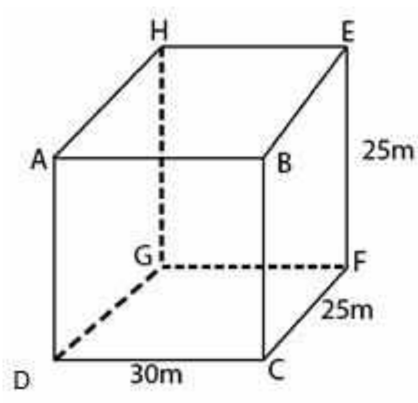
= [2(30×25+30×25+25×25)]

= [2(750+750+625)]

= (2×2125) = 4250

The total surface area of the glass is 4250 cm^2

(ii)



From the figure, the tape is required along sides AB, BC, CD, DA, EF, FG, GH, HE AH, BE, DG, and CF.

Total length of tape = 4(l+b+h)

= [4(30+25+25)] (after substituting the values)

= 320

Therefore, 320 cm tape is required for all 12 edges.

7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm, and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Solution:

Let l, b and h be the length, breadth and height of the box.

Bigger Box

l = 25cm

b = 20 cm

h = 5 cm

Total surface area of bigger box = 2(lb+lh+bh)

= [2(25×20+25×5+20×5)]

= [2(500+125+100)]

= 1450 cm^2

Extra area required for overlapping 1450×5/100 cm^2

= 72.5 cm^2

While considering all overlaps, the total surface area of the bigger box.

= (1450+72.5) cm^2 = 1522.5 cm^2

Area of cardboard sheet required for 250 such bigger boxes

= (1522.5×250) cm^2 = 380625 cm^2

Smaller Box

Similarly, total surface area of the smaller box = [2(15×12+15×5+12×5)] cm^2

= [2(180+75+60)] cm^2

= (2×315) cm^2

= 630 cm^2

Therefore, the extra area required for overlapping 630×5/100 cm^2 = 31.5 cm^2

The total surface area of 1 smaller box while considering all overlaps

= (630+31.5) cm^2 = 661.5 cm^2

Area of cardboard sheet required for 250 smaller boxes = (250×661.5) cm^2 = 165375 cm^2

In Short

Box	Dimensions (in cm)	Total surface area (in cm^2)	Extra area required for overlapping (in cm^2)	Total surface area for all overlaps (in cm^2)	Area for 250 such boxes (in cm^2)

Bigger Box	l = 25 b = 20 c = 5	1450	1450×5/100 = 72.5	(1450+72.5) = 1522.5	(1522.5×250) = 380625
Smaller Box	l = 15 b = 12 h =5	630	630×5/100 = 31.5	(630+31.5) = 661.5	(250×661.5) = 165375

Now, total cardboard sheet required = (380625+165375) cm^2

= 546000 cm^2

Given, cost of 1000 cm^2 cardboard sheet = Rs. 4

Therefore, the cost of 546000 cm^2 cardboard sheet =Rs. (546000×4)/1000 = Rs. 2184

Therefore, the cost of cardboard required for supplying 250 boxes of each kind will be Rs. 2,184.

8. Praveen wanted to make a temporary shelter for her car by making a box-like structure with a tarpaulin that covers all four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how many tarpaulins would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?

Solution:

Let l, b and h be the length, breadth and height of the shelter.

Given:

l = 4m

b = 3m

h = 2.5m

Tarpaulins will be required for the top and four wall sides of the shelter.

Using formula, area of tarpaulin required = 2(lh+bh)+lb

On putting the values of l, b and h, we get

= [2(4×2.5+3×2.5)+4×3] m^2

= [2(10+7.5)+12]m^2

= 47m^2

Therefore, 47 m^2 of tarpaulin will be required.

EXERCISE 13.2 PAGE NO: 216

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder (Assume π =22/7).

Solution:

Height of cylinder, h = 14cm

Let the diameter of the cylinder be d.

The curved surface area of cylinder = 88 cm^2

We know that the formula to find the Curved surface area of the cylinder is 2πrh.

So 2πrh =88 cm^2 (r is the radius of the base of the cylinder)

2×(22/7)×r×14 = 88 cm^2

2r = 2 cm

d =2 cm

Therefore, the diameter of the base of the cylinder is 2 cm.

2. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square metres of the sheet are required for the same? Assume π = 22/7

Solution:

Let h be the height and r be the radius of a cylindrical tank.

Height of cylindrical tank, h = 1m

Radius = half of diameter = (140/2) cm = 70cm = 0.7m

Area of sheet required = Total surface area of tank = 2πr(r+h) unit square

= [2×(22/7)×0.7(0.7+1)]

= 7.48 square metres

Therefore, 7.48 square metres of the sheet are required.

3. A metal pipe is 77 cm long. The inner diameter of a cross-section is 4 cm, the outer diameter being 4.4cm (see fig. 13.11). Find its

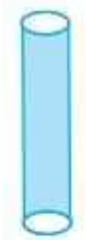


Fig. 13.11

(i) inner curved surface area

(ii) outer curved surface area

(iii) total surface area

(Assume π=22/7)

Solution:

Let r_1 and r_2 inner and outer radii of the cylindrical pipe.

r_1 = 4/2 cm = 2 cm

r_2 = 4.4/2 cm = 2.2 cm

Height of cylindrical pipe, h = length of cylindrical pipe = 77 cm

(i) Curved surface area of the outer surface of pipe = $2\pi r_1 h$

= $2\times(22/7)\times2\times77$ cm^2

= 968 cm^2

(ii) Curved surface area of the outer surface of pipe = $2\pi r_2 h$

= $2\times(22/7)\times2.2\times77$ cm^2

= $(22\times22\times2.2)$ cm^2

= 1064.8 cm^2

(iii) Total surface area of pipe = inner curved surface area+ outer curved surface area+ area of both circular ends of pipe

= $2r_1h+2r_2h+2\pi(r_1^2-r_2^2)$

= $9668+1064.8+2\times(22/7)\times(2.2^2-2^2)$

= 2031.8+5.28

= 2038.08 cm^2

Therefore, the total surface area of the cylindrical pipe is 2038.08 cm^2.

4. The diameter of a roller is 84 cm, and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2 (Assume π = 22/7).

Solution:

A roller is shaped like a cylinder.

Let h be the height of the roller and r be the radius.

h = Length of roller = 120 cm

Radius of the circular end of roller = r = (84/2) cm = 42 cm

Now, CSA of roller = 2πrh

= 2×(22/7)×42×120

= 31680 cm^2

Area of field = 500×CSA of roller

= (500×31680) cm^2

= 15840000 cm^2

= 1584 m^2.

Therefore, the area of the playground is 1584 m^2.

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.

(Assume π = 22/7)

Solution:

Let h be the height of a cylindrical pillar and r be the radius.

Given:

Height cylindrical pillar = h = 3.5 m

Radius of the circular end of pillar = r = diameter/2 = 50/2 = 25cm = 0.25m

CSA of pillar = 2πrh

= 2×(22/7)×0.25×3.5

= 5.5 m^2

Cost of painting 1 m^2 area = Rs. 12.50

Cost of painting 5.5 m^2 area = Rs (5.5×12.50)

= Rs.68.75

Therefore, the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2 is Rs 68.75.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height. (Assume π = 22/7)

Solution:

Let h be the height of the circular cylinder and r be the radius.

The radius of the base of the cylinder, r = 0.7m

CSA of cylinder = $2\pi rh$

CSA of cylinder = 4.4m^2

Equating both equations, we have

2×(22/7)×0.7×h = 4.4

Or h = 1

Therefore, the height of the cylinder is 1 m.

7. The inner diameter of a circular well is 3.5m. It is 10m deep. Find

(i) its inner curved surface area.

(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.

(Assume π = 22/7)

Solution:

Inner radius of circular well, r = 3.5/2m = 1.75m

Depth of circular well, say h = 10m

(i) Inner curved surface area = $2\pi rh$

= (2×(22/7)×1.75×10)

= 110 m^2

Therefore, the inner curved surface area of the circular well is 110 m^2.

(ii) Cost of plastering 1 m^2 area = Rs.40

Cost of plastering 110 m^2 area = Rs (110×40)

= Rs.4400

Therefore, the cost of plastering the curved surface of the well is Rs. 4,400.

8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. (Assume π = 22/7)

Solution:

Height of cylindrical pipe = Length of cylindrical pipe = 28m

Radius of circular end of pipe = diameter/ 2 = 5/2 cm = 2.5cm = 0.025m

Now, CSA of cylindrical pipe = 2πrh, where r = radius and h = height of the cylinder

= 2×(22/7)×0.025×28 m^2

= 4.4m^2

The area of the radiating surface of the system is 4.4m^2.

9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5m high.

(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank? (Assume π = 22/7)

Solution:

Height of cylindrical tank, h = 4.5m

Radius of the circular end , r = (4.2/2)m = 2.1m

(i) The lateral or curved surface area of the cylindrical tank is 2πrh.

= 2×(22/7)×2.1×4.5 m^2

= (44×0.3×4.5) m^2

= 59.4 m^2

Therefore, the CSA of the tank is 59.4 m^2.

(ii) Total surface area of tank = 2πr(r+h)

= 2×(22/7)×2.1×(2.1+4.5)

= 44×0.3×6.6

= 87.12 m^2

Now, Let S m^2 steel sheet be actually used in making the tank.

S(1 -1/12) = 87.12 m^2

This implies, S = 95.04 m^2

Therefore, 95.04m^2 steel was used in actuality while making such a tank.

10. In fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth.

The frame has a base diameter of 20 cm and a height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required to cover the lampshade. (Assume π = 22/7)

Fig. 13.12

Solution:

Say h = height of the frame of the lampshade, which looks like a cylindrical shape.

r = radius

Total height is h = (2.5+30+2.5) cm = 35cm and

r = (20/2) cm = 10cm

Use the curved surface area formula to find the cloth required for covering the lampshade, which is 2πrh.

= (2×(22/7)×10×35) cm^2

= 2200 cm^2

Hence, 2200 cm^2 cloth is required to cover the lampshade.

11. The students of Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? (Assume π =22/7)

Solution:

The radius of the circular end of the cylindrical penholder, r = 3cm

Height of penholder, h = 10.5cm

Surface area of a penholder = CSA of pen holder + area of base of penholder

$= 2\pi rh+\pi r^2$

$= 2\times(22/7)\times3\times10.5+(22/7)\times3^2= 1584/7$

Therefore, the Area of cardboard sheet used by one competitor is 1584/7 cm^2

So, the Area of cardboard sheet used by 35 competitors = $35\times1584/7$ = 7920 cm^2

Therefore, a 7920 cm^2 cardboard sheet will be needed for the competition.

EXERCISE 13.3 PAGE NO: 221

1. Diameter of the base of a cone is 10.5 cm, and its slant height is 10 cm. Find its curved surface area. (Assume π=22/7)

Solution:

Radius of the base of cone = diameter/ 2 = (10.5/2)cm = 5.25cm

The slant height of the cone, say l = 10 cm

CSA of the cone is = πrl

= (22/7)×5.25×10 = 165 cm^2

Therefore, the curved surface area of the cone is 165 cm^2.

2. Find the total surface area of a cone, if its slant height is 21 m and the diameter of its base is 24 m. (Assume π = 22/7)

Solution:

Radius of cone, r = 24/2 m = 12m

Slant height, l = 21 m

Formula: Total Surface area of the cone = πr(l+r)

Total Surface area of the cone = (22/7)×12×(21+12) m^2

= 1244.57m^2

3. Curved surface area of a cone is 308 cm^2, and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

(Assume π = 22/7)

Solution:

The slant height of the cone, l = 14 cm

Let the radius of the cone be r.

(i) We know the CSA of cone = πrl

Given: Curved surface area of a cone is 308 cm^2

(308) = (22/7)×r×14

308 = 44 r

r = 308/44 = 7 cm

The radius of a cone base is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base (πr^2)

Total surface area of cone = $308+(22/7)\times7^2 = 308+154 = 462$ cm^2

Therefore, the total surface area of the cone is 462 cm^2.

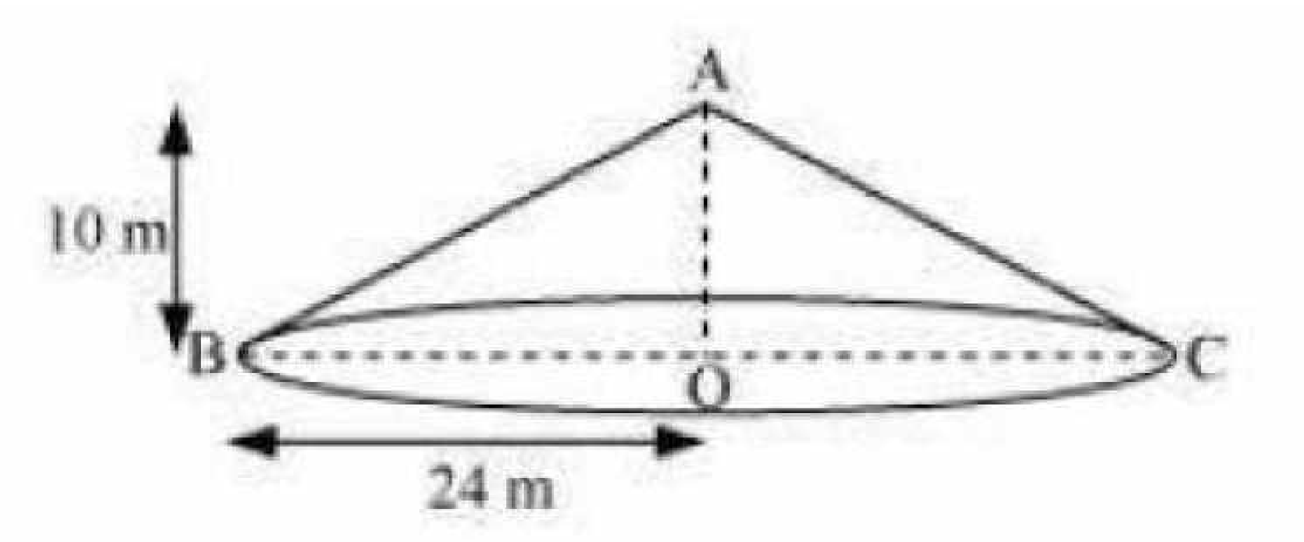
4. A conical tent is 10 m high, and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

(Assume π=22/7)

Solution:



Let ABC be a conical tent.

Height of conical tent, h = 10 m

Radius of conical tent, r = 24m

Let the slant height of the tent be l.

(i) In the right triangle ABO, we have

$AB^2 = AO^2+BO^2$(using Pythagoras' theorem)

$l^2 = h^2+r^2$

$= (10)^2+(24)^2$

$= 676$

$l = 26$ m

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = πrl

$= (22/7)\times24\times26\ m^2$

Cost of 1 m^2 canvas = Rs 70

Cost of $(13728/7)m^2$ canvas is equal to Rs $(13728/7)\times70$ = Rs 137280

Therefore, the cost of the canvas required to make such a tent is Rs 137280.

5. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14]

Solution:

Height of the conical tent, h = 8m

Radius of the base of the tent, r = 6m

Slant height of the tent, $l^2 = (r^2+h^2)$

$l^2 = (6^2+8^2) = (36+64) = (100)$

or $l = 10$ m

Again, CSA of conical tent = πrl

$= (3.14\times6\times10)\ m^2$

$= 188.4m^2$

Let the length of the tarpaulin sheet required be L.

As 20 cm will be wasted,

The effective length will be (L-0.2m).

The breadth of tarpaulin = 3m (given)

Area of sheet = CSA of the tent

$[(L–0.2)\times3] = 188.4$
$L-0.2 = 62.8$

L = 63 m

Therefore, the length of the required tarpaulin sheet will be 63 m.

6. The slant height and base diameter of the conical tomb are 25m and 14 m, respectively. Find the cost of whitewashing its curved surface at the rate of Rs. 210 per 100 m^2. (Assume π = 22/7)

Solution:

Slant height of the conical tomb, l = 25m

Base radius, r = diameter/2 = 14/2 m = 7m

CSA of the conical tomb = πrl

= (22/7)×7×25 = 550

CSA of the conical tomb= 550m^2

Cost of whitewashing 550 m^2 area, which is Rs (210×550)/100

= Rs. 1155

Therefore, the cost will be Rs. 1155 while whitewashing the tomb.

7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume π =22/7)

Solution:

Radius of the conical cap, r = 7 cm

Height of the conical cap, h = 24cm

Slant height, $l^2 = (r^2+h^2)$

$= (7^2+24^2)$

= (49+576)

= (625)

Or l = 25 cm

CSA of 1 conical cap = πrl

= (22/7)×7×25

= 550 cm^2

CSA of 10 caps = (10×550) cm^2 = 5500 cm^2

Therefore, the area of the sheet required to make 10 such caps is 5500 cm^2.

8. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{(1.04)} = 1.02$)

Solution:

Given:

Radius of cone, r = diameter/2 = 40/2 cm = 20cm = 0.2 m

Height of cone, h = 1m

Slant height of cone is l, and $l^2 = (r^2+h^2)$

Using given values, $l^2 = (0.2^2+1^2)$

= (1.04)

Or l = 1.02 m

Slant height of the cone is 1.02 m.

Now,

CSA of each cone = πrl

= (3.14×0.2×1.02)

= 0.64056 m

CSA of 50 such cones = (50×0.64056) = 32.028

CSA of 50 such cones = 32.028 m^2

Again,

Cost of painting 1 m^2 area = Rs 12 (given)

Cost of painting 32.028 m^2 area = Rs (32.028×12)

= Rs.384.336

= Rs.384.34 (approximately)

Therefore, the cost of painting all these cones is Rs. 384.34.

EXERCISE 13.4 PAGE NO: 225

1. Find the surface area of a sphere of radius

(i) 10.5cm (ii) 5.6cm (iii) 14cm

(Assume π=22/7)

Solution:

Formula: Surface area of a sphere (SA) = $4\pi r^2$

(i) Radius of a sphere, r = 10.5 cm

SA = $4\times(22/7)\times10.5^2 = 1386$

Surface area of a sphere is 1386 cm^2

(ii) Radius of a sphere, r = 5.6cm

Using formula, SA = $4\times(22/7)\times5.6^2 = 394.24$

Surface area of a sphere is 394.24 cm^2

(iii) Radius of a sphere, r = 14cm

SA = $4\pi r^2$

= $4\times(22/7)\times(14)^2$

= 2464

Surface area of a sphere is 2464 cm^2

2. Find the surface area of a sphere of diameter

(i) 14cm (ii) 21cm (iii) 3.5cm

(Assume π = 22/7)

Solution:

(i) Radius of sphere, r = diameter/2 = 14/2 cm = 7 cm

Formula for the surface area of sphere = $4\pi r^2$

= $4\times(22/7)\times7^2 = 616$

Surface area of a sphere is 616 cm^2

(ii) Radius (r) of sphere = 21/2 = 10.5 cm

Surface area of a sphere = $4\pi r^2$

$= 4\times(22/7)\times10.5^2 = 1386$

Surface area of a sphere is 1386 cm^2

Therefore, the surface area of a sphere having a diameter 21 cm is 1386 cm^2

(iii) Radius(r) of a sphere = 3.5/2 = 1.75 cm

Surface area of a sphere = $4\pi r^2$

$= 4\times(22/7)\times1.75^2 = 38.5$

Surface area of a sphere is 38.5 cm^2

3. Find the total surface area of a hemisphere of radius 10 cm. [Use π=3.14]

Solution:

Radius of the hemisphere, r = 10cm

Formula: Total surface area of the hemisphere = $3\pi r^2$

$= 3\times3.14\times10^2 = 942$

The total surface area of the given hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Solution:

Let r_1 and r_2 be the radii of the spherical balloon and spherical balloon when air is pumped into it, respectively. So,

$r_1 = 7$cm

$r_2 = 14$ cm

Now, Required ratio = (initial surface area)/(Surface area after pumping air into balloon)

$= 4\pi r_1^2/4\pi r_2^2$

$= (r_1/r_2)^2$

$= (7/14)^2 = (1/2)^2 = ¼$

Therefore, the ratio between the surface areas is 1:4.

5. A hemispherical bowl made of brass has an inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. (Assume π = 22/7)

Solution:

Inner radius of hemispherical bowl, say r = diameter/2 = (10.5)/2 cm = 5.25 cm

Formula for the surface area of hemispherical bowl = $2\pi r^2$

= $2\times(22/7)\times(5.25)^2$ = 173.25

Surface area of the hemispherical bowl is 173.25 cm^2

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 1 cm^2 area = Rs 16 /100

Cost of tin-plating 173.25 cm^2 area = Rs. $(16\times173.25)/100$ = Rs 27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl at the rate of Rs 16 per 100 cm^2 is Rs **27.72.**

6. Find the radius of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let the radius of the sphere be r.

Surface area of sphere = 154 (given)

Now,

$4\pi r^2 = 154$

$r^2 = (154\times7)/(4\times22) = (49/4)$

r = (7/2) = 3.5

The radius of the sphere is 3.5 cm.

7. The diameter of the moon is approximately one-fourth of the diameter of the earth.

Find the ratio of their surface areas.

Solution:

If the diameter of the earth is said d, then the diameter of the moon will be d/4 (as per the given statement).

Radius of earth = d/2

Radius of moon = ½×d/4 = d/8

Surface area of moon = $4\pi(d/8)^2$

Surface area of earth = $4\pi(d/2)^2$

$$\text{Ratio of their Surface areas} = \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = 4/64 = 1/16$$

The ratio between their surface areas is 1:16.

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume π =22/7)

Solution:

Given:

Inner radius of the hemispherical bowl = 5cm

Thickness of the bowl = 0.25 cm

Outer radius of the hemispherical bowl = (5+0.25) cm = 5.25 cm

Formula for outer CSA of the hemispherical bowl = $2\pi r^2$, where r is the radius of the hemisphere.

= $2\times(22/7)\times(5.25)^2$ = 173.25 cm^2

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see fig. 13.22). Find

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in(i) and (ii).

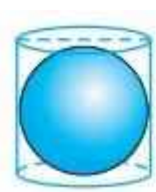


Fig. 13.22

Solution:

(i) Surface area of the sphere = $4\pi r^2$, where r is the radius of sphere

(ii) Height of the cylinder, h = r+r =2r

The radius of the cylinder = r

CSA of the cylinder formula = $2\pi rh = 2\pi r(2r)$ (using value of h)

$= 4\pi r^2$

(iii) Ratio between areas = (Surface area of sphere)/(CSA of Cylinder)

$= 4\pi r^2/4\pi r^2 = 1/1$

The ratio of the areas obtained in (i) and (ii) is 1:1.

EXERCISE 13.5

1. A matchbox measures 4 cm×2.5cm×1.5cm. What will be the volume of a packet containing 12 such boxes?

Solution:

Dimensions of a matchbox (a cuboid) are l×b×h = 4 cm×2.5 cm×1.5 cm, respectively

Formula to find the volume of matchbox = l×b×h = (4×2.5×1.5) = 15

Volume of matchbox = 15 cm^3

Now, volume of 12 such matchboxes = (15×12) cm^3 = 180 cm^3

Therefore, the volume of 12 matchboxes is 180 cm^3.

2. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m^3= 1000 l)

Solution:

Dimensions of a cuboidal water tank are l = 6 m and b = 5 m, and h = 4.5 m

Formula to find the volume of the tank, V = l×b×h

Putting the values, we get

V = (6×5×4.5) = 135

The volume of the water tank is 135 m^3

Again,

We are given that the amount of water that 1m^3 volume can hold = 1000 l

Amount of water, 135 m^3volume hold = (135×1000) litres = 135000 litres

Therefore, given cuboidal water tank can hold up to 135000 litres of water.

3. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Solution:

Given:

Length of the cuboidal vessel, l = 10 m

Width of the cuboidal vessel, b = 8m

Volume of the cuboidal vessel, V = 380 m^3

Let the height of the given vessel be h.

Formula for volume of a cuboid, V = l×b×h

Using the formula, we have

l×b×h = 380

10×8×h= 380

Or h = 4.75

Therefore, the height of the vessels is 4.75 m.

4. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m^3.

Solution:

The given pit has its length(l) as 8m, width (b)as 6m and depth (h)as 3 m.

Volume of cuboidal pit = l×b×h = (8×6×3) = 144 (using formula)

The required Volume is 144 m^3

Now,

Cost of digging per m^3 volume = Rs 30

Cost of digging 144 m^3 volume = Rs (144×30) = Rs 4320

5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Solution:

The length (l) and depth (h) of the tank are 2.5 m and 10 m, respectively.

To find: The value of breadth, say b.

Formula to find the volume of a tank = l×b×h = (2.5× b×10) m^3= 25b m^3

The capacity of tank= 25b m^3, which is equal to 25000b litres

Also, the capacity of a cuboidal tank is 50000 litres of water (Given).

Therefore, 25000 b = 50000

This implies that b = 2

Therefore, the breadth of the tank is 2 m.

6. A village, having a population of 4000, requires 150 litres of water per head per day.

It has a tank measuring 20 m×15 m×6 m. For how many days will the water in this tank last?

Solution:

Length of the tank = l = 20 m

Breadth of the tank = b = 15 m

Height of the tank = h = 6 m

Total population of a village = 4000

Consumption of water per head per day = 150 litres

Water consumed by the people in 1 day = (4000×150) litres = 600000 litres …(1)

Formula to find the capacity of the tank, C = l×b×h

Using the given data, we have

C = (20×15×6) m^3= 1800 m^3

Or C = 1800000 litres

Let water in this tank last for d days.

Water consumed by all people in d days = Capacity of the tank (using equation (1))

600000 d =1800000

d = 3

Therefore, the water in this tank will last for 3 days.

7. A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates, each measuring 1.5m×1.25 m×0.5 m, that can be stored in the godown.

Solution:

From the statement, we have

Length of the godown = 40 m

Breadth = 25 m

Height = 15 m

Whereas,

Length of the wooden crate = 1.5 m

Breadth = 1.25 m

Height = 0.5 m

The godown and wooden crate are in cuboidal shape. Find the volume of each using the formula V = lbh

Now,

Volume of godown = $(40\times25\times15)\ m^3 = 15000\ m^3$

Volume of a wooden crate = $(1.5\times1.25\times0.5)\ m^3 = 0.9375\ m^3$

Let us consider that, n wooden crates can be stored in the godown, then

Volume of n wooden crates = Volume of godown

$0.9375\times n = 15000$

Or $n = 15000/0.9375 = 16000$

Hence, the number of wooden crates that can be stored in the godown is 16,000.

8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Solution:

Side of a cube = 12cm (Given)

Find the volume of the cube.

Volume of cube = $(\text{Side})^3 = (12)^3 cm^3 = 1728 cm^3$

Surface area of a cube with side 12 cm = $6a^2 = 6(12)^2 cm^2$ …(1)

The cube is cut into eight small cubes of equal volume; say, the side of each cube is p.

Volume of a small cube = p^3

Surface area = $6p^2$ …(2)

Volume of each small cube = $(1728/8)\ cm^3 = 216\ cm^3$

Or $(p)^3 = 216\ cm^3$

Or p = 6 cm

Now, Surface areas of the cubes ratios = (Surface area of the bigger cube)/(Surface area of the smaller cubes)

From equations (1) and (2), we get

Surface areas of the cubes ratios = $(6a^2)/(6p^2) = a^2/p^2 = 12^2/6^2 = 4$

Therefore, the required ratio is 4 : 1.

9. A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Solution:

Given:

Depth of river, h = 3 m

Width of river, b = 40 m

Rate of water flow = 2km per hour = 2000m/60min = 100/3 m/min

Now, volume of water flowed in 1 min = $(100/3) \times 40 \times 3 = 4000m^3$

Therefore, 4000 m^3 water will fall into the sea in a minute.

EXERCISE 13.6 PAGE NO: 230

1. The circumference of the base of the cylindrical vessel is 132cm, and its height is 25cm.

How many litres of water can it hold? ($1000\ cm^3 = 1L$) (Assume $\pi = 22/7$)

Solution:

Circumference of the base of cylindrical vessel = 132 cm

Height of vessel, h = 25 cm

Let r be the radius of the cylindrical vessel.

Step 1: Find the radius of the vessel.

We know that the circumference of the base = $2\pi r$, so

$2\pi r = 132$ (given)

$r = (132/(2\pi))$

$r = 66\times7/22 = 21$

The radius is 21 cm.

Step 2: Find the volume of the vessel.

Formula: Volume of cylindrical vessel = $\pi r^2 h$

$= (22/7)\times21^2\times25$

$= 34650$

Therefore, the volume is $34650\ cm^3$

Since **$1000\ cm^3 = 1L$**

So, Volume = 34650/1000 L= 34.65L

Therefore, the vessel can hold 34.65 litres of water.

2. The inner diameter of a cylindrical wooden pipe is 24cm, and its outer diameter is 28 cm. The length of the pipe is 35cm. Find the mass of the pipe, if $1cm^3$ of wood has a mass of 0.6g. (Assume $\pi = 22/7$)

Solution:

Inner radius of cylindrical pipe, say r_1 = $diameter_1$/ 2 = 24/2 cm = 12cm

Outer radius of cylindrical pipe, say r_2 = $diameter_2$/ 2 = 28/2 cm = 14 cm

Height of pipe, h = Length of pipe = 35cm

Now, the Volume of pipe = $\pi(r_2^2-r_1^2)h$ cm^3

Substitute the values.

Volume of pipe = 110×52 cm^3 = 5720 cm^3

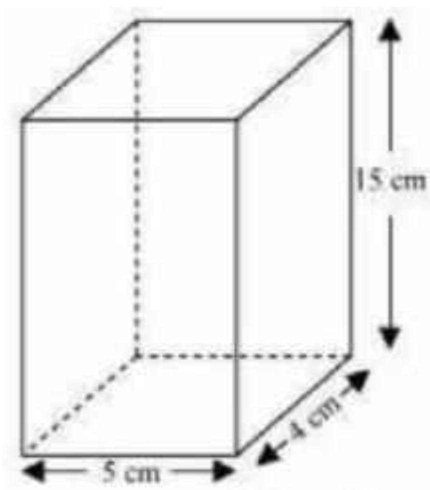
Since **Mass of 1 cm^3 wood = 0.6 g**

Mass of 5720 cm^3 wood = (5720×0.6) g = 3432 g or 3.432 kg.

3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5cm and width 4cm, having a height of 15 cm and (ii) a plastic cylinder with a circular base of diameter 7cm and height 10cm. Which container has greater capacity, and by how much? (Assume π=22/7)

Solution:

(i) Tin can will be cuboidal in shape.



Dimensions of the tin can are

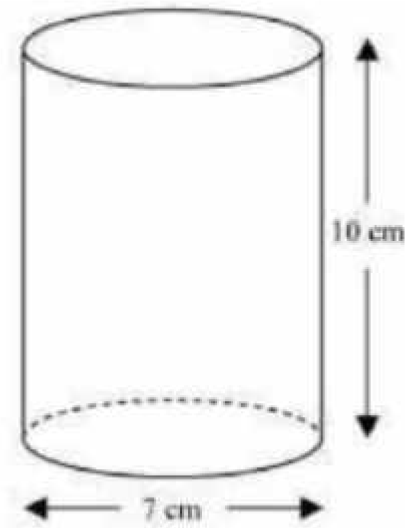
Length, l = 5 cm

Breadth, b = 4 cm

Height, h = 15 cm

Capacity of tin can = l×b×h= (5×4×15) cm^3 = 300 cm^3

(ii) Plastic cylinder will be cylindrical in shape.



Dimensions of the plastic can are

Radius of the circular end of plastic cylinder, r = 3.5cm

Height , H = 10 cm

Capacity of the plastic cylinder = $\pi r^2 H$

Capacity of the plastic cylinder = $(22/7)\times(3.5)^2\times10 = 385$

Capacity of the plastic cylinder is 385 cm^3

From the results of (i) and (ii), the plastic cylinder has more capacity.

Difference in capacity = (385-300) cm^3 = 85cm^3

4. If the lateral surface of a cylinder is 94.2cm^2 and its height is 5cm, then find

(i) radius of its base (ii) its volume.[Use π= 3.14]

Solution:

CSA of cylinder = 94.2 cm^2

Height of cylinder, h = 5cm

(i) Let the radius of the cylinder be r.

Using the CSA of the cylinder, we get

$2\pi rh = 94.2$

$2\times3.14\times r\times5 = 94.2$

r = 3

The radius is 3 cm.

(ii) Volume of cylinder

The formula for the volume of the cylinder = $\pi r^2 h$

Now, $\pi r^2 h = (3.14\times(3)^2\times5)$ (using the value of r from (i))

= 141.3

Volume is 141.3 cm^3

5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) inner curved surface area of the vessel

(ii) radius of the base

(iii) capacity of the vessel

(Assume π = 22/7)

Solution:

(i) Rs 20 is the cost of painting 1 m^2 area.

Rs 1 is the cost to paint 1/20 m^2 area.

So, Rs 2200 is the cost of painting = $(1/20\times2200)$ m^2

= 110 m^2 area

The inner surface area of the vessel is 110m^2.

(ii) Radius of the base of the vessel, let us say r.

Height (h) = 10 m and

Surface area formula = $2\pi rh$

Using the result of (i),

$2\pi rh = 110$ m^2

$2\times22/7\times r\times10 = 110$

r = 1.75

The radius is 1.75 m.

(iii) Volume of vessel formula = $\pi r^2 h$

Here r = 1.75 and h = 10

Volume = $(22/7)\times(1.75)^2\times10 = 96.25$

The volume of vessel is 96.25 m^3

Therefore, the capacity of the vessel is 96.25 m^3 or 96250 litres.

6. The capacity of a closed cylindrical vessel of height 1m is 15.4 litres. How many square metres of the metal sheet would be needed to make it? (Assume π = 22/7)

Solution:

Height of cylindrical vessel, h = 1 m

Capacity of cylindrical vessel = 15.4 litres = 0.0154 m^3

Let r be the radius of the circular end.

Now,

Capacity of cylindrical vessel = $(22/7)\times r^2\times1 = 0.0154$

After simplifying, we get r = 0.07 m

Again, the total surface area of the vessel = $2\pi r(r+h)$

$= 2\times22/7\times0.07(0.07+1)$

$= 0.44\times1.07$

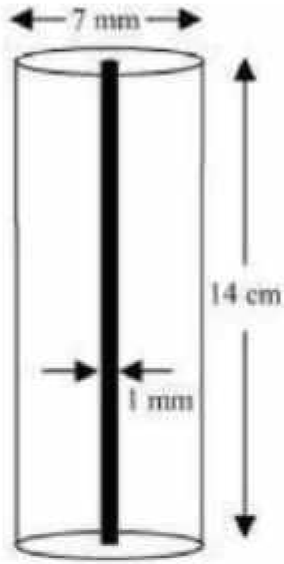
$= 0.4708$

Total surface area of the vessel is 0.4708 m^2

Therefore, 0.4708 m^2 of the metal sheet would be required to make the cylindrical vessel.

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm, and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. (Assume π = 22/7)

Solution:



Radius of pencil, r_1 = 7/2 mm = 0.7/2 cm = 0.35 cm

Radius of graphite, r_2 = 1/2 mm = 0.1/2 cm = 0.05 cm

Height of pencil, h = 14 cm

Formula to find the volume of wood in pencil = $(r_1^2-r_2^2)h$ cubic units

Substituting values, we have,

= $[(22/7)\times(0.35^2-0.05^2)\times14]$

= 44×0.12

= 5.28

This implies that the volume of wood in pencil = 5.28 cm^3

Again,

Volume of graphite = r_2^2h cubic units

Substituting the values, we have,

= $(22/7)\times0.05^2\times14$

= 44×0.0025

= 0.11

So, the volume of graphite is 0.11 cm^3.

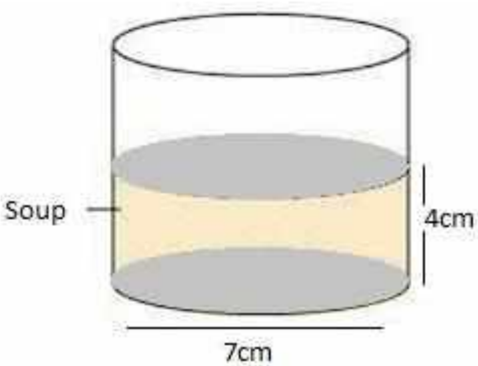
8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? (Assume π = 22/7)

Solution:

Diameter of the cylindrical bowl = 7 cm

Radius of the cylindrical bowl, r = 7/2 cm = 3.5 cm

Bowl is filled with soup to a height of4cm, so h = 4 cm



Volume of the soup in one bowl= $\pi r^2 h$

$(22/7)\times 3.5^2\times 4 = 154$

Volume of the soup in one bowl is 154 cm^3

Therefore,

Volume of the soup given to 250 patients = (250×154) cm^3= 38500 cm^3

= 38.5litres.

EXERCISE 13.7 PAGE NO: 233

1. Find the volume of the right circular cone with

(i) radius 6cm, height 7 cm (ii) radius 3.5 cm, height 12 cm (Assume π = 22/7)

Solution:

Volume of cone = (1/3) $\pi r^2 h$ cube units

Where r be radius and h be the height of the cone

(i) Radius of the cone, r = 6 cm

Height of the cone, h = 7cm

Ley V be the volume of the cone, so we have

V = (1/3)×(22/7)×36×7

= (12×22)

= 264

The volume of the cone is 264 cm^3.

(ii) Radius of the cone, r = 3.5cm

Height of the cone, h = 12cm

Volume of the cone = (1/3)×(22/7)×3.5^2×7 = 154

Hence,

The volume of the cone is 154 cm^3.

2. Find the capacity in litres of a conical vessel with

(i) radius 7cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

(Assume π = 22/7)

Solution:

(i) Radius of the cone, r =7 cm

Slant height of the cone, l = 25 cm

Height of cone, $h = \sqrt{l^2 - r^2}$

$h = \sqrt{25^2 - 7^2}$

$h = \sqrt{625 - 49}$

or h = 24

Height of the cone is 24 cm

Now,

Volume of the cone, V = (1/3) $\pi r^2 h$ (formula)

V = (1/3)×(22/7) ×7^2×24

= (154×8)

= 1232

So, the volume of the vessel is 1232 cm^3

Therefore, the capacity of the conical vessel = (1232/1000) liters (because 1L = 1000 cm^3)

= 1.232 Liters.

(ii) Height of the cone, h = 12 cm

Slant height of the cone, l = 13 cm

Radius of cone, $r = \sqrt{l^2 - h^2}$

$r = \sqrt{13^2 - 12^2}$

$r = \sqrt{169 - 144}$

r = 5

Hence, the radius of the cone is 5 cm.

Now, Volume of the cone, V = (1/3)$\pi r^2 h$

V = (1/3)×(22/7)×52×12 cm^3

= 2200/7

Volume of the cone is 2200/7 cm^3

Now, Capacity of the conical vessel= 2200/7000 litres (1L = 1000 cm^3)

= 11/35 litres

3. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

Solution:

Height of the cone, h = 15 cm

Volume of cone =1570 cm^3

Let r be the radius of the cone

As we know, volume of the cone, V = (1/3) $\pi r^2 h$

So, (1/3) $\pi r^2 h$ = 1570

$(1/3)\times3.14\times r^2\times15 = 1570$

$r^2 = 100$

r = 10

Radius of the base of the cone 10 cm.

4. If the volume of a right circular cone of height 9cm is 48πcm^3, find the diameter of its base.

Solution:

Height of cone, h = 9cm

Volume of cone =48π cm^3

Let r be the radius of the cone.

As we know, volume of the cone, V = (1/3) $\pi r^2 h$

So, $1/3\ \pi\ r^2(9) = 48\ \pi$

$r^2 = 16$

r = 4

Radius of the cone is 4 cm.

So, diameter = 2×Radius = 8

Thus, diameter of the base is 8cm.

5. A conical pit of a top diameter 3.5m is 12m deep. What is its capacity in kilolitres?

(Assume π = 22/7)

Solution:

Diameter of conical pit = 3.5 m

Radius of conical pit, r = diameter/ 2 = (3.5/2)m = 1.75m

Height of pit, h = Depth of pit = 12m

Volume of cone, V = (1/3) $\pi r^2 h$

V = (1/3)×(22/7) ×$(1.75)^2$×12 = 38.5

Volume of the cone is 38.5 m^3

Hence, capacity of the pit = (38.5×1) kiloliters = 38.5 kiloliters.

6. The volume of a right circular cone is 9856cm^3. If the diameter of the base is 28cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

(Assume π = 22/7)

Solution:

Volume of a right circular cone = 9856 cm^3

Diameter of the base = 28 cm

(i) Radius of cone, r = (28/2) cm = 14 cm

Let the height of the cone be h

Volume of cone, V = (1/3) $\pi r^2 h$

(1/3) $\pi r^2 h$ = 9856

(1/3)×(22/7) ×14×14×h = 9856

h = 48

The height of the cone is 48 cm.

(ii) Slant height of cone, $l = \sqrt{r^2 + h^2}$

$$l = \sqrt{14^2 + 48^2} = \sqrt{196 + 2304} = 50$$

Slant height of the cone is 50 cm.

(iii) curved surface area of cone = πrl

= (22/7)×14×50

= 2200

Curved surface area of the cone is 2200 cm^2.

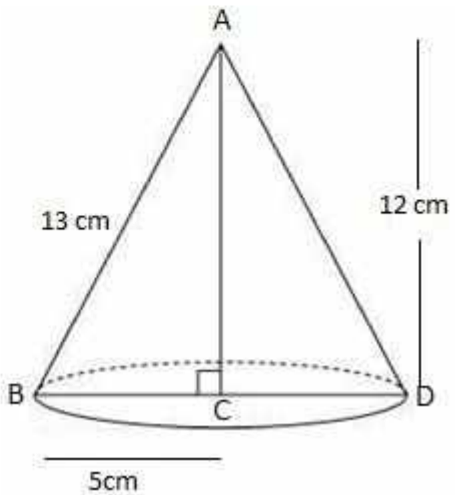
7. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Solution:

Height (h)= 12 cm

Radius (r) = 5 cm, and

Slant height (l) = 13 cm



Volume of cone, V = (1/3) $\pi r^2 h$

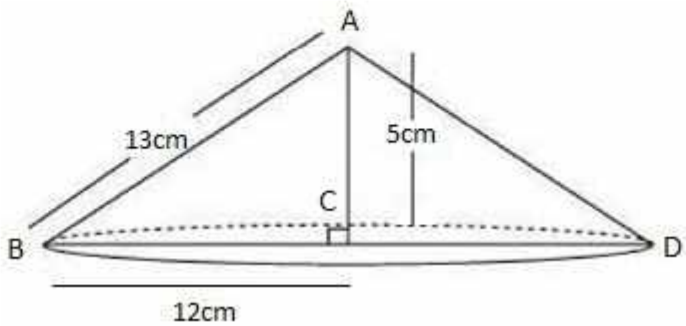
V = $(1/3)\times\pi\times 5^2\times 12$

= 100π

Volume of the cone so formed is 100π cm^3.

8. If the triangle ABC in Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Solution:



A right-angled ΔABC is revolved about its side 5cm, a cone will be formed of radius as 12 cm, height as 5 cm, and slant height as 13 cm.

Volume of cone = (1/3) $\pi r^2 h$, where r is the radius and h is the height of the cone.

= (1/3)×π×12×12×5

= 240 π

The volume of the cones formed is 240π cm^3.

So, the required ratio = (the result of question 7) / (the result of question 8) = (100π)/(240π) = 5/12 = 5:12.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas.

(Assume π = 22/7)

Solution:

Radius (r) of heap = (10.5/2) m = 5.25

Height (h) of heap = 3m

Volume of heap = (1/3)$\pi r^2 h$

= (1/3)×(22/7)×5.25×5.25×3

= 86.625

The volume of the heap of wheat is 86.625 m^3.

Again,

Area of canvas required= CSA of cone = πrl, where $l = \sqrt{r^2 + h^2}$
After substituting the values, we have
CSA of cone = $[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + 3^2}]$

= (22/7)×5.25×6.05

= 99.825

Therefore, the area of the canvas is 99.825 m^2.

EXERCISE 13.8

1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

(Assume π =22/7)

Solution:

(i) Radius of the sphere, r = 7 cm

Using, Volume of the sphere = (4/3) πr^3

= $(4/3)\times(22/7)\times 7^3$

= 4312/3

Hence, volume of the sphere is 4312/3 cm^3

(ii) Radius of the sphere, r = 0.63 m

Using, volume of sphere = (4/3) πr^3

= $(4/3)\times(22/7)\times 0.63^3$

= 1.0478

Hence, volume of the sphere is 1.05 m^3 (approx).

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

(Assume π =22/7)

Solution:

(i) Diameter = 28 cm

Radius, r = 28/2 cm = 14cm

Volume of the solid spherical ball = (4/3) πr^3

Volume of the ball = $(4/3)\times(22/7)\times 14^3$ = 34496/3

Hence, volume of the ball is 34496/3 cm^3

(ii) Diameter = 0.21 m

Radius of the ball =0.21/2 m= 0.105 m

Volume of the ball = (4/3)πr^3

Volume of the ball = (4/3)× (22/7)×0.105^3 m^3

Hence, volume of the ball = 0.004851 m^3

3. The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7)

Solution:

Given,

Diameter of a metallic ball = 4.2 cm

Radius(r) of the metallic ball, r = 4.2/2 cm = 2.1 cm

Volume formula = 4/3 πr^3

Volume of the metallic ball = (4/3)×(22/7)×2.1 cm^3

Volume of the metallic ball = 38.808 cm^3

Now, using the relationship between density, mass and volume,

Density = Mass/Volume

Mass = Density × volume

= (8.9×38.808) g

= 345.3912 g

Mass of the ball is 345.39 g (approx).

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Let the diameter of the earth be "d". Therefore, the radius of the earth will be d/2.

Diameter of the moon will be d/4, and the radius of the moon will be d/8.

Find the volume of the moon.

Volume of the moon = (4/3) πr^3= (4/3) π $(d/8)^3$ = 4/3π(d^3/512)

Find the volume of the earth

Volume of the earth = (4/3) πr^3= (4/3) π $(d/2)^3$ = 4/3π(d^3/8)

Fraction of the volume of the earth is the volume of the moon

Volume of the moon/ volume of the earth $= \dfrac{\frac{4}{3}\pi(\frac{d^3}{512})}{\frac{4}{3}\pi(\frac{d^3}{8})} = 8/512 = 1/64$

Answer: Volume of the moon is of the 1/64 volume of the earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7)

Solution:

Diameter of the hemispherical bowl = 10.5 cm

Radius of the hemispherical bowl, r = 10.5/2 cm = 5.25 cm

Formula for volume of the hemispherical bowl = (2/3) πr^3

Volume of the hemispherical bowl = $(2/3)\times(22/7)\times5.25^3 = 303.1875$

Volume of the hemispherical bowl is 303.1875 cm^3

Capacity of the bowl = (303.1875)/1000 L = 0.303 litres(approx.)

Therefore, the hemispherical bowl can hold 0.303 litres of milk.

6. A hemispherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7)

Solution:

Inner Radius of the tank, (r) = 1m

Outer Radius (R) = 1.01m

Volume of the iron used in the tank = (2/3) $\pi(R^3 - r^3)$

Put values,

Volume of the iron used in the hemispherical tank = $(2/3)\times(22/7)\times(1.01^3 - 1^3) = 0.06348$

So, volume of the iron used in the hemispherical tank is 0.06348 m^3.

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let r be the radius of a sphere.

Surface area of the sphere = $4\pi r^2$

$4\pi r^2 = 154$ cm^2 (given)

$r^2 = (154\times7)/(4\times22)$

$r = 7/2$

The radius is 7/2 cm.

Now,

Volume of the sphere = $(4/3)\pi r^3$

Volume of the sphere = $(4/3)\times(22/7)\times(7/2)^3 = 179\frac{2}{3}$

Volume of the sphere is $179\frac{2}{3}$ cm^3

8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 4989.60. If the cost of white-washing is 20 per square metre, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Solution:

(i) Cost of whitewashing the dome from inside = Rs 4989.60

Cost of whitewashing $1m^2$ area = Rs 20

CSA of the inner side of dome = 498.96/2 m^2 = 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of the inner side of dome = 249.48 m^2 (from (i))

Formula to find CSA of a hemisphere = $2\pi r^2$

$2\pi r^2 = 249.48$

$2\times(22/7)\times r^2 = 249.48$

$r^2 = (249.48\times7)/(2\times22)$

$r^2 = 39.69$

$r = 6.3$

So, the radius is 6.3 m.

Volume of air inside the dome = Volume of hemispherical dome

Using the formula, the volume of the hemisphere = $2/3\ \pi r^3$

$= (2/3)\times(22/7)\times6.3\times6.3\times6.3$

$= 523.908$

$= 523.9$(approx.)

Answer: The volume of air inside the dome is 523.9 m^3.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

Solution:

Volume of the solid sphere = $(4/3)\pi r^3$

Volume of twenty seven solid sphere = $27\times(4/3)\pi r^3 = 36\pi r^3$

(i) New solid iron sphere radius = r'

Volume of this new sphere = $(4/3)\pi(r')^3$

$(4/3)\pi(r')^3 = 36\pi r^3$

$(r')^3 = 27r^3$

$r' = 3r$

Radius of the new sphere will be 3r (thrice the radius of the original sphere)

(ii) Surface area of the iron sphere of radius r, S $=4\pi r^2$

Surface area of the iron sphere of radius $r' = 4\pi (r')^2$

Now

$S/S' = (4\pi r^2)/(4\pi (r')^2)$

$S/S' = r^2/(3r')^2 = 1/9$

The ratio of S and S' is 1: 9.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume $\pi = 22/7$)

Solution:

Diameter of the capsule = 3.5 mm

Radius of the capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm

Volume of the spherical capsule = 4/3 πr^3

Volume of the spherical capsule = $(4/3)\times(22/7)\times(1.75)^3 = 22.458$

Answer: The volume of the spherical capsule is 22.46 mm^3.

EXERCISE 13.9

PAGE NO: 236

1. A wooden bookshelf has external dimensions as follows: Height = 110cm, Depth = 25cm,

Breadth = 85cm (see fig. 13.31). The thickness of the plank is 5cm everywhere. The external faces are to be polished, and the inner faces are to be painted. If the rate of polishing is 20 paise per cm² and the rate of painting is 10 paise per cm², find the total expenses required for polishing and painting the surface of the bookshelf.

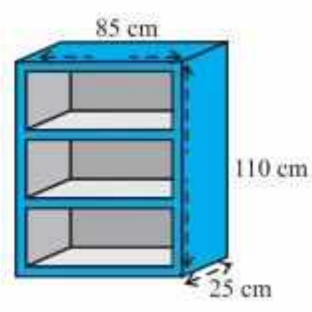


Fig. 13.31

Solution:

External dimensions of book self:

Length, l = 85cm

Breadth, b = 25 cm

Height, h = 110 cm

External surface area of the shelf while leaving out the front face of the shelf.

= lh+2(lb+bh)

= [85×110+2(85×25+25×110)] = (9350+9750) = 19100

External surface area of the shelf is 19100 cm^2

Area of front face = [85×110-75×100+2(75×5)] = 1850+750

So, the area is 2600 cm^2

Area to be polished = (19100+2600) cm^2 = 21700 cm^2.

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs. (21700×0.20) = Rs 4340

Dimensions of the row of the bookshelf

Length(l) = 75 cm

Breadth (b) = 20 cm and

Height(h) = 30 cm

Area to be painted in one row= 2(l+h)b+lh = [2(75+30)× 20+75×30] = (4200+2250) = 6450

So, the area is 6450 cm^2.

Area to be painted in 3 rows = (3×6450)cm^2 = 19350 cm^2.

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs (19350 x 0.1) = Rs 1935

Total expense required for polishing and painting = Rs. (4340+1935) = Rs. 6275

Answer: The cost for polishing and painting the surface of the bookshelf is Rs. 6275.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in fig. 13.32. Eight such spheres are used forth is the purpose and are to be painted silver. Each support is a cylinder of radius 1.5cm and height 7cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per $cm^{2,}$ and black paint costs 5 paise per cm^2.

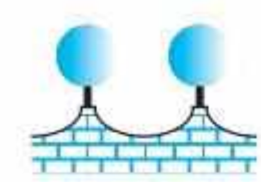


Fig. 13.32

Solution:

Diameter of the wooden sphere = 21 cm

Radius of the wooden sphere, r = diameter/ 2 = (21/2) cm = 10.5 cm

Formula: Surface area of the wooden sphere = $4\pi r^2$

= 4×(22/7)×$(10.5)^2$ = 1386

So, the surface area is 1386 cm^3

Radius of the circular end of cylindrical support = 1.5 cm

Height of the cylindrical support = 7 cm

Curved surface area = $2\pi rh$

= 2×(22/7)×1.5×7 = 66

So, CSA is 66 cm^2

Now,

Area of the circular end of cylindrical support = πr^2

= $(22/7)\times 1.5^2$

= 7.07

Area of the circular end is 7.07 cm^2

Again,

Area to be painted silver = [8 ×(1386-7.07)] = 8×1378.93 = 11031.44

Area to be painted is 11031.44 cm^2

Cost for painting with silver colour = Rs(11031.44×0.25) =Rs 2757.86

Area to be painted black = (8×66) cm^2 = 528 cm^2

Cost for painting with black colour =Rs (528×0.05) = Rs26.40

Therefore, the total painting cost is

= Rs(2757.86 +26.40)

= Rs 2784.26

3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Solution:

Let the diameter of the sphere be "d".

Radius of the sphere, $r_1 = d/2$

New radius of the sphere, say $r_2 = (d/2)\times(1-25/100) = 3d/8$

Curved surface area of the sphere, $(CSA)_1 = 4\pi r_1^2 = 4\pi\times(d/2)^2 = \pi d^2$ …(1)

Curved surface area of the sphere when the radius is decreased $(CSA)_2 = 4\pi r_2^2 = 4\pi\times(3d/8)^2 = (9/16)\pi d^2$ …(2)

From equations (1) and (2), we have

Decrease in surface area of sphere = $(CSA)_1 - (CSA)_2$

= $\pi d^2 - (9/16)\pi d^2$

= $(7/16)\pi d^2$

Percentage decrease in surface area of sphere = $\dfrac{(CSA)_1 - (CSA)_2}{(CSA)_1} \times 100$

$= (7d^2/16d^2)\times 100 = 700/16 = 43.75\%$.

Therefore, the percentage decrease in the surface area of the sphere is 43.75%.